Trishna's

IIT JEE Super Course in Physics

Optics and Modern Physics

ALWAYS LEARNING

PEARSON

Super Course in Physics

OPTICS AND MODERN PHYSICS

for the IIT-JEE

Volume 5

Trishna Knowledge Systems

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Preface

The IIT-JEE, the most challenging amongst national level engineering entrance examinations, remains on the top of the priority list of several lakhs of students every year. The brand value of the IITs attracts more and more students every year, but the challenge posed by the IIT-JEE ensures that only the *best* of the aspirants get into the IITs. Students require thorough understanding of the fundamental concepts, reasoning skills, ability to comprehend the presented situation and exceptional problem-solving skills to come on top in this highly demanding entrance examination.

The pattern of the IIT-JEE has been changing over the years. Hence an aspiring student requires a step-by-step study plan to master the fundamentals and to get adequate practice in the various types of questions that have appeared in the IIT-JEE over the last several years. Irrespective of the branch of engineering study the student chooses later, it is important to have a sound conceptual grounding in Mathematics, Physics and Chemistry. A lack of proper understanding of these subjects limits the capacity of students to solve complex problems thereby lessening his/her chances of making it to the top-notch institutes which provide quality training.

This series of books serves as a source of learning that goes beyond the school curriculum of Class XI and Class XII and is intended to form the backbone of the preparation of an aspiring student. These books have been designed with the objective of guiding an aspirant to his/her goal in a clearly defined step-by-step approach.

- **Master the Concepts and Concept Strands!**
  This series covers all the concepts in the latest IIT-JEE syllabus by segregating them into appropriate units. The theories are explained in detail and are illustrated using solved examples detailing the different applications of the concepts.
- **Let us First Solve the Examples—Concept Connectors!**
  At the end of the theory content in each unit, a good number of "Solved Examples" are provided and they are designed to give the aspirant a comprehensive exposure to the application of the concepts at the problem-solving level.
- **Do Your Exercise—Daily!**
  Over 200 unsolved problems are presented for practice at the end of every chapter. Hints and solutions for the same are also provided. These problems are designed to sharpen the aspirant's problem-solving skills in a step-by-step manner.
- **Remember, Practice Makes You Perfect!**
  We recommend you work out ALL the problems on your own – both solved and unsolved – to enhance the effectiveness of your preparation.

A distinct feature of this series is that unlike most other reference books in the market, this is not authored by an individual. It is put together by a team of highly qualified faculty members that includes IITians, PhDs etc from some of the best institutes in India and abroad. This team of academic experts has vast experience in teaching the fundamentals and their application and in developing high quality study material for IIT-JEE at T.I.M.E. (Triumphant Institute of Management Education Pvt. Ltd), the number 1 coaching institute in India. The essence of the combined knowledge of such an experienced team is what is presented in this self-preparatory series. While the contents of these books have been organized keeping in mind the specific requirements of IIT-JEE, we are sure that you will find these useful in your preparation for various other engineering entrance exams also.

We wish you the very best!

CHAPTER 1

OPTICS

CHAPTER OUTLINE

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INTRODUCTION

Optics is the branch of Physics, which deals with the nature, sources, properties and effects of light. Optics is classified into **Geometrical Optics** (also called Ray optics) and **Physical Optics** (or Wave Optics).

Geometrical optics is the study of simple properties of light and optical instruments assuming rectilinear propagation of light (i.e., light travels in straight lines).

Optical medium is one through which light can pass. It may or may not contain matter.

Transparent medium is one through which most of the incident light passes.

Translucent medium is one through which only a small portion of the incident light passes.

Opaque medium is one through which no light can pass.

A ray or **ray of light** is the straight line, which represents the direction of propagation of light.

A collection of rays of light is called **a beam of light**. A beam of light could be **converging** (meeting at a point), **diverging** (emanating from a point) or **parallel**. See Fig. 1.1.

A medium that has same composition of matter throughout (hence the same density at all points) is called a **homogeneous medium.**

A medium having the same physical properties in all directions is called an **isotropic medium**. Light travels with the same speed in all directions in an isotropic medium. Example, glass.

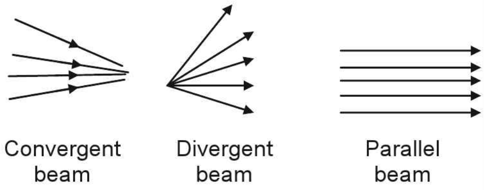


Fig. 1.1

If the physical properties of a medium are different in different directions, it is called an **anisotropic medium**. These are usually crystalline materials. Example, quartz, calcite. In these media, light travels with different speeds in different directions.

The point where the light rays coming from a source meet or appear to diverge from, after undergoing reflection, refraction or both, is called the **image point** of the source.

If the rays actually converge or meet at a point after reflection/refraction, the **image is real** and it can be caught on a screen. **Real images** of real objects **are inverted**.

If the rays appear to diverge from a point, after reflection, refraction, that point is the **virtual image** point of the source. It cannot be caught on a screen. **Virtual images** of real objects **are erect**.

REFLECTION

When light travelling in one medium (called incident medium), reaches the boundary with another medium and the bounding surface throws the incident light partly or wholly back into the incident medium, the phenomenon of reflection is said to have taken place at the boundary separating the two media. Thus reflection is a boundary phenomenon.

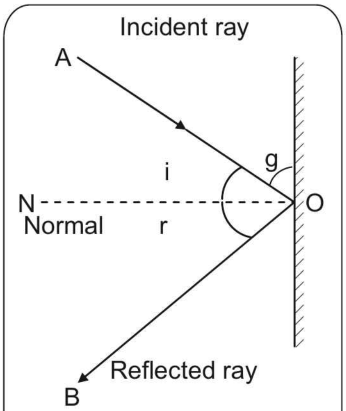


Fig. 1.2

The reflecting surface is called a mirror or reflector, which can be either a plane surface (called plane mirror) or a curved surface (called spherical, ellipsoidal or parabolic mirrors depending upon the shape of the curved surface). Fig. 1.2 illustrates reflection at a plane mirror.

O = Point of incidence

ON = Normal to the point of incidence

AO = Incident ray

OB = Reflected ray

i = Angle of incidence

r = Angle of reflection

g = Glancing angle

$= 90° - i$

The angle which the incident ray makes with the normal at the point of incidence is called angle of incidence (i) The angle which the reflected ray makes with the normal at the point of incidence is called angle of reflection (r). The angle which the incident ray makes with the plane reflecting surface is called glancing angle (g).

$$g = 90° - i$$

Laws of reflection

(i) The incident ray, the reflected ray and the normal at the point of incidence, all lie in the same plane.

(ii) The angle of incidence (i) is equal to the angle of reflection (r) i.e., i = r.

The above laws hold good for all reflecting surfaces, whether plane or curved.

Notes:

(1) If $\angle i = 0$, the ray is incident normally on the reflecting surface. For normal incidence, $\angle r = 0$ as per laws of reflection. Hence for i = 0 (normal incidence), the ray retraces its path after reflection.

(2) Since the incident ray and reflected ray travel through the same medium, there is no change in the frequency, wavelength and speed of the ray after reflection.

(3) The intensity of the reflected ray may be less than the intensity of the incident ray due to the decrease in the amplitude. [$\because$ Intensity μ (amplitude)^2].

(4) If a parallel beam of light is incident on an irregular reflecting surface, the light rays are reflected in random directions but for each ray, laws of reflection can be applied. Such a reflection is called irregular reflection or diffuse reflection.

Real object and virtual object

A point object from which incident rays actually diverge is called a real object (O).

Note:
Real object and incident rays from it will be on the same side of the reflecting (or refracting) surface. See Fig. 1.3.

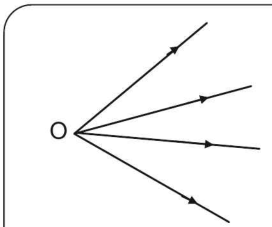


Fig. 1.3

The point to which incident rays appear to converge is called a virtual object (O'). See Fig. 1.4.

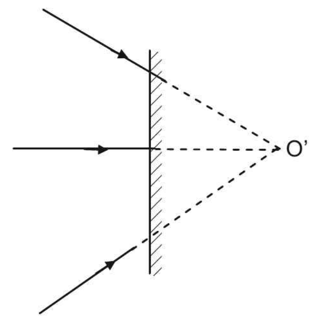


Fig. 1.4

Note:
Virtual object and incident rays will be on the opposite sides of the reflecting (or refracting) surface. See Fig. 1.4.

Real image and virtual image

The point to which reflected or refracted rays actually converge is called a real image (I) point. See Fig. 1.5.

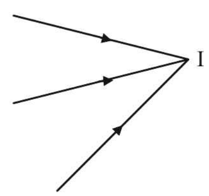


Fig. 1.5

Note:
Real image and the reflected (or refracted) rays towards it will be on the same side of reflecting (or refracting) surface.

The point from which reflected rays (or refracted rays) appear to diverge is called a virtual image (I') point. See Fig. 1.6.

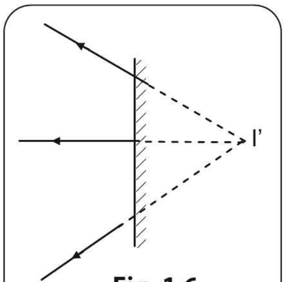


Fig. 1.6

Note:
Virtual image and the reflected (or refracted) rays will be on the opposite sides of the reflecting (or refracting) surface.

Reflection from plane mirror (Image formation)

Two rays are sufficient to locate the image of a point object by reflection. As shown in Fig. 1.7, we take one ray at normal incidence, i.e., i = 0, and another at any arbitrary i emanating from the point object O as shown. The one which is at normal incidence is reflected back along the line of incidence, but the ray which is incident at angle i is reflected at an angle i. They do not meet in front of the mirror. But the ray at angle i, when extended behind the mirror, meets the extension of the normal ray at a point I, which is the virtual image of O.

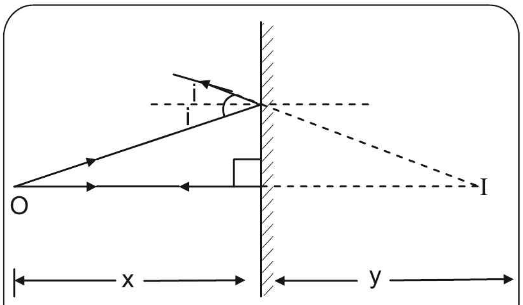


Fig. 1.7

Consider reflection at a plane mirror as shown in Fig. 1.8. The object is at a distance x and image is at a distance y. By the properties of congruent triangles, we can prove x = y. Now consider an extended object OO' such that it is parallel to the surface of the mirror. The point O' is at the same object distance x. The image of O' is I' at same y. Image of the object OO' is therefore II', as shown in Fig. 1.8. (An infinite number of point objects between O and O' will have their corresponding point images between I and I')

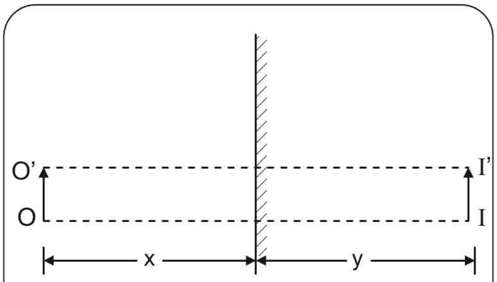


Fig. 1.8

By elementary geometry, it can be proved that OO' = II'

$$\textit{Lateral magnification}\ \mathrm{m} = \frac{\text{image dimension}}{\text{object dimension}} = 1$$

(In this case dimension is length or height).

By elementary geometry, we can show that

$$\mathrm{m} = 1.$$

If the object OO' is perpendicular to the surface of the mirror as shown in Fig. 1.9, we can show easily that the image II' is also perpendicular to the surface of the mirror and the *longitudinal magnification*,

$$\mathrm{m}' = -1.$$

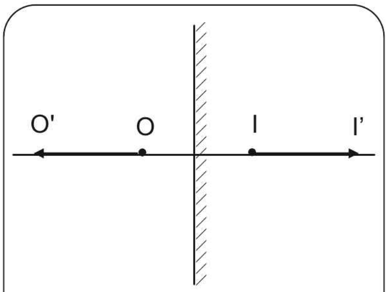


Fig. 1.9

If the object is a plane area as shown in Fig 1.10, the image formed behind the mirror can be seen to be laterally inverted. Image of the left hand looks like the right hand.

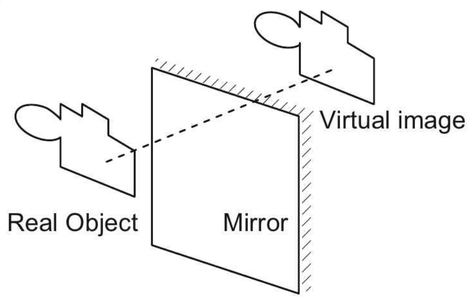


Fig. 1.10

Image formed by plane mirrors kept inclined at an angle (θ)

Consider two plane mirrors kept inclined to each other at an angle θ, with their reflecting surfaces facing each other.

Then, due to multiple reflections that take place, more than one image are formed.

The number of images formed (n) is given by

(i) $n = \dfrac{360°}{\theta} - 1$, if $\dfrac{360°}{\theta}$ is an even integer.

(ii) When $\dfrac{360°}{\theta}$ is an odd integer, the number of images formed (n) is given by

(a) $n = \dfrac{360°}{\theta}$, when the object is placed unsymmetrical to the mirrors (i.e., the object is not placed along the bisector of θ).

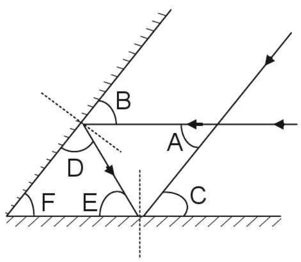
(b) $n = \dfrac{360°}{\theta} - 1$, when the object is placed symmetrical to the mirrors (i.e., the object is placed along the bisector of θ).

(iii) If the mirrors are parallel to each other, number of images formed is infinite (∞).

CONCEPT STRANDS

Concept Strand 1

Two plane mirrors are placed at an angle θ between them. A beam of light parallel to one of the mirrors falls on the other mirror and after two successive reflections emerges parallel to the latter. Find the angle between the mirrors.



Solution

By geometry

$$\angle A = \angle B = \angle C$$

But $\quad \angle B = \angle D = \angle F$

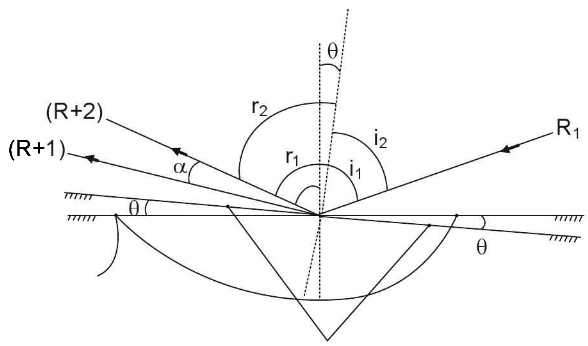
$$\angle C = \angle E = \angle F$$

$\therefore \quad \angle D = \angle E = \angle F$

$\therefore \quad \angle F = 60°$

Concept Strand 2

A ray is reflected from a plane mirror. Without changing the direction of the incident ray, the mirror is rotated through an angle θ. Through what angle does the reflected ray rotate?



Solution

From the figure

$$i_1 - i_2 = \theta \qquad \text{— (1)}$$

$$\theta + r_1 - \alpha = r_2 \Rightarrow r_1 - r_2 = \alpha - \theta$$

$$i_1 - i_2 = \alpha - \theta \qquad \text{— (2)}$$

$$(1) \equiv (2)$$

$\therefore \quad \alpha - \theta = \theta \Rightarrow \alpha = 2\theta$

Deviation (δ) produced by a plane mirror

The angular separation between the direction of the incident ray and the direction of the reflected ray is called the deviation produced by the mirror. See Fig. 1.11.

Here, $\delta = \angle A'OB$

$= \angle A'OA - \angle AOB$

$= 180° - 2i \qquad \text{— (1)}$

But $g = 90° - i$ (g = glancing angle)

$\Rightarrow \quad i = 90° - g$. Substituting in (1)

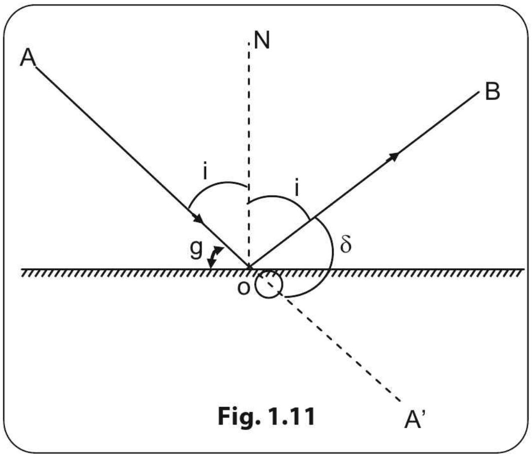


Fig. 1.11

$$\therefore \quad \delta = 180° - 2\,(90° - g)$$
$$= 2g$$
$$\delta = 2\,g$$

where, g = glancing angle

Characteristics of the image formed by a plane mirror

(i) The size of the image is same as that of the object. i.e., |magnification| = 1 for plane mirrors.
(ii) The image is as far behind the mirror as the object is in front of it.
(iii) The image of a real object is virtual, erect and laterally inverted.
(iv) If an object moves with a speed u towards a fixed mirror, the image also moves towards the mirror with speed u. The speed of the image relative to the object is 2u.
(v) If a plane mirror moves with a speed u towards or away from a fixed object, the image appears to move towards or away from the object with a speed 2u.
(vi) If the mirror is moved towards or away from the object by a distance d, the image moves towards or away from the object by 2d.
(vii) When a plane mirror is rotated through an angle θ, keeping the incident ray fixed, the reflected ray turns through an angle 2θ.
(viii) If g is the glancing angle (angle between incident ray and the plane mirror), then the deviation produced by plane mirror (d) is given by d = 2 g.

Reflection from spherical mirror

A spherical mirror is a polished curved surface that forms part of a sphere. Two types of spherical mirrors in common use are (i) concave mirror and (ii) convex mirror. The curvature of these mirrors with respect to the incident ray is as shown in Fig. 1.12.

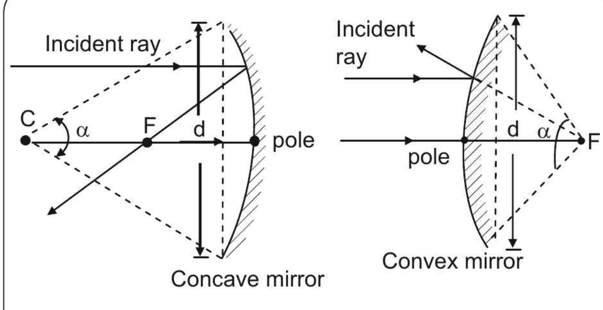


Fig. 1.12

Pole (P) of a spherical mirror is the geometrical centre of the spherical mirror.

Centre of curvature (C) of a spherical mirror is the centre of the sphere of which the mirror forms a part. The distance of the centre of curvature from the pole is called the *radius of curvature* (R). *Principal* ***a***xis of the mirror is the line passing through the pole and the centre of curvature. A ray of light parallel to the principal axis and incident close to the pole is called a *paraxial ray*. *Focus* (F) is the point on the principal axis where a paraxial ray, after reflection, would pass through or appear to pass through (Fig. 1.12). The distance of the focus from the pole is known as the *focal length* (f).

The chord-length of the circular boundary of the reflecting surface is called aperture (d) and the angle subtended by the reflecting surface at the centre of sphere is called angular aperture (α).

New cartesian sign convention

(i) All distances are measured along the principal axis of the mirror, from the pole (P) of the mirror.
(ii) The distance to the object from the pole, along the principal axis is denoted by u, the distance to the image from the pole is denoted by v and the focal length is denoted by f.
(iii) All distances measured in the direction of the incident ray are treated as positive and all distances measured opposite to the direction of the incident ray are treated as negative.
(iv) All heights are measured from the principal axis. Heights above the principal axis are treated as positive

and heights below the principal axis are treated as negative.

(v) The focal length of a concave mirror is always treated as negative while the focal length of a convex mirror is always treated as positive.

Note:
If an object is placed in front of a spherical mirror, u will be negative for real objects and u will be positive for virtual objects.

Rules for image formation

(i) A ray passing parallel to the principal axis, after reflection from the mirror, passes or appears to pass through its focus.

(ii) A ray passing through or directed towards the focus, after reflection from the spherical mirror becomes parallel to the principal axis (by the principle of reversibility of light rays).

(iii) A ray passing through or directed towards the centre of curvature, after reflection from the spherical mirror, retraces its path as it is incident normally on the spherical mirror ($i = 0$).

Image formation by spherical mirrors

Relation between focal length and radius of curvature is shown in Fig. 1.13 and Fig. 1.14.

Concave mirror

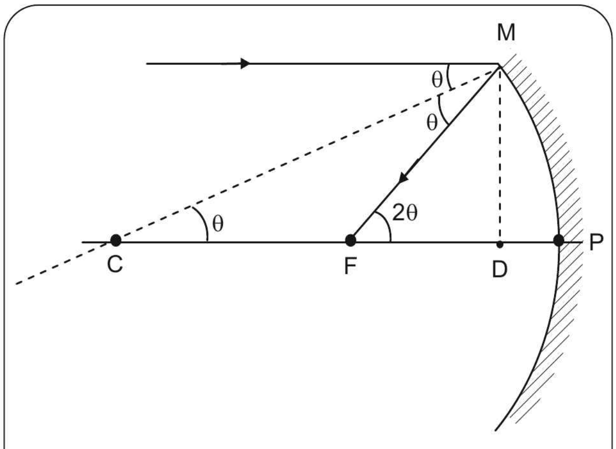


Fig. 1.13

Convex mirror

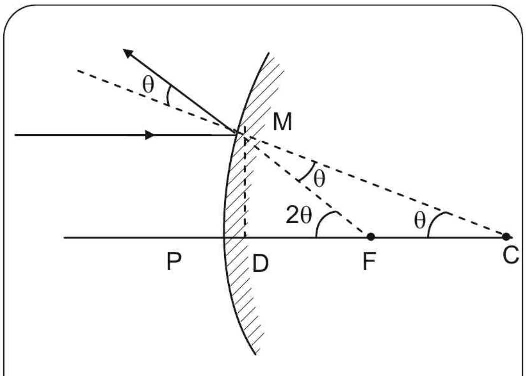


Fig. 1.14

Figures 1.13 and 1.14 show the relationship between the radius of curvature and the focal length in a concave mirror and a convex mirror, respectively. From the figures,

$$\tan\theta = \frac{MD}{CD}, \tan 2\theta = \frac{MD}{FD}.$$

For paraxial ray θ is small (for small apertures)

$\therefore \quad \tan\theta = \theta, \tan 2\theta = 2\theta$

$\Rightarrow \quad CD = 2FD$ and $CD = CP = R$

$FD = FP = f$

$$R = 2f$$

Mirror formula

Concave mirror

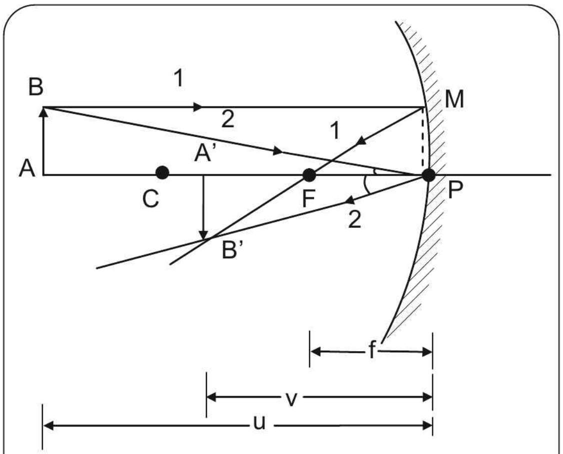


Fig. 1.15

An object AB of height h is placed at a distance u from the pole P in front of a concave mirror of focal length f as shown in Fig. 1.15. Consider two rays emanating from B. One of them travels parallel and close to the axis (paraxial) and falls at point M and the other falls at the point P. After reflection, they meet at B' at a distance v from the pole. B' is the image of point B of the object. By similar construction, images of the other points of the object can be obtained. In order to find the position and size of the image from the pole, consider triangles FA'B' and FPM.

Since the ray BM is paraxial,

$\angle MPF \approx 90°$ and $PM \approx AB$.

$$\therefore \quad \frac{FA'}{A'B'} = \frac{PF}{PM} \Rightarrow \frac{PA' - PF}{A'B'} = \frac{PF}{PM}$$

denoting $A'B'$ by $-h'$, and AB by $+h$

$PA' = -v$

$PF = -f$

$PM \approx AB = +h.$

$$\therefore \quad -\frac{-v+f}{-h'} = \frac{-f}{+h} \qquad \text{— (1)}$$

consider triangles PA'B' and PAB,

$$\frac{PA'}{A'B'} = \frac{PA}{AB} \Rightarrow \frac{-v}{-h'} = \frac{-u}{+h} \sim \qquad \text{— (2)}$$

$$(1) \Rightarrow \frac{-v+f}{-f} = \frac{-h'}{+h}$$

$$(2) \Rightarrow \frac{-v}{-u} = \frac{-h'}{+h}$$

$$\therefore \quad \frac{-v+f}{-f} = \frac{-v}{-u} \Rightarrow \frac{v-f}{f} = \frac{v}{u} \Rightarrow$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

This mirror formula relates the distances of the object and the image to the focal length.

From (3) $\frac{h'}{h} = -\frac{v}{u}$

$\therefore$ Lateral magnification, $m = \frac{h'}{h} = \frac{-v}{u}$

This is the lateral magnification formula for a curved mirror.

For plane mirror, $R = 2f = \infty$

$$\Rightarrow \frac{1}{v} + \frac{1}{u} = 0$$

$$\Rightarrow \quad \frac{1}{v} = -\frac{1}{u} \text{ or}$$

$$v = -u$$

$$m = -\frac{v}{u} = 1$$

Convex mirror

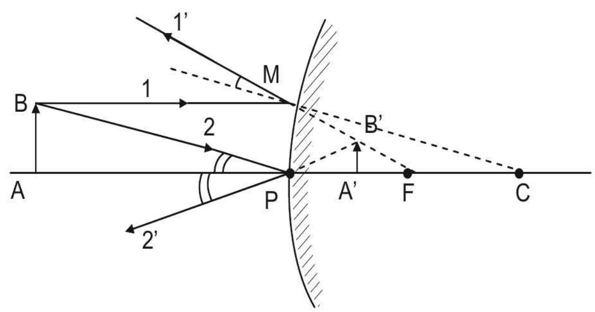


Fig. 1.16

Consider a paraxial ray shown as 1 in Fig. 1.16, emanating from the point B of the object falling at the point M on the convex mirror. It gets reflected along 1' and appears to be reflected from the focus F. Ray labelled 2 in the figure falls at the pole and gets reflected along 2'. The two reflected rays do not meet in front of the mirror but appear to meet at B'. Likewise A' is the image of A. Therefore A'B' is the virtual, erect, diminished image of AB. Consider v, u, f, h' same as in previous case, with proper sign.

$\angle MPF \approx 90°$ and $MP \approx AB$

Triangles A'FB' and PFM are similar.

$$\frac{A'F}{A'B'} = \frac{PF}{PM} \Rightarrow \frac{PF - PA'}{A'B'} = \frac{PF}{AB}$$

$$\Rightarrow \quad \frac{+f-(+v)}{+h'} = \frac{+f}{+h} \qquad \text{—(1)}$$

Triangles PA'B' and PAB are similar.

$$\frac{PA'}{A'B'} = \frac{PA}{AB} \Rightarrow \frac{+v}{+h'} = \frac{-u}{+h}$$

$$\Rightarrow \quad \frac{f-v}{f} = \frac{h'}{h} \qquad \text{— (2)}$$

$$(1) \Rightarrow \frac{1}{v'} - \frac{1}{-60} = \frac{1}{+40} \Rightarrow v = +120$$

$$(2) \Rightarrow \frac{-v}{u} = \frac{h'}{h} \because \frac{f-v}{f} = \frac{-v}{u} \Rightarrow$$

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

and the lateral magnification is, from (3),

$$m = \frac{h'}{h} = -\frac{v}{u}$$

Same formulae hold good for all mirrors. m is lateral magnification. The longitudinal magnification m', can be obtained as follows:

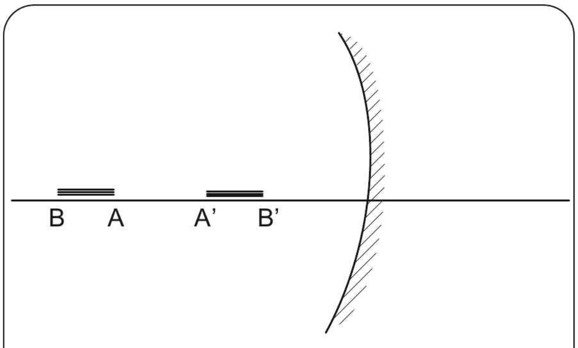


Fig. 1.17

The mirror formula, $\frac{1}{v} + \frac{1}{u}$ = constant, implies that when object distance is more, image distance will be less. A'B' is the image of AB as shown in Fig. 1.17.

But what is $\frac{A'B'}{AB}$, i.e., longitudinal magnification m'?

If u, v refer to A, A' and u', v' refer to B, B', respectively then

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{1}{v'} + \frac{1}{u'}$$

Rearranging, $$\frac{1}{v} - \frac{1}{v'} = \frac{1}{u'} - \frac{1}{u}$$

$$\Rightarrow \frac{v' - v}{vv'} = -\frac{(u' - u)}{uu'}$$

$$\Rightarrow \frac{v' - v}{u' - u} = m' = -\frac{vv'}{uu'} \approx -\frac{v^2}{u^2}$$

(for small object length) $\Rightarrow$

$$m' = -\frac{v^2}{u^2} = -m^2$$

This can also be derived by calculus method:

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Differentiating,

$$-\frac{dv}{v^2} - \frac{du}{u^2} = 0 \Rightarrow \frac{dv}{du} = -\frac{v^2}{u^2}$$

$$\Rightarrow \qquad m' = -\frac{v^2}{u^2} = -m^2.$$

Note:

(a) Suppose the length is not small (as compared to object distance), then we should not use the above m' formula, but independently calculate v and v' and take

$$m' = \frac{v' - v}{u' - u}.$$

(b) If the object is inclined to the axis as shown by the arrow ℓ, in Fig. 1.18, magnification is obtained as follows:

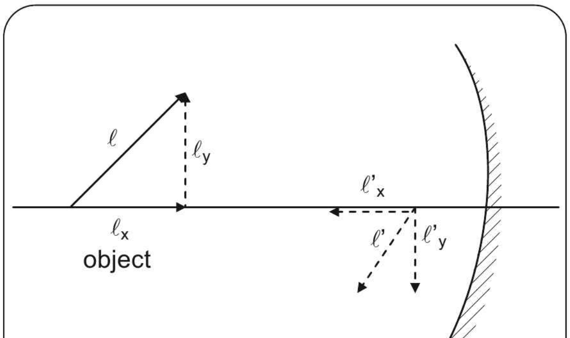


Fig. 1.18

The arrow ℓ' is the image of ℓ

$$\ell'_x = \ell_x . m'$$

$$\ell'_y = \ell_y . m.$$

$$\ell' = \sqrt{\ell'^2_x + \ell'^2_y}$$

$$= \sqrt{(\ell'_x)^2 \times (m')^2 + \ell y^2 m^2}$$

$$= m\sqrt{\ell_x^2 m^2 + \ell_y^2} \qquad (\because (m')^2 = m^4)$$

$$\therefore \quad \text{magnification } \frac{\ell'}{\ell} = \frac{m\sqrt{\ell_x^2 m^2 + \ell_y^2}}{\sqrt{\ell_x^2 + \ell_y^2}}$$

The above result can be used only if $\ell_x << u$.

CONCEPT STRAND

Concept Strand 3

Fill in the following table, each column of which refers to a spherical mirror. Check your results by ray diagrams. Distances are in centimeter. If a number has no plus or minus sign, it may have either sign. All objects are real.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
Type	**Concave**						**convex**	
f	−20		−20			20		
R					+40		40	
v					+10		4	
u	−10	−10	−30	−60				−24
m		+1		−0.5		+0.1		0.5
Real image		No						
Erect image								No
Enlarged								

Solution

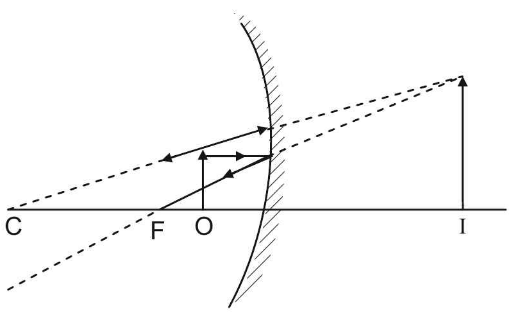
(i) Concave mirror

$\Rightarrow$ f = −20 $\Rightarrow$ R = −40

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}+\frac{1}{-10}=\frac{1}{-20}$$

$\Rightarrow$ v = +20 $\Rightarrow$ virtual image

$$m=-\frac{v}{u}=\frac{-20}{-10}=+2$$



$\Rightarrow$ erect image, enlarged

Ray diagram check:

(ii) Given u = −10, m = +1

$\Rightarrow$ $m=-\frac{v}{u} \Rightarrow +1=-\frac{v}{-10}$

$\Rightarrow$ v = +10 . (confirms virtual image)

$$\therefore \frac{1}{f}=\frac{1}{v}+\frac{1}{u}=\frac{1}{+10}+\frac{1}{-10}=0$$

$\Rightarrow$ f = infinity

$\Rightarrow$ plane mirror $\Rightarrow$ R = ∞

m = +1 $\Rightarrow$ Erect

equal in height to object.

(iii) f = −20 $\Rightarrow$ concave

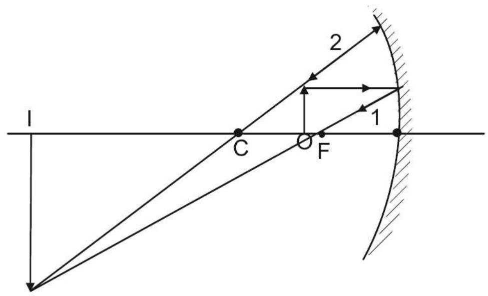
$\Rightarrow$ R = −40

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{-20}-\frac{1}{-30} \Rightarrow v=-60$$

$$\Rightarrow \text{Real, } m=-\frac{v}{u}=\frac{-(-60)}{(-30)}=-2$$

$\Rightarrow$ Inverted, enlarged.

Ray diagram:



(iv) Given u = −60, m = −0.5

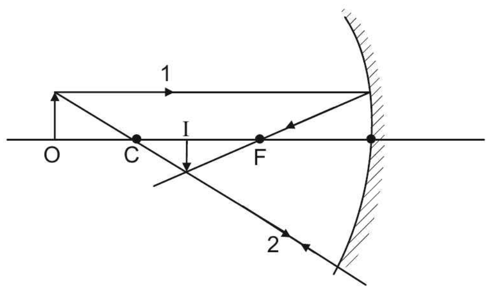
$\Rightarrow$ inverted, diminished

$$-0.5=-\frac{v}{-60} \Rightarrow v=-30 \Rightarrow \text{Real}$$

$$\frac{1}{f}=\frac{1}{-30}+\frac{1}{-60} \Rightarrow f=-20$$

$\Rightarrow$ concave and R = − 40

Ray diagram:



(v) Given v = +10, R = +40

$\Rightarrow$ f = +20

$\Rightarrow$ Convex mirror

and v = +10

$\Rightarrow$ Virtual image

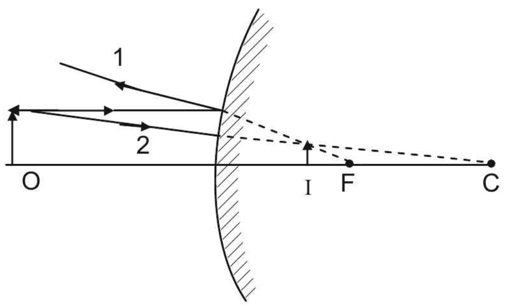
$$\frac{1}{+20} = \frac{1}{+10} + \frac{1}{u}$$

$\Rightarrow$ u = −20

$$\Rightarrow m = -\frac{10}{-20} = +0.5$$

$\Rightarrow$ Erect, diminished, virtual image

Ray diagram:



(vi) Given f = 20 (± sign not given) and m = +0.1

Image is erect and diminished.

Therefore, it cannot be a concave mirror, because an erect image given by a concave mirror will always be enlarged (i.e., |m| >1) and of course it will be virtual.

(Even if you do not know this you can arrive at it by the mirror formula.

Let us assume concave mirror, f = −20.

$$\text{Since } m = +0.1 = -\frac{v}{u}$$

$\Rightarrow$ u = −10 v

$$\Rightarrow \frac{1}{-20} = \frac{1}{v} + \frac{1}{-10v}$$

$\Rightarrow$ v = 18

$\Rightarrow$ u = −180

$\Rightarrow$ virtual object ; this cannot be because all objects are considered as real in problem

$\therefore$ f = +20

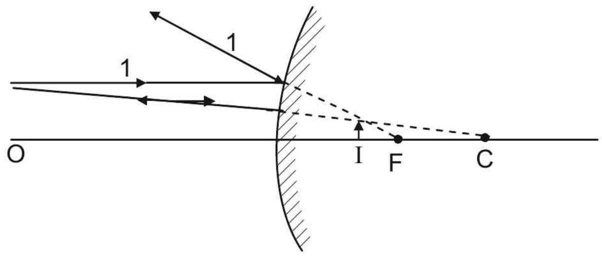
$\Rightarrow$ R = +40, and

$\therefore$ v = +18

$\Rightarrow$ u= −180

$\Rightarrow$ virtual image, erect, diminished.

Ray diagram:

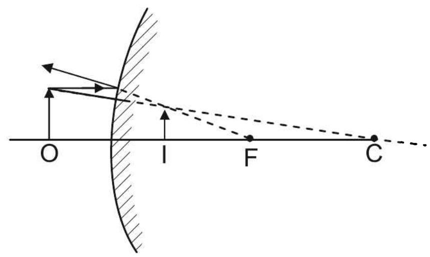


(vii) Given that the mirror is convex. $\therefore$ R = +40, f = +20, v has to be +4, virtual, erect.

$$\frac{1}{+20} = \frac{1}{+4} + \frac{1}{u} \Rightarrow u = -5$$

$$\Rightarrow m = -\frac{4}{-5} = +0.8\text{, diminished}$$

Ray diagram:



(viii) Given that the image is inverted

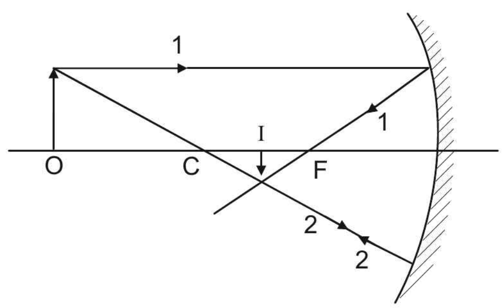
$\Rightarrow$ concave mirror and m = −0.5

$$\Rightarrow -0.5 = -\frac{v}{-24} \Rightarrow v = -12$$

$$\text{and } \frac{1}{f} = \frac{1}{-12} + \frac{1}{-24}$$

$\Rightarrow$ f = −8 $\Rightarrow$ R = −16

Ray diagram:



So the solution table is

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)
Type	Concave	Plane	Concave	Concave	Convex	Convex	Convex	Concave
f	–20	∞	–20	–20	+20	+20	+20	–8
R	–40	∞	–40	–40	+40	+40	+40	–16
v	+20	+10	–60	–30	+10	+18	+4	–12
u	–10	–10	–30	–60	–20	–180	–5	–24
m	+2	+1	–2	–0.5	+0.5	+0.1	+0.8	–0.5
Real image	No	No	Yes	Yes	No	No	No	Yes
Erect image	Yes	Yes	No	No	Yes	Yes	Yes	No
Enlarged	Yes	Size unchanged	Yes	No	No	No	No	No

Combination of mirrors

If an object is placed between two mirrors, multiple images are formed as in the case of plane mirrors. Image due to one mirror serves as object for the other mirror and so on. Note that coordinate axes will change from one mirror to the other, as illustrated in concept strand 1.4.

CONCEPT STRAND

Concept Strand 4

An object of height 1.5 mm is kept between two coaxially mounted spherical mirrors one concave and the other convex, both of focal length 15 cm and separated by 40 cm. The object is at 20 cm from the pole of the concave mirror. Considering the first reflection at the concave mirror and a second reflection at the convex mirror, find the position, nature and size of the final image formed.

Solution

For the first reflection

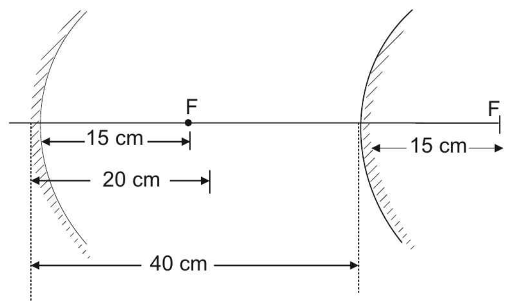
$u = -20$ cm, $f = -15$ cm

$$v = \frac{uf}{u-f} = -60 \text{ cm} \qquad m_1 = -\frac{v}{u} = -\frac{60}{20} = -3$$

(image is real inverted and of size 4.5 mm)

For the second reflection

$u = (60 - 40) = 20$ cm ($\because$ the image due to the first mirror is formed at a distance of 20 cm from the pole of the second mirror in the direction of the incident ray)



$f = 15$ cm ($\because$ convex mirror)

$$v = \frac{uf}{u-f} = 60 \text{ cm}$$

$m_2 = -\frac{60}{20} = -3$; total magnification is $m = m_1m_2 = 9$; image is virtual, erect and of size 13.5 mm)

Power of a mirror

The power of a mirror, is defined as

$$P = \frac{1}{-f}\left(f \text{ in metre}\right)$$

the unit being the diopter (D). For a concave mirror P is positive and for a convex mirror P is negative. For plane mirror P is zero.

$$P = -\frac{1}{f} = -\frac{1}{2R}$$

where R = radius of curvature of mirror. The power of a mirror is the same in all media.

Summary of results

A. Concave Mirror

Sl.No.	Position of object	Position of image	Size of image	Nature of image
1	At infinity	At focus F on same side	Point size	Real and inverted
2	Beyond 2F	Between F and 2F on the same side	Smaller than object size	- do -
3	At 2F	At 2F on the same side	Same size as the object	- do -
4	Between F and 2F	Beyond 2F on the same side	Larger than object size	- do -
5	At F	At infinity	Highly enlarged	- do -
6	Between F and pole (P)	Behind the mirror (opposite side of object)	Enlarged	Virtual and erect

B. Convex Mirror

Sl.No.	Position of object	Position of image	Size of image	Nature of image
1	Anywhere (for real objects)	Behind the mirror (on the opposite side of object)	Smaller than the object	Virtual and erect
2	At infinity (∞)	At F (on the opposite side of object)	Highly diminished	- do -

1. For a concave mirror of focal length 'f', if the distance of real object O from the principal focus F is 'a' and the distance of real image I from the principal focus F is 'b', by substituting $|v| = b + f$ and $|u| = a + f$ in the mirror formula, the magnitude of magnification m is given by

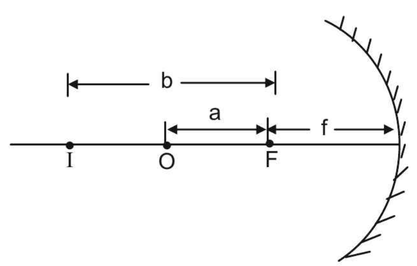


Fig. 1.19

$$m = \frac{f}{a} = \frac{b}{f}$$

and

$$f = \sqrt{ab} \text{ (Newton's formula)}$$

2. The lateral magnification m of a mirror is given by

$$m = -\frac{v}{u} = \frac{f}{(f-u)} = \frac{f-v}{f}$$

where u, v and f are used with signs as per New Cartesian sign convention.

If m is positive, the image is erect. If m is negative, the image is inverted.

This gives

$$u = \frac{f(m-1)}{m},$$

$v = f(1 - m)$ and

$$f = \frac{v}{1-m} = \frac{mu}{m-1}$$

where f, m, u and v are used with signs as per Cartesian sign conventions.

3. If only the magnitudes of m, v, u and f are considered (i.e., all values of m, u, v and f are treated as positive values only). Then

$$m = \frac{v}{u} = \frac{f}{|u-f|} = \frac{|v-f|}{f}$$

$$u = \frac{f(m+1)}{m}$$

$$v = f(m+1)$$

and

$$f = \frac{mu}{(m+1)} = \frac{v}{(m+1)}$$

Interpretation of the mirror formulae

As per the sign convention we follow, for a real object (in front of the mirror), we have the following cases as shown in tabular form.

For real objects

u	v	Image position	Lateral magnification $m = -\frac{v}{u}$	Nature of image
		Concave mirror		
–ve	–ve	In front	–ve	Real, laterally inverted
–ve	+ve	Behind	+ve	Virtual erect
		Convex mirror		
–ve	+ve	Behind	+ve	Virtual erect

Important notes for image formed by mirrors

(i) If $|m| < 1$, image size is diminished, if $|m| > 1$, image size is enlarged.

(ii) $m' = -m^2$ is always negative, i.e., image is longitudinally inverted.

(iii) If the object itself is inverted, image will be erect.

(iv) For real objects, the real images are formed infront of mirror and virtual images are formed behind the mirror.

(v) Real images of real objects are inverted while virtual images of real objects are erect.

REFRACTION

The phenomenon of change in the direction of propagation of light (or bending of light) at the boundary separating two optically different media, is called refraction of light. Hence refraction of light is a boundary phenomenon. The medium in which the light ray was travelling before reaching the boundary is called the incident medium. The medium in which the light ray travels, after crossing the boundary, is called the refracting medium. The boundary, separating the two media, is called the refracting surface. A ray of light travelling in the incident medium is called the incident ray. The ray of light travelling in the refracting medium is called refracted ray and the point on the boundary (refracting surface) where the incident ray falls is called point of incidence. It is also the point of refraction.

Refraction of light from optically denser to rarer medium

This type of refraction is shown in Fig. 1.20 below. The refracted ray deviates away from the normal at the point of refraction.

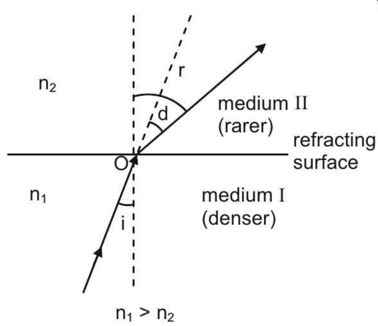


Fig. 1.20

Refraction of light from optically rarer to denser medium

This type of refraction is shown in Fig. 1.21 below. The refracted ray deviates towards the normal at the point of refraction.

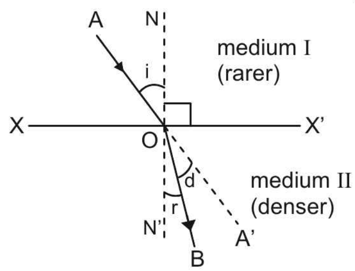


Fig. 1.21

AO = Incident ray

OB = Refracted ray

O = Point of incidence = point of refraction.

XX' = Refracting surface

NON' = Normal to the refracting surface at the point of incidence

AOA' = direction of incident light ray

$\angle$AON = i = angle of incidence

$\angle$BON' = r = angle of refraction.

$\angle$A'OB = d = angular deviation due to refraction = (i – r)

The plane angle between by the incident ray and the normal to the refracting surface at the point of incidence, measured inside the incident medium, is called the angle of incidence. The plane angle between by the refracted ray and the normal to the refracting surface at the point of refraction, measured inside the refracting medium, is called the angle of refraction.

Note:

(1) The point of incidence and point of refraction (O) are same for a given ray, for a given pair of media.

(2) Angular deviation produced by the refracting surface is given by d = (i – r).

(3) If the incident ray falls normally on the refracting surface (i.e., i = 0), then it goes without any deviation into the refracting medium. i.e., d = 0 and r = 0. Hence no bending occurs for such rays. No bending occurs for normal incidence.

Reflection and refraction are natural phenomena which are attributed to waves. In a homogeneous medium (medium having same composition) which is isotropic (having same physical properties in all direction), a wave travels with constant speed. In different media, the same wave travels with different speeds. The speed of a wave in a medium (v) is the product of its wavelength (λ) in the medium and its frequency (f).

$$v = \lambda f$$

Since the frequency and phase of the wave does not change from medium to medium (the frequency of the wave depends upon the frequency of oscillation of the source of disturbance which produced the wave), the speed of the wave in a medium will change only if the wavelength (λ) also changes. The wavelengths of a given wave in different media are different; hence the speed of the wave in different media are different. This causes the wave to bend when it travels from one medium to another medium, with the bending occurring at the boundary of the two media. This phenomenon is called refraction. Since light is an electromagnetic wave, refraction phenomenon occurs for light also.

Consider a wave travelling with a speed 'v_1' in medium 1. If the same wave travels with a faster speed 'v_2' in another medium 2. (i.e., $v_2 > v_1$), then medium 2 is considered as a rarer medium with respect to medium 1. If the same wave travels with a slower speed 'v_3' in another medium 3 (i.e., $v_3 < v_1$), then medium 3 is considered as a denser medium with respect to medium 1.

The speed of light is fastest in free space (or vacuum) and is the same for all frequencies and all wavelengths. The maximum speed of light is approximately equal to $c = 3.0 \times 10^8$ m s^{-1} in vacuum.

Different colours of light are due to different wavelengths. Light always travels slower in all media other than vacuum (free space).

Absolute refractive index

If 'v' is the speed of light in a medium, the ratio

$$\frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$$

is called the **absolute refractive index (n)** of the medium.

$$n = \frac{c}{v}$$

Note:

The absolute refractive index of vacuum is $\frac{c}{c} = 1.0$. It is the minimum value of absolute refractive index.

Since the speed of light in all media is less than the speed of light in free space, all media are denser for light compared to vacuum. Hence the absolute refractive index for every medium, other than free space, will be greater than 1. For a given pair of media, the medium in which light travels faster will be the rarer medium and the medium in which light travels slower will be the denser medium.

Relative refractive index

If 'v_1' and 'v_2' are the speeds of light in medium 1 and medium 2 respectively, the ratio $\frac{v_1}{v_2}$ is called the relative refractive index (${}_1n_2$) of medium 2 with respect to medium 1.

$$\text{i.e., } {}_1n_2 = \frac{v_1}{v_2}$$

If ${}_1n_2$ is greater than 1 (${}_1n_2 > 1$), then $v_1 > v_2$ and medium 1 is rarer and medium 2 is denser. Similarly $\frac{v_2}{v_1}$ is called the relative refractive index (${}_2n_1$) of medium 1 with respect to medium 2.

$${}_2n_1 = \frac{v_2}{v_1}$$

But
$${}_2n_1 = \frac{v_2}{v_1} = \frac{1}{(v_1/v_2)} = \frac{1}{{}_1n_2}$$

$$\therefore \quad {}_2n_1 = \frac{1}{{}_1n_2}$$

Also
$${}_1n_2 = \frac{v_1}{v_2} = \frac{\lambda_1 f}{\lambda_2 f}$$

$${}_1n_2 = \frac{\lambda_1}{\lambda_2}$$

This means that if the wavelength of light in medium 1 (λ_1) is longer than the wavelength of light in medium 2 (λ_2), then medium 1 is a rarer medium and medium 2 is a denser medium.

If 'n' is the absolute refractive index of a medium and a light of wavelength 'λ' in free space enters this medium, the wavelength of light in the medium (λ_m) will be obtained from

$$n = \frac{\lambda}{\lambda_m}$$

$$\lambda_m = \frac{\lambda}{n}$$

If 'n_1' is the absolute refractive index of medium 1 ($n_1 = \frac{c}{v_1}$) and 'n_2' is the absolute refractive index of medium 2 ($n_2 = \frac{c}{v_2}$), then medium 1 will be a rarer medium with respect to medium 2 if $n_2 > n_1$. Relative refractive index of medium 2 with respect to medium 1 is given by

$${}_1n_2 = \frac{v_1}{v_2} = \frac{cv_1}{cv_2} = \frac{(c/v_2)}{(c/v_1)} = \frac{n_2}{n_1}\ (>1, \text{ if } n_2 > n_1)$$

We know that if ${}_1n_2 > 1$, medium 2 is denser than medium 1.

Similarly, ${}_2n_1 = \frac{n_1}{n_2}$ (<1, if $n_1 < n_2$).

Also ${}_1n_2 \times {}_2n_1 = \frac{n_2}{n_1} \times \frac{n_1}{n_2} = 1$ ($\because$ $n_2 \neq 0$ and $n_1 \neq 0$)

$\therefore {}_2n_1 = \frac{1}{{}_1n_2}$; Also ${}_2n_3$ can be calculated as follows:

$${}_1n_2 \times {}_2n_3 \times {}_3n_1 = \frac{v_1}{v_2} \times \frac{v_2}{v_3} \times \frac{v_3}{v_1} = 1$$

$$\therefore {}_2n_3 = \frac{1}{{}_1n_2 \times {}_3n_1} = \frac{{}_1n_3}{{}_1n_2} \qquad \left(\because \frac{1}{{}_3n_1} = {}_1n_3\right)$$

The following points are to be noted:

1. The term 'refractive index of a medium, means the same as absolute refractive index of that medium. It is the refractive index of the medium with respect to vacuum or free space.
2. The absolute refractive index of vacuum is 1 and it is the lowest value of absolute refractive index.
3. The relative refractive index of vacuum with respect to a medium of absolute refractive index 'n' is given by $\frac{1}{n}$. This value will be always positive but less than 1.
4. The relative refractive index of one medium with respect to another will be always positive and be less than 1 or greater than 1.
5. Refractive index is a ratio of speeds. Hence it is expressed without any unit and has no dimension.

CONCEPT STRANDS

Concept Strand 5

The absolute refractive index of a medium is 2. What is the speed of light inside that medium?

Solution

$$n = \frac{c}{v} \rightarrow v = \frac{c}{n} = \frac{3 \times 10^8}{2} \qquad (\because c = 3 \times 10^8 \text{ m s}^{-1})$$

$$= 1.5 \times 10^8 \text{ m s}^{-1}$$

$\therefore$ speed of light in the medium = 1.5×10^8 m s^{-1}

Concept Strand 6

Light of wavelength 7000 Å in vacuum is passed through a medium of refractive index 1.5.

(a) What is the frequency of the light in the medium?
(b) What is the wavelength of light in the medium?

Solution

(a) Frequency in medium is the same as frequency in vacuum as frequency does not change with medium.

$$\therefore f = \frac{c}{\lambda} = \frac{3 \times 10^8}{7000 \times 10^{-10}} \text{Hz}$$

$$= 4.29 \times 10^{14} \text{ Hz}$$

(b) $$n = \frac{\lambda}{\lambda_m} \rightarrow \lambda_m = \frac{\lambda}{n} = \frac{7000 \times 10^{-10}}{1.5}$$

$$= 4667 \text{ Å}$$

Hence the wavelength of light in the medium is 4667 Å.

Concept Strand 7

The speed of light in water is 2.25×10^8 m s^{-1} and in glass is 2×10^8 m s^{-1}. Calculate the

(a) Absolute refractive index of water.
(b) Absolute refractive index of glass.
(c) The refractive index of glass with respect to water.
(d) The refractive index of water with respect to glass.

Solution

Speed in water $v_w = 2.25 \times 10^8$ m s^{-1} and
speed in glass $v_g = 2 \times 10^8$ m s^{-1}

(a) $$n_w = \frac{c}{v_w} = \frac{3 \times 10^8}{2.25 \times 10^8} = 1.33$$

$\therefore$ Absolute refractive index of water is 1.33

(b) $$n_g = \frac{c}{v_g} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

$\therefore$ Absolute refractive index of glass is 1.5

(c) $${}_w n_g = \frac{n_g}{n_w} = \frac{(3/2)}{(4/3)} = \frac{9}{8}(> 1)$$

Hence glass is optically denser than water.

(d) $${}_g n_w = \frac{1}{{}_w n_g} = \frac{1}{(9/8)} = \frac{8}{9}(< 1)$$

Hence water is optically rarer than glass.

Concept Strand 8

Calculate the time taken by light to pass normally through a glass plate of thickness 4 mm, given refractive index of glass = 1.5.

Solution

Time $$t = \frac{\text{distance (d)}}{\text{speed (v)}};$$

$$d = 4 \text{ mm} = 4 \times 10^{-3} \text{ m}$$

Speed $$v = \frac{c}{n} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m s}^{-1}$$

$\therefore$ Time taken $$t = \frac{4 \times 10^{-3}}{2 \times 10^8} = 2 \times 10^{-11} \text{ s}$$

Concept Strand 9

A light wave of frequency 5×10^{14} Hz passes normally through water of refractive index $\frac{4}{3}$. Calculate the number of light waves contained in 3 m depth of water.

Solution

$$f = 5 \times 10^{14} \text{ Hz}; \; n = \frac{4}{3}$$

The number of wavelengths in 1 m of the medium is called the wave number $(\bar{\upsilon})$.

$\bar{\upsilon} = \dfrac{1}{\lambda_m}$ is the number of waves contained in unit length, where λ_m = wavelength of light in medium.

∴ No. of waves contained in 3 m depth $= 3\bar{\upsilon}$

$$= 3 \times \frac{1}{\lambda_m} = \frac{3}{\lambda_m}$$

Wavelength in vacuum

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^{14}} = 0.6 \times 10^{-6}\,\text{m}$$

∴ Wavelength in medium

$$\lambda_m = \frac{\lambda}{n} = \frac{0.6 \times 10^{-6}}{(4/3)}$$

$$= 0.45 \times 10^{-6} = 4500\ \text{Å}$$

∴ No. of waves in 3 m depth

$$= \frac{3}{\lambda_m} = \frac{3}{0.45 \times 10^{-6}}$$

$$= 6.67 \times 10^6$$

Laws of refraction

The laws of refraction were established on the basis of observations and experiments.

1. The incident ray, the refracted ray and the normal to the refracting surface at the point of incidence lie in the same plane.
2. For a given pair of media and for a given wavelength of light, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant. This is also known as Snell's law. In the mathematical form, Snell's law can be written as:

$$\frac{\sin i}{\sin r} = \text{constant}$$

where i = angle of incidence,
r = angle of refraction (See Fig. 1.20)

This constant is the relative refractive index of the refracting medium with respect to incident medium ($_i n_r$) which we defined earlier in terms of the speed of light in the two media.

$\dfrac{\sin i}{\sin r} = {}_i n_r = \dfrac{n_2}{n_1} = {}_1 n_2$, where 1 and 2 denote the incident and refracting media, respectively.

$$n_1 \sin i = n_2 \sin r$$

is the general form of law of refraction.

sine of angle of incidence × refractive index of incident medium	=	sine of angle of refraction × refractive index of refracting medium

CONCEPT STRANDS

Concept Strand 10

For a given pair of media, when angle of incidence is 40° then angle of refraction is 25° Find the value of angle of refraction when angle of incidence becomes 50° in the same medium. Also calculate deviation produced in the two cases.

Solution

$$\frac{\sin i_1}{\sin r_1} = \frac{\sin i_2}{\sin r_2} \Rightarrow$$

$$\frac{\sin 45°}{\sin 25°} = \frac{\sin 50°}{\sin r_2} \Rightarrow$$

$$\sin r_2 = \frac{\sin 25° \times \sin 50°}{\sin 40°} = 0.5036$$

$$r_2 = \sin^{-1}(0.5036) = 30°14^1$$

$$d\ (\text{I case}) = i - r = 40° - 25° = 15^0$$

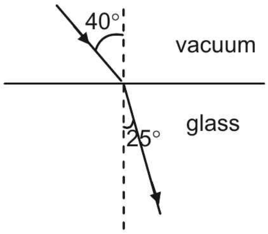
$$d\ (\text{II case}) = i - r = 50° - 30°\,14^1$$

$$= 19°\,46^1$$

Concept Strand 11

For a ray entering glass from vacuum the angle of incidence is 40° and the angle of refraction is 25°. Find the refractive index of glass.

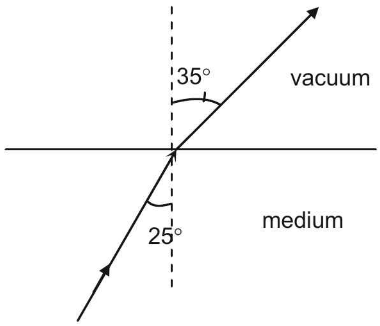
Solution



$$n = \frac{\sin i}{\sin r} = \frac{\sin 40°}{\sin 25°} = \frac{0.6428}{0.4226} = 1.52$$

Concept Strand 12

A ray gets refracted from a transparent medium to vacuum as shown in the figure. Find the refractive index of the medium.



Solution

To calculate refractive index of the medium, one has to consider the refraction from vacuum to medium. For this imagine the path of the ray reversed
i.e., $i = 35°$ and $r = 25°$

$$n = \frac{\sin i}{\sin r} = \frac{\sin 35°}{\sin 25°} = 1.36$$

Concept Strand 13

A ray of light enters glass from vacuum. If the angle of incidence is 45° find the angle of refraction. (Refractive index of glass 1.5)

Solution

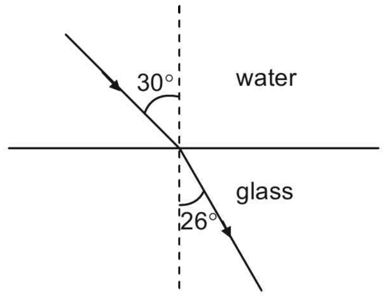
$$n = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin i}{n}$$

$$= \frac{\sin 45°}{1.5} = 0.4714$$

$\therefore \quad r = \sin^{-1}(0.4714) = 28°7'$

Concept Strand 14

Using the data given in the figure, calculate refractive index of glass with respect to water.



Solution

$${}_w n_g = \frac{\sin i}{\sin r}$$

$$= \frac{\sin 30°}{\sin 26°} = 1.14$$

The absolute refractive index of some mediums are as given below.

Sl. No.	Medium	Absolute refractive index (n)
1.	Vacuum	1.0
2.	Air	1.003 (for practical purposes, it is taken as 1.0)
3.	Glass	$\frac{3}{2} = 1.5$
4.	Water	$\frac{4}{3} = 1.33$
5.	Diamond	2.42

Principle of reversibility of light rays

When a ray of light, travelling through different media, undergoing refraction at various boundaries, is made to finally reverse its direction (say, by normal reflection), then it will retrace its path. This phenomenon is called principle of reversibility of light. The position of the object and image are interchangeable for mirrors, lenses etc., due to this phenomenon.

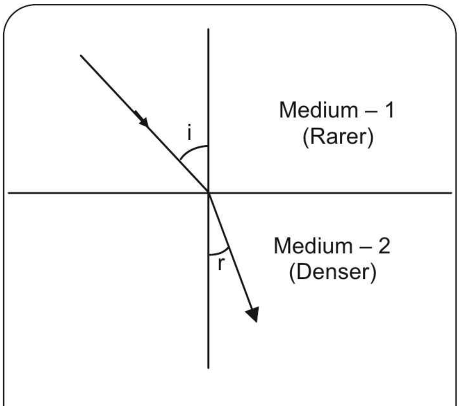


Fig. 1.22

When a ray of light travels from medium 1 to medium 2, we have

$$ {}_1n_2 = \frac{\sin i}{\sin r} = \frac{n_2}{n_1} \quad \text{---- (i)}$$

When the path of the ray is reversed, it travels from medium 2 to medium 1. Again, as per Snell's law, we have

$$ {}_2n_1 = \frac{\sin r}{\sin i} = \frac{n_1}{n_2} \quad \text{---- (ii)}$$

$$\text{(i)} \times \text{(ii)} \Rightarrow {}_1n_2 \times {}_2n_1 = 1$$

$$\Rightarrow {}_1n_2 = \frac{1}{{}_2n_1}$$

as was already found. This is due to principle of reversibility of light.

Deviation of a ray due to refraction (δ)

The angular separation between the direction of the incident ray and the refracted ray, measured inside the medium, is called the deviation (δ).

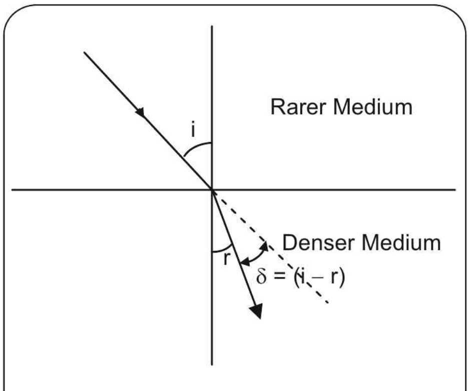


Fig. 1.23

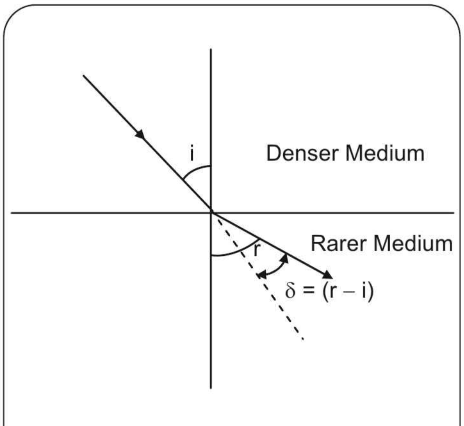


Fig. 1.24

When light travels from rarer to denser medium,

Deviation $\delta = (i - r)$ is towards normal

When light travels from denser to rarer medium,

deviation $\delta = (r - i)$ is away from normal

In both cases, the magnitude of the deviation,

$$\delta = |i - r|$$

Refraction through a transparent slab (lateral shift)

Consider a transparent slab of thickness t, and refractive index n. A monochromatic beam of light falls on one side at an angle of incidence i as shown in Fig. 1.25. Emergent ray will be parallel to incident ray, but there will be a lateral shift S of the incident ray. At the first interface,

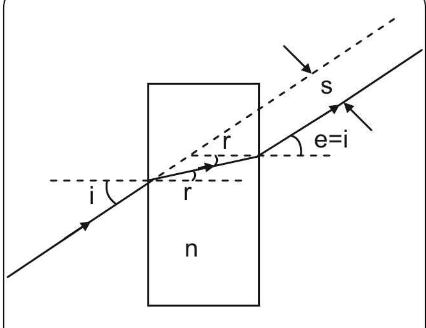


Fig. 1.25

1 sin i = n sin r and at the second interface

n sin r = 1 sin e

where, r is the angle of refraction at the first interface and e, the angle of refraction at the second interface. $\therefore$ e = i

From Fig. 1.26, lateral shift is calculated as follows:

$$AD = t;\ AB = \frac{AD}{\cos r} = \frac{t}{\cos r}$$

$$\text{Lateral shift } S = BC = AB \sin(i - r) = \frac{t\sin(i-r)}{\cos r}$$

$$\text{i.e., } S = \frac{t \sin(i-r)}{\cos r}$$

It may be noted that S_{max} = t for i = 90° (grazing incidence) and S_{min} = 0 for i = 0 (normal incidence)

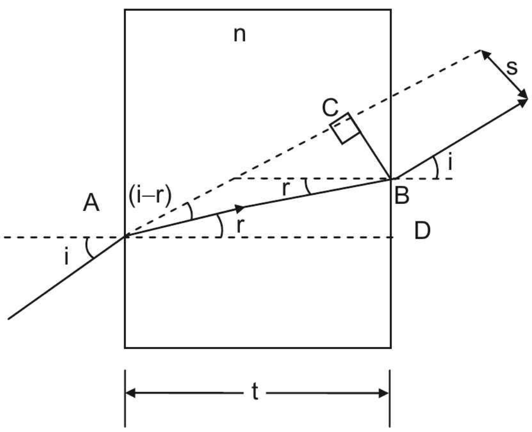


Fig. 1.26

Special case:

(i) **small i**

$$t\frac{\sin(i-r)}{\cos r} = \frac{t[\sin i \cos r - \cos i \sin r]}{\cos r}$$

[r small $\Rightarrow$ cos r $\approx$1] ; i small $\Rightarrow$ cos i $\approx$1]

$$\therefore \quad S = t(\sin i - \sin r) = t \sin i \left[1 - \frac{\sin r}{\sin i}\right]$$

$$\Rightarrow \quad S = t \sin i\left[1 - \frac{1}{n}\right] = ti\left(1 - \frac{1}{n}\right) \quad [\text{i small} \Rightarrow \sin i = i]$$

$$\Rightarrow \quad S = ti\frac{(n-1)}{n}$$

(Note: use formula $S = t\dfrac{\sin(i-r)}{\cos r}$ unless it is given that i = small)

(ii) When i is not small, it can be shown that

$$S = \frac{t\sin(i-r)}{\cos r} = t\sin i\left[1 - \frac{\cos i}{\sqrt{n^2 - \sin^2 i}}\right] \text{ or}$$

$$S = t\sin i\left[1 - \sqrt{\frac{1-\sin^2 i}{n^2 - \sin^2 i}}\right]$$

Refraction through multiple slabs

Consider a parallel stack of transparent slabs of indices of refraction n_1, n_2, …

A monochromatic light beam falls on one side of the stack as shown in Fig. 1.27.

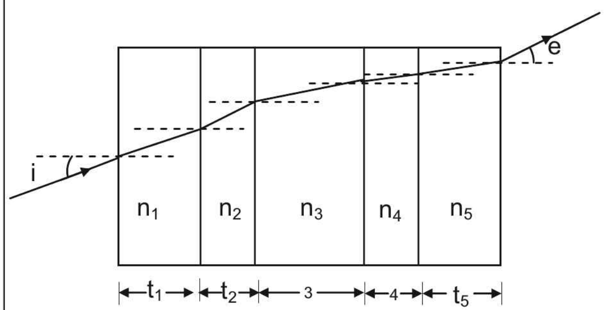


Fig. 1.27

It can be easily proved that the emergent ray is parallel to incident ray, i.e., e = i.

$1 \sin i = n_1 \sin r_1$

$n_1 \sin r_1 = n_2 \sin r_2$

....

....

$n_5 \sin r_5 = 1. \sin e \Rightarrow i = e$

Apparent depth and normal shift

When an object in one medium is viewed by an observer in another medium, the object appears to be at a distance 'd_a' (called apparent distance) and may be less or more than the actual distance 'd' separating them. If object is in an optically denser medium and observer is in an optically rarer medium, the apparent depth is less than the actual depth. i.e., $d_a < d$.

Example, A pond appears to be of less depth when viewed from outside (i.e., from air). A fish in the pond appears to be at less depth than the actual depth when viewed from outside the water. If the object is in the optically rarer medium and observer is in the optically denser medium, then the apparent distance is more than the actual distance i.e., $d_a > d$.

Example, an aeroplane flying in the sky appears to be at greater height than its actual height when viewed from ground. The above phenomenon is true whether the observation is done at an angle or done normally (i.e., at angle of incidence = 0°). If the observation is done normally, the difference between the actual distance 'd' and apparent distance 'd_a' is called the *normal shift*.

∴ Normal shift N_s = Actual normal distance – apparent normal distance

$$= (d - d_a)$$

∴ Normal shift (N_s) is negative if object is in rarer medium and observer is in denser medium (∵ $d_a > d$).

Expression for normal shift

A. Object in denser medium and observer in rarer medium

O = Actual position of object

O' = Apparent position of object

OA = d = Actual depth of object from refracting surface

O'A = d_a = Apparent depth of object from refracting surface

OO' = N_s = Normal shift

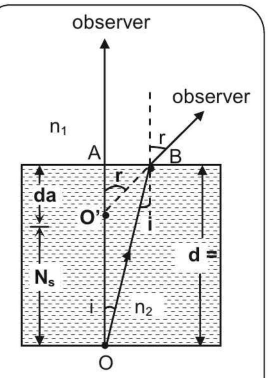


Fig. 1.28

Consider an object O in an optically denser medium (say water) observed from an optically rarer medium (say air), as shown in Fig. 1.28.

Ray OA is incident normally on the refracting surface and goes undeviated.

Ray OB is incident at a small angle of incidence 'i' and gets refracted away from normal at a small angle of refraction 'r'. When the refracted ray is extended backwards, it meets ray OA at point O'. Hence the virtual image of the object O is formed at O'. i.e., to the observer in the rarer medium, the object appears to be at O'.

From ΔOAB,

$$\tan i = \frac{AB}{OA} = \frac{AB}{d}$$

and tan i = sin i (for small angle i)

From ΔO'AB, $\tan r = \dfrac{AB}{O'A} = \dfrac{AB}{d_a}$ and

tan r = sin r (for small angle r)

$$\therefore \frac{\sin i}{\sin r} = \frac{AB}{d} \div \frac{AB}{d_a} = \frac{d_a}{d} \qquad \text{— (1)}$$

But from Snell's law for refraction,

$$\frac{\sin i}{\sin r} = \frac{\text{Refractive index of refracting medium}}{\text{Refraction index of incident medium}}$$

$$= \frac{n_1}{n_2} = {}_2n_1 = \frac{1}{{}_1n_2}\left(\because {}_2n_1 = \frac{1}{{}_1n_2}\right) \qquad \text{— (2)}$$

Substituting the value from (2) in (1), we get

$$\frac{1}{{}_1n_2} = \frac{d_a}{d} \text{ or } d_a = \frac{d}{{}_1n_2}$$

Hence

Apparent depth

$$(d_a) = \frac{\text{Actual depth (d) or real depth}}{\text{Relative refractive index of denser medium with respect to rarer medium}}$$

Normal shift $N_S = OO' = OA - O'A$

$$= d - d_a$$

$$= d - \frac{d}{{}_1n_2}$$

$$N_S = d\left(1 - \frac{1}{{}_1n_2}\right)$$

This is the required expression for normal shift and apparent depth.

Note:

(a) If the absolute refractive index of denser medium is 'n' and the rarer medium is air, then

$$d_a = \frac{d}{n} \text{ and } N_s = d\left(1 - \frac{1}{n}\right)$$

(b) If there are different media of absolute refractive indices n_1, n_2, n_3 and actual depth of these media are d_1, d_2 and d_3, then for an observer in air, apparent depth

$$d_a = \frac{d_1}{n_1} + \frac{d_2}{n_2} + \frac{d_3}{n_3} \quad \text{and}$$

Normal shift (N_S) is given by:

$$N_s = d_1\left(1 - \frac{1}{n_1}\right) + d_2\left(1 - \frac{1}{n_2}\right) + d_3\left(1 - \frac{1}{n_3}\right)$$

B. Object in rarer medium and observer in denser medium

When an object O in a rarer medium (say air) is observed from within a denser medium (say water), the image of O appears to be raised upto I as shown in Fig. 1.29. Then

Real height = AO = h

Apparent height = AI = h_a

$\therefore$ Apparent shift = OI = $h_a - h$

Since refraction is taking place from a rarer medium to a denser medium, as per Snell's law,

$$n = \frac{\sin i}{\sin r} \Rightarrow n = \frac{\tan i}{\tan r} \quad (\because \text{ i and r are small angles})$$

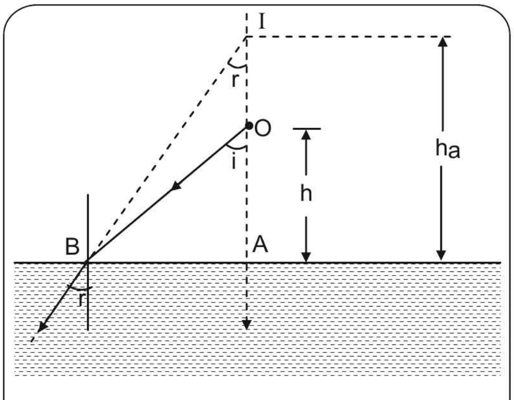


Fig. 1.29

$$\therefore \quad n = \frac{AB}{AO} \times \frac{AI}{AB} = \frac{AI}{AO}$$

$$\therefore \quad n = \frac{\text{Apparent height}}{\text{Real height}} \Rightarrow AI = n(AO)$$

$\therefore$ Apparent height,

$$h_a = nh$$

$$\therefore \quad S = \text{Apparent shift OI} = AI - AO = nh - h$$

$$S = h\,(n - 1)$$

Critical angle (C)

If a ray of light coming from an optically denser medium (of absolute refractive index n_2) reaches the boundary with an optically rarer medium (of absolute refractive index n_1), it bends away from the normal in the rarer medium as shown in Fig. 1.30.

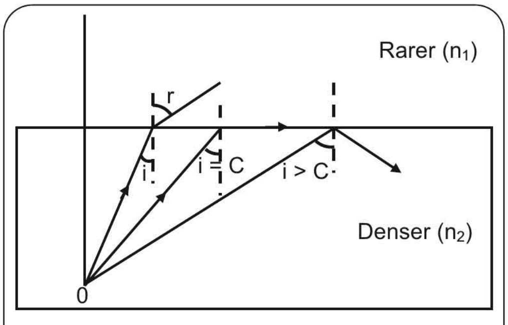


Fig. 1.30

As per Snell's law, $n_2 \sin i = n_1 \sin r$ or

$$\frac{\sin i}{\sin r} = \frac{n_1}{n_2} = \frac{1}{(n_2 / n_1)} = \frac{1}{{}_1n_2}$$

If for an angle of incidence C inside the denser medium, the angle of refraction 'r' is 90°, then we have

$$\frac{\sin C}{\sin 90°} = \frac{1}{{}_1n_2}$$

Since sin 90° = 1, we have

$$\sin C = \frac{1}{{}_1n_2}$$

The refracted ray goes tangential to the refracting surface. Angle C is called the critical angle for the refracting surface separating medium 2 and medium 1.

Hence critical angle for a refracting surface separating a denser and a rarer medium is defined as the angle of incidence in the denser medium for which the refracted ray just grazes the refracting surface at an angle of refraction of 90° in the refracting medium.

If 'n' is the absolute refractive index of the denser medium and the refracting medium (rarer medium) is air, then we have $\sin C = \frac{1}{n}$, where C = critical angle for the refracting surface.

$$C = \sin^{-1}\frac{1}{n}$$

∴ Larger the value of 'n', smaller will be the value of C and vice versa.

Note:

If rarer medium is not specified, it will be considered as vacuum when only the critical angle of a dense medium alone is given.

The absolute refractive index 'n' and corresponding critical angle C (with vacuum or air) for some materials are listed below.

n and C of some materials

Material	Refractive index (n)	Critical angle (C)
Glass	1.5	$41^0\, 48'$
Water	1.33	$48^0\, 45'$
Diamond	2.42	$24^0\, 24'$

Critical angle of a medium for different colours of light

White light is made up of seven major colours violet, indigo, blue, green, yellow, orange and red (VIBGYOR). In vacuum, all these colours have the same speed $c = 3.0 \times 10^8$ m s^{-1}. Their wavelengths vary from 4000 Å for violet to 7000 Å for red (Å = 10^{-10} m). The frequencies of each wavelength in vacuum can be obtained from the relation $f = \frac{c}{\lambda}$, wheref = frequency. This frequency does not change from medium to medium. In any other medium, different colours of light (characterized by their wavelength in that medium or frequency in that medium) travel with different speeds. Consequently, the absolute refractive index of a medium is different for different colours of light. If 'λ' is the wavelength of electromagnetic radiation in vacuum, the refractive index of a medium for this wavelength (n_λ) can be obtained from **Cauchy's formula** which is

$$n_\lambda = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^4}$$

where A, B and C are constants. A has no unit and no dimension, B has unit (m^2) and C has unit (m^4).

Since A>> B >> C, Cauchy's formula for visible light of wavelength 'λ' can be written as

$$n_\lambda = A + \frac{B}{\lambda^2}$$

where, A and B are constants.

$$\therefore n_\lambda \propto \frac{1}{\lambda^2}. \text{ Since critical angle } C \propto \frac{1}{n_\lambda}, \text{ we get}$$

$$C \propto \lambda^2$$

∴ Larger the wavelength of light, larger will be the critical angle and vice-versa.

So for a given medium, C for red light will be greatest and C for violet light will be least ($\because \lambda_{red} > \lambda_{violet}$). Also $n_{violet} > n_{red}$ for any medium other than vacuum.

Total internal reflection

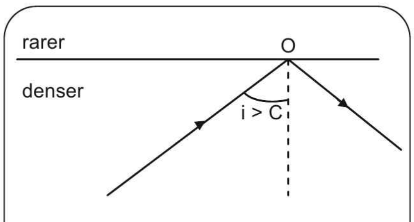


Fig. 1.31

When a light ray coming from a denser medium is incident at the surface separating it from a rarer medium, at an angle of incidence greater than the critical angle, (Fig. 1.31) no part of light is refracted, but the entire light energy is reflected back to the denser medium according to the laws of reflection. This phenomenon is known as total internal reflection

Note:
If the angle of incidence i in the above case is less than critical angle C, there is partial reflection as well as partial refraction. The percentage of energy refracted decreases as angle of incidence i increases and becomes zero when i exceeds critical angle C.

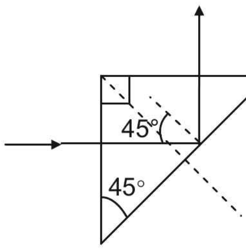
Conditions for total internal reflection

(i) The light ray should fall on the surface of separation from the side of denser medium.
(ii) The angle of incidence should be greater than the critical angle.

CONCEPT STRANDS

Concept Strand 15

A ray of light falls normal to one face of a right angled isosceles prism and emerges normal to the other face. What is the minimum index of refraction of the glass?



Solution

$$45° > \theta_c \Rightarrow \sin 45^\circ > \sin\theta_c = \frac{1}{n}$$

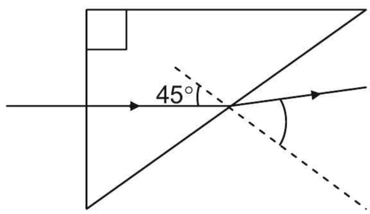
$$\Rightarrow n > \frac{1}{\sin 45} = 1.414$$

Concept Strand 16

What happens if the above prism (n = 1.50) is immersed in water (n = 4/3)?

Solution

$$\theta_{c\,glass/water} = \sin^{-1}\frac{1}{{}^{3}\!/_{2} \times {}^{3}\!/_{4}} = 62.7° > 45^\circ$$



Hence no total internal reflection, but only refraction takes place

$$1.5 \sin 45 = \frac{4}{3}\sin r \Rightarrow r = 52.7°$$

PRISMS

A simple prism is a homogeneous transparent refracting medium bounded by at least two non-parallel plane surfaces inclined at some angle.

A prism can have any number of surfaces but the surface on which light is incident and the surface from which light emerges (or comes out) must be plane surfaces and non-parallel.

Since the two refracting surfaces are at an angle, the deviation produced by one refracting surface gets added to the deviation produced by the other refracting surface, so that the incident ray and emergent ray are in different directions resulting in a net deviation of the incident ray.

If the refracting surfaces were parallel as in the case of a rectangular glass slab, the deviation produced by these surfaces will be in opposite directions and cancel each other, when the incident medium and emergent medium are same. There is only a lateral shift of the ray and no deviation.

The two non-parallel plane surfaces of a prism participating in refraction of light are called the refracting surfaces and the line of intersection of the two refracting surfaces is called the refracting edge. The angle between the two refracting surfaces measured inside the material of prism is called the angle of prism or refracting angle of prism and is denoted by A (Fig. 1.32).

A section of the prism perpendicular to the refracting edge, is called the principal section of the prism.

Some of the commonly used prisms are equilateral prism, right angled prism and right angled isosceles prism. These have principal sections which are equilateral triangle, right angled triangle and right angled isosceles triangle respectively.

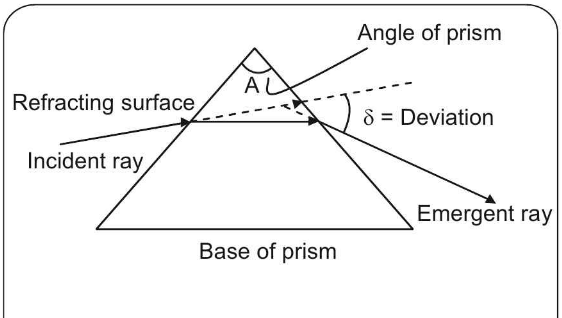


Fig. 1.32

To derive an expression for the d eviation (d) produced by a prism

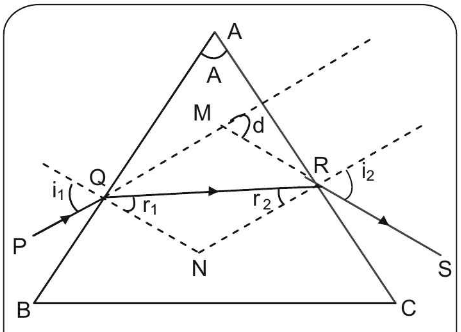


Fig. 1.33

ABC is the principal section of a prism (Fig. 1.33). BC is the base and angle A opposite to it is the refracting angle. PQ is the incident ray, which after refraction has its path QR in the prism. On emerging from the other face it comes out along RS.

QN and RN are the normals to the refracting faces. At Q, i_1 is the angle of incidence and r_1 the angle of refraction. At R, r_2 is the angle of incidence and i_2 the angle of emergence

The emergent ray when produced backwards meets the incident ray at M. 'd' is the deviation produced for the ray by the prism

To show that $A = r_1 + r_2$

Sum of the four angles of quadrilateral NQAR = 360°

$\Rightarrow \angle NQA + \angle QAR + \angle ARN + \angle RNQ = 360°$

$\Rightarrow 90^0 + A + 90^0 + \angle RNQ = 360°$

$\Rightarrow A = 180° - \angle RNQ$ — (1)

Sum of the three angles of Δ RNQ is 180°

$\Rightarrow \angle RNQ + r_1 + r_2 = 180°$

$\Rightarrow r_1 + r_2 = 180° - \angle RNQ$ — (2)

From (1) and (2)

$A = r_1 + r_2$

If $r_1 > A_i$ Then $A = r_1 - r_2$

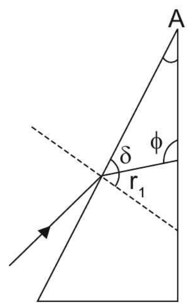


Fig. 1.34

$A = 10°$

Let $r_1 = 30°$

$\therefore \ \delta = 60°$

$r_2 \therefore \phi = 180° - (60° + 10°)$

$= 110°$

$\therefore \ r_2 + 90° = \phi = 110°$

$r_2 = 20°$

$r_1 - r_2 = 30 - 20 = 10° = A$

To show that $d = i_1 + i_2 - A$

$$\begin{aligned} d &= \text{exterior angle of } \Delta\, MQR \\ &= \text{sum of interior opposite angles} \\ &= \angle MQR + \angle QRM = (i_1 - r_1) + (i_2 - r_2) \\ &= i_1 + i_2 - (r_1 + r_2) \\ &= i_1 + i_2 - A \\ d &= i_1 + i_2 - A \end{aligned}$$

Notes:

(1) Deviation produced by the first face is $d_1 = i_1 - r_1$ and that by the second face is $d_2 = i_2 - r_2$.
(2) $d = d_1 + d_2$, is the total deviation produced by the prism
(3) Using Principle of reversibility of rays it can be seen that
  (a) i_1 and i_2 can be interchanged.
  (b) when i_1 and i_2 are interchanged r_1 and r_2 get interchanged but deviation d remains the same.
(4) Thus for the same deviation, there are two possible angles of incidence i_1 and i_2.
(5) The refracted ray is bent towards the base of the prism. (However, if the material of the prism is rarer than that of surrounding, the deviation will be in the opposite direction).
(6) Deviation (d) depends on
  (i) the angle (A) of the prism
  (ii) the angle of incidence i_1
  (iii) the angle of emergence i_2, which in turn depends on the refractive index of the material of the prism. It can be shown that when refractive index (n) of the material of the prism increases deviation (d) increases
  (iv) wavelength of light used

Variation of deviation (d) with angle of incidence

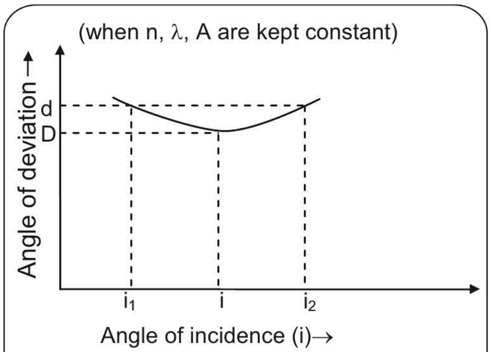


Fig. 1.35

Fig. 1.35 shows the variation of deviation with the angle of incidence.
The following points may be noted:

(i) As the angle of incidence increases from low value, deviation (d) decreases first, reaches a minimum value D, and thereafter increases.
(ii) In general, for a given deviation (d) there are two possible angles of incidence i_1 and i_2.
(iii) When i_1 is the angle of incidence then i_2 will be the angle of emergence and vice versa.
(iv) When i_1 increases, then i_2 decreases and vice versa. When d attains its minimum value D, then $i_1 = i_2$ which implies that $r_1 = r_2$. In other words the ray is symmetric with respect to the first and the second faces.
(v) The refractive index (n) of the material of the prism can be expressed in terms of D and A as follows:

$$A = r_1 + r_2 \qquad \text{— (1)}$$

$$\text{and } d = i_1 + i_2 - A \qquad \text{— (2)}$$

When the ray undergoes minimum deviation

$$d = D;$$
$$i_1 = i_2 = i \text{ (say)};$$
$$r_1 = r_2 = r \text{ (say)}$$

Now from (1) $A = 2r$

$$\Rightarrow r = \frac{A}{2} \text{ and from (2)}$$

$$D = 2i - A$$

$$\Rightarrow i = \frac{D + A}{2}$$

Hence $$n = \frac{\text{Sin } i}{\text{Sin } r} \Rightarrow$$

$$n = \frac{\text{Sin } \dfrac{D + A}{2}}{\text{Sin } \dfrac{A}{2}}$$

where, n = refractive index of material of prism
A = refracting angle of the prism
D = angle of minimum deviation

CONCEPT STRAND

Concept Strand 17

For a prism, refracting angle is 65° and angle of minimum deviation is 40°. Find

(i) Refractive index of the material of the prism.
(ii) Angle of incidence and angle of emergence corresponding to minimum deviation.
(iii) Angle of refraction corresponding to minimum deviation.

Solution

(i) $n = \dfrac{\text{Sin}\dfrac{A+D}{2}}{\text{Sin}\dfrac{A}{2}} = \dfrac{\text{Sin}\dfrac{65^0+40}{2}}{\text{Sin}\dfrac{65^0}{2}} = \dfrac{\text{Sin } 52.5^0}{\text{Sin } 32.5^0} = 1.48$

(ii) $i_1 = \dfrac{D+A}{2}$

$= \dfrac{40+65}{2}$

$= 52.5°$

$i_2 = i_1 = 52.5°$

(iii) $r_1 = \dfrac{A}{2}$

$= 32.5°$

Note:
When there is minimum deviation, the deviation produced by the first face $d_1 = (i_1 - r_1)$ is the same as that produced by the second face $d_2 = (i_2 - r_2)$

The path of a ray undergoing minimum deviation through an equilateral prism

In an equilateral prism, the three angles are 60° each.

i.e., $A = 60°$

$r_1 + r_2 = 60°$

When there is minimum deviation $r_1 = r_2 = 30°$. Simple geometry shows that the path of the ray through prism is parallel to the base (Fig. 1.36)

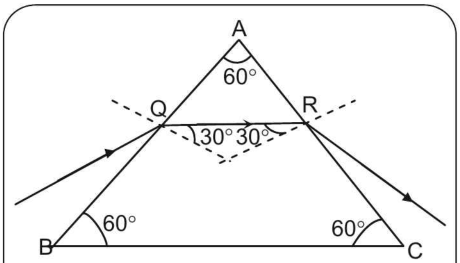


Fig. 1.36

This is also there for isoseles prism!

A +2 (i – r) = 180

A +2 (Q) = 180

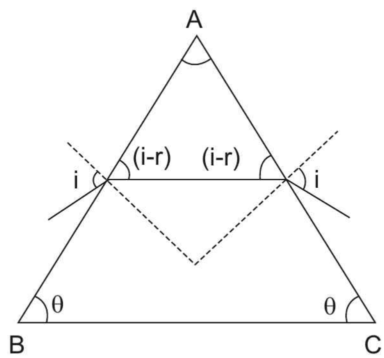


Fig. 1.37

(i – r) = Q

PQ || A B

Grazing incidence and grazing emergence

If the angle of incidence is 90° (i.e., $i_1 = 90°$), it is called grazing incidence. The incident ray travels along the refracting surface of the prism and the angle of refraction r_1 = critical angle C for material of prism. The angular deviation for such a ray is maximum when it passes through that prism. $d_{max} = i_1 + i_2 - A$.

$$d_{max} = 90° + i_2 - A$$

If the ray emerges from the prism along the second refracting surface with, $i_2 = 90°$, then it is known as grazing emergence. The angle of incidence r_2 at the second refracting surface is equal to the critical angle C for the material of the prism.

Small angled prisms

A prism is said to be small angled if the refracting angle A is small. A prism whose refracting angle is equal to or less than 10^0, is called a thin prism.

$$A = r_1 + r_2$$

Small value for A
$\Rightarrow$ small values for r_1 and r_2

$$n = \frac{\text{Sin } i_1}{\text{Sin } r_1} = \frac{\text{Sin } i_2}{\text{Sin } r_2}$$

Small values of r_1 and r_2
$\Rightarrow$ small values for i_1 and i_2

Sin $i_1 \approx i_1$; Sin $i_2 \approx i_2$;

Sin $r_1 \approx r_1$ and Sin $r_2 \approx r_2$

$$n = \frac{i_1}{r_1} = \frac{i_2}{r_2} \Rightarrow i_1 = nr_1 \text{ and } i_2 = nr_2$$

$$\begin{aligned} d &= i_1 + i_2 - A \\ &= nr_1 + nr_2 - A \\ &= n(r_1 + r_2) - A \\ &= nA - A \\ d &= (n-1)A \end{aligned}$$

Note:
For a small angled prism deviation produced is independent of the angle of incidence. Also it is proportional to the angle of the prism

CONCEPT STRANDS

Concept Strand 18

A prism of angle 60° is of a material of refractive index $\sqrt{2}$. Calculate angles of minimum deviation and maximum deviation.

Solution

(i) $A = 60^\circ, n = \sqrt{2}$

At δ_{min}, $r = \dfrac{A}{2} \Rightarrow r = 30°$

$$n = \frac{\sin i}{\sin r} \Rightarrow \sqrt{2} = \frac{\sin i}{\sin 30°} \Rightarrow \sin i = \frac{1}{\sqrt{2}} \Rightarrow i = 45°$$

$$\delta_{min} = 2i - A = 90° - 60° = 30°$$

$$\theta_c = \sin^{-1}\frac{1}{\sqrt{2}} = 45° \Rightarrow A > \theta_c$$

(ii) Now, since $r_1 = \theta_c$ and $A = r_1 + r_2$,

We have $r_2 = A - r_1 = A - \theta_c$

$\therefore$ $i_2 = n \sin r_2 = n \sin (A - \theta_c)$

$\therefore$ Applicable formula is

$$\delta_{max} = 90^\circ + \sin^{-1}[n \sin(A - \theta_c)] - A$$
$$= 90° + \sin^{-1}[\sqrt{2}.\ \text{Sin}(60° - 45°)] - 60° = 90° + 22° - 60° = 52°$$

(We may also calculate δ_{max} without the use of this formula:

For δ_{max}, $i_1 = 90°$

$\therefore$ $r_1 = \theta_c = 45°$

$\Rightarrow$ $r_2 = A - r_1 = 60° - 45° = 15°$

$\therefore$ $\dfrac{\sin i_2}{\sin 15°} = \sqrt{2}$

$\Rightarrow$ $\sin i_2 = \sqrt{2}\sin 15° = 0.37 \Rightarrow i_2 = 22°$

$\therefore$ $\delta_{max} = i_1 + i_2 - A$
$= 90° + 22° - 60° = 52°$)

Concept Strand 19

If a prism $\left(n = \sqrt{2}\right)$ has D = 2A, what is A?

Solution

$$\sqrt{2} = \frac{\sin\dfrac{A+D}{2}}{\sin\dfrac{A}{2}} = \frac{\sin\dfrac{3A}{2}}{\sin\dfrac{A}{2}}$$

let $\theta = \dfrac{A}{2}$

$$\sqrt{2} = \frac{\sin 3\theta}{\sin\theta} = 3 - 4\sin^2\theta$$

$\Rightarrow$ $\theta = 39°4' \Rightarrow A = 2\theta = 78°\ 8'$

Monochromatic light

Light consisting of only one wavelength is called monochromatic light.

For example, light from a sodium vapour lamp is nearly monochromatic. Its wavelength is about 5893 Å (average of D_1 and D_2 lines, 5895.9 Å and 5890 Å, respectively). For all practical purposes, it can be considered as monochromatic. Light from a laser is monochromatic.

Composite Light

Light consisting of more than one wavelength is called composite light.

White light is an example of composite light. It has 7 major constituents. They are Violet, Indigo, Blue, Green, Yellow, Orange and Red. (VIBGYOR)

Note:
As one moves from violet end to red end:

(1) The wavelength in vacuum increases from 4000 Å to 8000 Å
(2) Frequency decreases
(3) Velocity of light in a given medium increases
(4) Deviation produced by a prism decreases
(5) Refractive index of the medium decreases

Dispersion

Dispersion is the process of splitting a composite light into its constituent wavelengths. The coloured pattern obtained as a result is called spectrum.

Rainbow is formed because of dispersion of solar light by water particles in the atmosphere.

One method of producing dispersion in the laboratory is by sending the composite light through a prism.

Dispersion of white light using a prism:

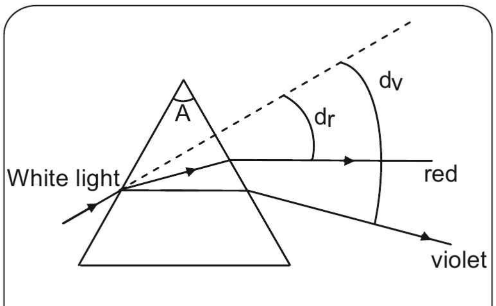


Fig. 1.38

When white light is sent through a prism different colours get deviated through different angles, maximum being for violet and minimum for red. This is due to the material of the prism having different refractive indices for different colours of light, $n_v > n_r$ ($\lambda_v < \lambda_r$) as per Cauchy's formula $n \propto \frac{1}{\lambda^2}$. As a result white light gets split into its constituent colours causing dispersion.

In Fig. 1.38, d_v is the deviation of violet and d_r the deviation of red light produced by the prism. Mean deviation (d) for the pair of violet and red is:

$$d = \frac{d_v + d_r}{2}$$

Angular dispersion (θ)

Angular dispersion, caused by the prism, of the pair of colours violet and red is $d_v - d_r$. The unit of angular dispersion is radian and it has no dimension.

$$\theta = d_v - d_r$$

Dispersive power of the material of the prism

Dispersive power (ω) of the material of the prism is the ratio of angular dispersion to mean deviation. It is a measure of the capability of the material of the prism to split composite light into its components.

$$\text{Dispersive power } (\omega) = \frac{\text{Angular dispersion}}{\text{Mean deviation}}$$

For the pair violet and red

$$\omega = \frac{d_v - d_r}{d}$$

Note:

(1) Dispersive power is a characteristic of the material of the prism and is independent of the geometry of the prism.
(2) Dispersive power depends on the pair of colours used. If not specifically mentioned one can take the pair as violet and red.
(3) Dispersive power being a ratio, has no unit and dimension.
(4) Dispersive power is greater for flint glass than for crown glass.

CONCEPT STRAND

Concept Strand 20

In one dispersion experiment

$d_v = 3.25°$, $d_b = 3°$ and

$d_r = 2.75°$. Calculate angular dispersion, mean deviation and dispersive power for

(i) violet – red pair and (ii) blue – red pair

Solution

Case I (violet – red pair)

Angular dispersion = dv – dr = 3.25° – 2.75° = 0.5°

$$\text{Mean deviation} = d = \frac{d_v + d_r}{2} = \frac{3.25° + 2.75°}{2} = 3°$$

$$\text{Dispersive power} = \frac{d_v - d_r}{d}$$

$$= \frac{0.5°}{3°} = 0.167$$

Case II (blue – red pair)

$$\text{Angular dispersion} = d_b - d_r = 3° - 2.75° = 0.25°$$

$$\text{Mean deviation} = d = \frac{d_b + d_r}{2} = \frac{3° + 2.75°}{2} = 2.875°$$

$$\text{Dispersive power} = \frac{d_b - d_r}{d} = \frac{0.25°}{2.875°} = 0.087$$

To express angular dispersion and dispersive power in terms of refractive indices of pair of colours considered

Let violet light and red light be the pair of colours considered. Let n_v and n_r be the refractive indices of the material of prism for these colours. Imagine that dispersion occurs through a small angled prism of angle A

$$d_v = (n_v - 1)\,A \text{ and } d_r = (n_r - 1)A$$

$$\text{Angular dispersion} = d_v - d_r$$

$$= (n_v - 1)A - (n_r - 1)A = (n_v - n_r)A$$

Mean deviation

$$= \frac{d_v + d_r}{2} = \frac{(n_v - 1)A + (n_r - 1)A}{2}$$

$$= \left(\frac{n_v + n_r}{2} - 1\right) A = (n\text{-}1)\,A, \text{ where } n = \frac{n_v + n_r}{2}$$

is the mean refractive index of the material of prism

$$\text{Dispersive power } \omega = \frac{d_v - d_r}{d} = \frac{(n_v - n_r)A}{(n - 1)A} = \frac{n_v - n_r}{n - 1}$$

$$\omega = \frac{n_v - n_r}{n - 1}$$

CONCEPT STRAND

Concept Strand 21

Calculate the dispersive power of the material of a prism with respect to the given pair of colours of light. $n_v = 1.538$ and $n_r = 1.527$

Solution

$$\text{Mean refractive index } n = \frac{n_v + n_r}{2}$$

$$= \frac{1.538 + 1.527}{2}$$

$$= 1.5325$$

$$\text{Dispersive power } \omega = \frac{n_v - n_r}{n - 1}$$

$$= \frac{1.538 - 1.527}{1.5325 - 1} = 0.021$$

Combining two prisms to produce dispersion without deviation (Direct vision spectrum)

Two prisms can be combined in such a way as to have dispersion but no deviation for the mean ray.

Two small angle prisms of different materials and suitable angles A and A' are combined as in the figure. Here A and A' are to be in opposite directions so that deviation produced by the two are opposite and hence cancel each other. Let n and n' be the refractive indices of the materials of the two prisms.

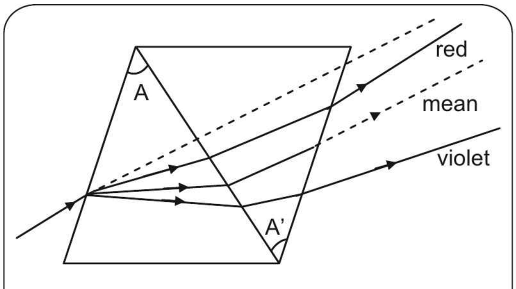


Fig. 1.39

Deviation for the mean ray by the first prism = (n – 1)A

Deviation for the mean ray by the second prism = (n' – 1)A'

Total deviation = (n – 1)A +(n' – 1)A' — (1)

Since the combination is to have no net deviation for the mean ray

$$(n - 1)A + (n' - 1)A' = 0 \Rightarrow$$

$$\frac{A}{A'} = -\frac{n' - 1}{n - 1}$$

This is the condition for no deviation.

Since there is no deviation, the mean ray will be parallel to the incident direction.

The negative sign for $\frac{A}{A'}$ shows that the deviations of the two prisms are in opposite directions and so the two prisms are to be kept in opposite directions.

The angular dispersion produced by the first prism is $(n_v - n_r)A$ and that by the second prism is $(n_v' - n_r')A'$.

Net angular dispersion (θ) by the combination is

$$\theta = (n_v - n_r)A + (n_v' - n_r')A'$$

Such combination of prisms are used in direct vision spectroscopes

Note:

If the two prisms are of the same material n' = n so that $\frac{A}{A'} = -1 \Rightarrow A = -A'$

In such case

Net angular dispersion = $(n_v - n_r)A - (n_v - n_r)A = 0$

Hence there will be no dispersion

Hence the prisms should be of different materials

Deviation without dispersion

One can also think of combination of two prisms to have deviation without dispersion. Such combination of prisms are called achromatic combination of prisms. In this case net angular dispersion is zero but there is deviation for mean ray

i.e., $(n_v' - n_r')A' + (n_v - n_r)A = 0 \Rightarrow \frac{A}{A'} = -\frac{n'_v - n'_r}{n_v - n_r}$

Net deviation for mean ray

$$d = (n - 1)A + (n' - 1)A'$$

CONCEPT STRANDS

Concept Strand 22

A crown glass prism of 11° is combined with a flint glass prism to form a combination giving dispersion without deviation. Given the refractive indices as

	Crown	Flint
Red	1.51	1.62
Blue	1.52	1.64

Find (1) The angle of the flint glass prism
(2) The net angular dispersion produced

Solution

Refractive index of mean ray (crown)

$$n = \frac{1.51 + 1.52}{2} = 1.515$$

Refractive index of mean ray(flint)

$$n' = \frac{1.62 + 1.64}{2} = 1.63$$

Condition for no deviation is $\frac{A}{A'} = -\frac{n'-1}{n-1}$

$$\Rightarrow A' = -\frac{(n-1)A}{n'-1}$$

$$= -\frac{(1.515-1)11}{(1.63-1)} = -8.99°$$

Angle of flint glass prism = 8.99°

Net angular dispersion = $(n_B - n_B)A + (n_B' - n_r')$ A'= (1.52 – 1.51)11 + (1.64 – 1.62) (–8.99) = –0.07°

$$\therefore \quad \text{Net deviation } d = (n-1)A - \frac{(n'-1)}{(n_B' - n_r')}(n_B - n_r)$$

$$A = (n-1)A\left[1 - \frac{\omega}{\omega'}\right], \text{ where}$$

n = refractive index of first prism for mean light

ω = dispersive power of material of first prism and

ω' = dispersive power of material of second prism

REFRACTION AT SPHERICAL SURFACES

Consider the interface between two media of refractive indices n_1 and n_2, ($n_2 > n_1$) as shown in Fig. 1.39. The interface is a section of spherical shape and can be either convex or concave for oncoming light ray. Let a beam of light fall on the interface from the medium of refractive index n_1. The spherical surface is characterized by

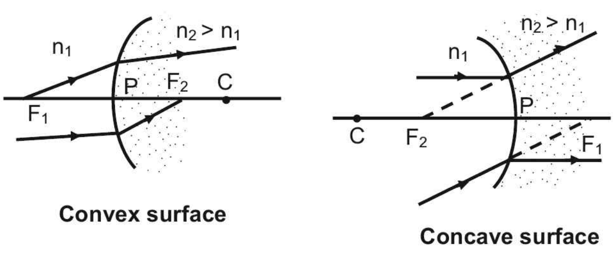


Fig. 1.40

C – Centre of curvature

R – Radius of curvature

P – Pole (centre of the spherical surface)

PC extended – Principal axis

F_1 and F_2 – Principal foci

First principal focus of a convex surface is a point on the principal axis from where the rays falling on the surface, after refraction, will travel parallel to the principal axis.

First principal focus of a concave surface is a point on the principal axis to which if a beam of light is directed, will, after refraction, travel parallel to the principal axis.

Second principal focus of a convex surface is the point on the principal axis to which a parallel beam of light, after refraction, will converge.

Second principal focus of a concave surface is the point on the principal axis from where a parallel beam of light, after refraction, will appear to come.

Derivation of refraction equation

Consider refraction at a convex surface as shown in Fig. 1.40.

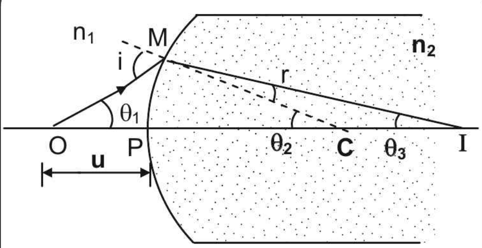


Fig. 1. 41

Sign conventions are the same as before, viz.,

(i) Distances are measured from the pole
(ii) Direction of incident ray is positive.
(iii) Rays of light are paraxial.
(iv) Angles i, r, θ_1, θ_2 and θ_3 are small.
(v) MP is almost perpendicular to the axis.

In Fig. 1.39, Δ MPO, has right angle at P

$$\Rightarrow \tan\theta_1 \approx \theta_1 = \frac{PM}{PO}$$

$$\tan\theta_2 \approx \theta_2 = \frac{PM}{PC}$$

$$i = \theta_1 + \theta_2 = PM\left(\frac{1}{PO} + \frac{1}{PC}\right) \qquad \text{—(1)}$$

$$\tan\theta_3 \approx \theta_3 = \frac{PM}{PI}$$

$$r = \theta_2 - \theta_3 = PM\left(\frac{1}{PC} - \frac{1}{PI}\right) \qquad \text{—(2)}$$

Snell's law: $n_1 \sin i = n_2 \sin r$

$\Rightarrow n_1 i = n_2 r$ ($\because$ paraxial rays)

$$\therefore\ n_1 (PM)\left(\frac{1}{PO} + \frac{1}{PC}\right) = n_2 (PM)\left(\frac{1}{PC} - \frac{1}{PI}\right)$$

Sign convention lead to PO $\Rightarrow -u$, PC $\Rightarrow R$ PI $\Rightarrow v$

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

[It is easy to remember this formula: object in medium 1, $\Rightarrow \frac{n_1}{u}$, Image in medium 2, $\Rightarrow \frac{n_2}{v}$]

Image is real if v is positive and virtual if v is negative. ***Lateral magnification***, m, can be obtained from the geometry of Fig. 1.42.

$n_1 \sin i = n_2 \sin r \equiv n_1 \tan i = n_2 \tan r$

$$\Rightarrow n_1 \cdot \frac{AB}{PA} = n_2 \frac{A'B'}{PA'}$$

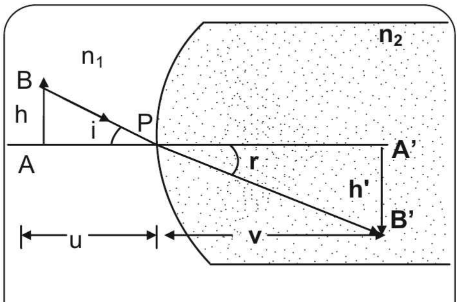


Fig. 1.42

$$\Rightarrow n_1\left(\frac{+h}{-u}\right) = n_2\left(\frac{-h'}{+v}\right) \Rightarrow$$

Lateral magnification,

$$m = \frac{h'}{h} = +\frac{n_1 v}{n_2 u}$$

Image is erect, if m is positive; inverted if m is negative; enlarged if $|m| > 1$; diminished if $|m| < 1$. ***Longitudinal magnification***, m', can be derived as follows:

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{-n_2 dv}{v^2} + n_1 \cdot \frac{du}{u^2} = 0$$

$$\Rightarrow \frac{dv}{du} = m' = +\frac{n_1 v^2}{n_2 u^2}$$

$$= \frac{n_1^2 v^2}{n_2^2 u^2} \cdot \frac{n_2}{n_1} \Rightarrow$$

Longitudinal magnification,

$$m' = \frac{n_2}{n_1} m^2$$

m' is always positive
$\Rightarrow$ object and image are in the same direction

$$\overrightarrow{AB} \qquad \overrightarrow{A'B'}$$

Note that all the fomulae are similar to those for spherical mirrors.

SUMMARY

Spherical mirror (Reflection)

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}=\frac{2}{R} \quad (\text{v : positve} \Rightarrow \text{(Real)})$$

$$m=\frac{-v}{u}$$

Spherical surface (Refraction)

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R} \quad (\text{v: positive} \Rightarrow \text{Real})$$

$$m=\frac{n_1 v}{n_2 u}$$

$$\begin{bmatrix} \text{m positive} \Rightarrow \text{Erect image} \\ \text{m negative} \Rightarrow \text{Inverted image} \end{bmatrix}$$

$$m'=\frac{-v^2}{u^2} \qquad\qquad m'=\frac{n_1 v^2}{n_2 u^2}$$

(m' negative ⇒ image longitudinally inverted)
(m' positive ⇒ image and object in same direction)

Power of a surface

The expression $\frac{n_2 - n_1}{R}$ is called the power of a refracting surface of a medium with refractive index n_2 and radius of curvature R, when light ray falls on the surface from medium with refractive index n_1.

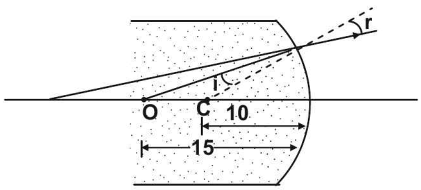
The SI unit of power is the diopter (D) = 1 m^{-1}. One diopter is the power of a surface of focal length 1 m. It is positive for a convex surface and negative for a concave surface.

CONCEPT STRANDS

Concept Strand 23

An object is immersed in a medium of refractive index n = 2.0. Object is 15 cm from the concave surface whose radius of curvature is 10 cm. Locate the image.

Solution



$$u = -15,\ n_1 = 2,\ n_2 = 1,\ R = -10$$

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

$$\frac{1}{v}-\frac{2}{-15}=\frac{1-2}{-10} \Rightarrow v=-30 \text{ cm (virtual)}$$

Concept Strand 24

What is the relationship between u and v if the refracting surface is plane?

Solution

$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$

For plane surface, $R = \infty \Rightarrow \frac{n_2}{v}=\frac{n_1}{u} \Rightarrow v=\frac{n_2}{n_1}u$

[we already know this from the apparent depth formula with object real; this gives always virtual image]

Concept Strand 25

To obtain a real image at infinity in a medium 1 (with refractive index n_1) out of a real object in a medium 2 (with refractive index n_2) and at a distance d from the interface, what should be the geometry of the interface?

Solution

$$\frac{n_1}{\infty}-\frac{n_2}{-d}=\frac{n_1-n_2}{R}$$

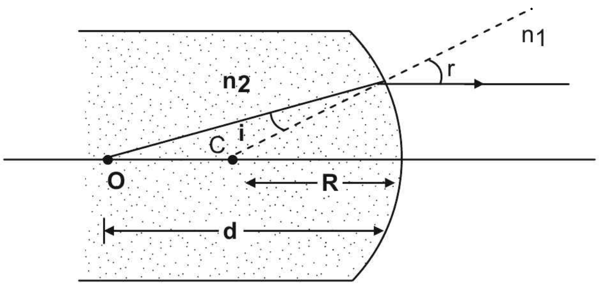
Clearly it cannot be a plane (whose $R = \infty$)

∴ surface can be only spherical, whose $R = \left(\frac{n_1-n_2}{n_2}\right)d$,

d, being modulus value, is always positive

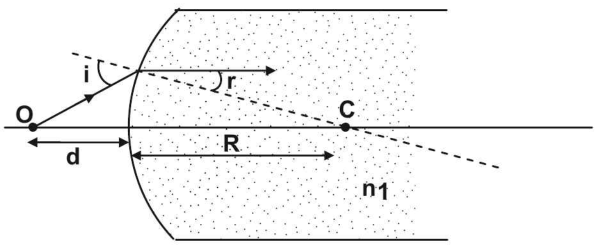
$\therefore$ R can be negative (concave) with $n_2 > n_1$

$$|R| = \left(\frac{n_2 - n_1}{n_2}\right)d$$



Or R can be positive $\Rightarrow$ (convex) when $n_1 > n_2$

$$R = \left(\frac{n_1 - n_2}{n_2}\right)d$$



LENSES

Lenses are made of a transparent refracting medium bounded by curved surfaces or combination of curved and plane surfaces. Some examples are shown below:

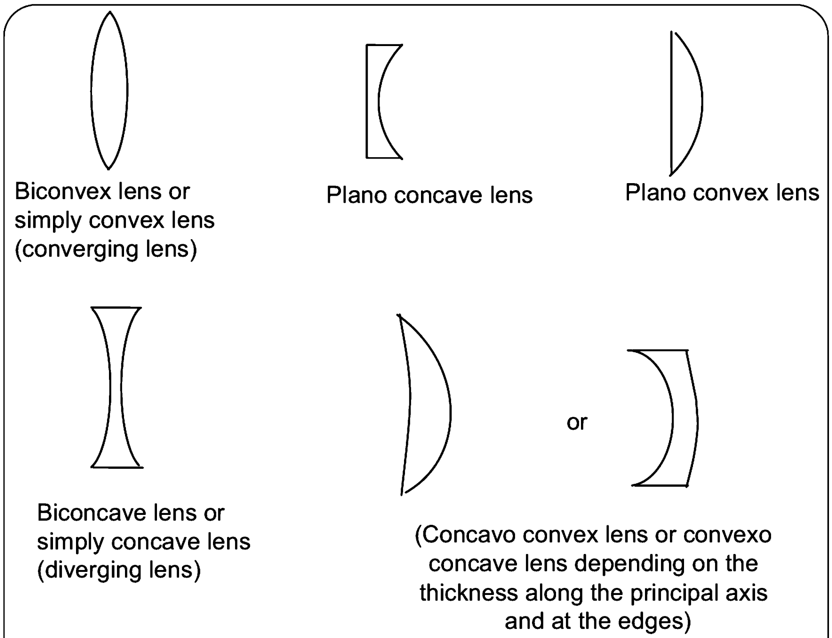


Fig. 1.43

We study only thin lenses so that we ignore the thickness of the lenses. (For thick lenses, we have to apply two-stage refractions).

The following are to be noted:

There are two centres of curvature, two radii of curvature and principal focus at which a parallel beam of light meets or appears to meet after refraction. Optic centre is the point on the principal axis within or outside the lens through which a refracted ray goes undeviated.

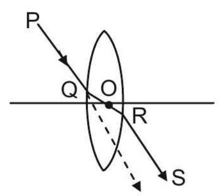


Fig. 1.44

Rules for image formation by a lens

(a) A ray passing through the optic centre of the lens proceeds undeviated through the lens.
(b) A ray passing parallel to the principal axis, after refraction through the lens, passes or appears to pass through the focus.
(c) A ray through the focus or directed towards the focus of the lens, after refraction through the lens, becomes parallel to the principal axis.

It may be noted that only two rays from the same point of an object are needed to locate the image of the object. If these two rays, after refraction through the lens, intersect at a point, a real image of the object is formed at the point of intersection. If the two rays appear to intersect at a point (after refraction from lens), the image of the object is virtual.

Thin lens formula

A lens whose thickness along its axis is negligible compared to the radii of curvature of its surfaces is called a thin lens. A thin lens is placed between two media of refractive indices n_1 and n_3. Consider a ray passing from medium 1 of refractive index n_1 through lens 2 of refractive index n_2 to medium 3 of refractive index n_3.

We study the image formation, for a convex lens.

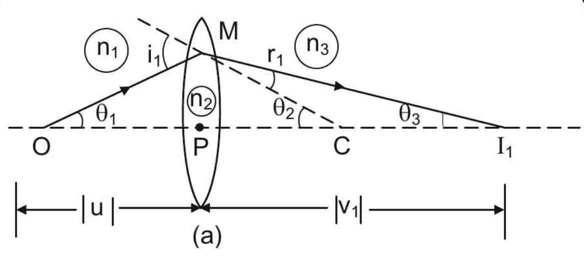


Fig. 1.45

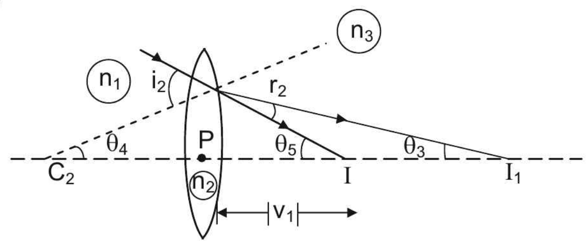


Fig. 1.46

Ray from the obejct at O gets refracted at the first surface, giving intermediate image I_1; I_1 is virtual object for refraction at the second surface, giving final image I at v, shown in Fig. 1.45 and Fig. 1.46. It may be noted that the small deviation of the ray is ignored due to the thinness of the lens.

So, for the refraction at the first surface,

$$n_1 i = n_2 r_1$$

$$\Rightarrow \quad n_1(\theta_1 + \theta_2) = n_2(\theta_2 - \theta_3)$$

$$\Rightarrow \quad n_1(PM)\left(\frac{1}{PO} + \frac{1}{PC_1}\right) = n_2(PM)\left(\frac{1}{PC_1} - \frac{1}{PI_1}\right) \quad \text{— (1)}$$

For refraction at the second surface,

$$n_2 i_2 = n_3 r_2$$

$$\Rightarrow \quad n_2(\theta_4 + \theta_3) = n_3(\theta_4 + \theta_5)$$

$$\Rightarrow \quad n_2(PM)\left(\frac{1}{PC_2} + \frac{1}{PI_1}\right) = n_3(PM)\left(\frac{1}{PC_2} + \frac{1}{PI}\right) \quad \text{— (2)}$$

Adding equations, (1) and (2)

$$n_2\left(\frac{1}{PC_1} + \frac{1}{PC_2}\right) = n_3\left(\frac{1}{PC_2} + \frac{1}{PI}\right) + n_1\left(\frac{1}{PO} + \frac{1}{PC_1}\right)$$

$$\Rightarrow \quad \frac{n_3}{PI} + \frac{n_1}{PO} = \frac{(n_2 - n_1)}{PC_1} - \frac{(n_3 - n_2)}{PC_2}$$

According to the sign convention,

$PI = v$

$PO = -u$

$PC_1 = R_1$

$PC_2 = -R_2$

Therefore,

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

which is the thin lens formula. Recalling that $P_1 = \frac{n_2 - n_1}{R_1}$ and $P_2 = \frac{n_3 - n_2}{R_2}$, the powers of the two surfaces, respectively, we may write

$$\frac{n_3}{v} - \frac{n_1}{u} = P_1 + P_2$$

The same formula holds good for a concave lens

If $n_3 = n_1$, (need not be air) formula becomes

$$\frac{n_1}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_1 - n_2}{R_2}$$

$$\Rightarrow \frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)_{n_3 = n_1}$$

This can also be written as (by dividing both sides by n_1)

$$\frac{1}{v} - \frac{1}{u} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where $n_{21} = \frac{n_2}{n_1}$

If $n_1 = 1$ (air), then

$$\frac{1}{v} - \frac{1}{u} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Now take the general formula.

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

Put $u = -\infty$; then v should be = f. Substituting in the general formula

$$\frac{n_3}{f} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

Therefore, $\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_3}{f}$

If $n_3 = n_1 = 1$ (air)

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

This is called ***Lensmaker's formula*** and

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

is the ***thin lens formula***.

Power of a lens

The expression $\frac{n_2 - n_1}{R}$ is called the power of a refracting surface of a medium with refractive index n_2 and radius of curvature R, when light ray falls on the surface from medium with refractive index n_1.

The formula for a lens can be written as

$\frac{n_1}{f} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. Here $\frac{n_1}{f}$ stands for power ⇒

$$P = \frac{n_1}{f}$$

P and f, here, are for n_1 medium. So we can better remember the formula as

$$P_{n1} = \frac{n_1}{f_{n1}} = (n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

So power in a medium is always $\frac{n_{med}}{f_{med}}$, in air $P = \frac{1}{f_{air}}$.

Hence $\qquad P = P_1 + P_2$

P_1 is the power of the first stage refraction,
P_2 is the power of the second stage refraction.

The power is usually printed on the lens and it always stands for power in air (n = 1). (When problems simply give power (or focal length) without mentioning immersed medium, take it as in air).

If 'f' is the focal length of the lens in air and the absolute refractive index of lens material is n_l, when this lens is immersed in a medium of refractive index n_m and if the focal length of the lens inside this medium is f_m, then it can be shown that

$$f_m = \frac{f\left[n_m(n_\ell - 1)\right]}{(n_\ell - n_m)}$$

If $(n_l - n_m)$ is positive (i.e., $n_l > n_m$), the nature of the lens inside the medium is the same as the nature of lens in air. Example, Convex lens in air is similar to convex lens in medium. If $(n_l - n_m)$ is negative (i.e., $n_l < n_m$), the nature of the lens inside the medium is opposite to the nature of the lens in air. Example Convex lens in air is similar to concave lens in medium.

If P is the power of lens in air and P_m is the power of lens inside the medium, then

$$P_m = \frac{n_m}{f_m} = \frac{(n_\ell - n_m)}{(n_\ell - 1)}P$$

$$\text{i.e., } P_m = \frac{(n_\ell - n_m)}{(n_\ell - 1)}P$$

Linear magnification of lens (m)

Linear magnification of a lens, $m = \dfrac{\text{height of image}(h_i)}{\text{height of object}(h_0)}$

It can be shown that for lens,

$$m = \frac{v}{u}$$

v and u are usual with signs as per New Cartesian sign convention.

If m is positive, the image is erect.

If m is negative, the image is inverted

We have $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$ (Lens formula).

$\therefore$ We get

$$m = \frac{v}{u} = \frac{f}{(f+u)} = \frac{f-v}{f}$$

where u, v and f are to be used with proper signs. **For convex lens, f is always positive and for concave lens, f is always negative.**

This gives

$$u = \frac{f(1-m)}{m},$$

$$v = f(1-m) \text{ and}$$

$$f = \frac{mu}{(1-m)} = \frac{v}{(1-m)}$$

where f, u, v and m are to be used with proper signs as per New Cartesian sign convention.

If only the magnitudes of u, v, f and m are considered (without bothering about their signs), then we get

$m = \dfrac{v}{u}$ and $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$ [Lens formula for magnitudes alone]

$\Rightarrow m = \dfrac{v}{u} = \dfrac{f}{|u-f|} = \dfrac{|v-f|}{f}$, signs not considered for u, v, f and m.

$\Rightarrow u = \dfrac{f(m+1)}{m}$, $v = f(1+m)$ and

$f = \dfrac{mu}{(m+1)} = \dfrac{v}{(1+m)}$, signs of u, v, f and m not taken into consideration.

Summary of the properties of images of real objects formed by lenses

A. Convex lens (Medium on either side of lens is taken as same)

Sl. No	Position of object	Position of image	Size of image	Nature of image
1.	At infinity	At focus F on other side	Point size	Real and inverted
2.	Beyond 2 F	Between F and 2 F on the other side	Smaller than object size	- do -
3.	At 2F	At 2F on the other side	Same size as object	- do -
4.	Between F and 2F	Beyond 2F on the other side	Larger than object size	- do -
5.	At F	At infinity	Highly enlarged	- do -
6.	Between F and optic centre	On the same side as object	Larger than object size	Virtual and erect

B. Concave lens (Medium on either side of lens is taken as same)

Sl. No	Position of image	Position of image	Size of image	Nature of image
1.	Anywhere (for real objects)	On the same side as object in between F and optic centre of lens	Smaller than the object	Virtual and erect
2	At infinity	At F on the same side	Highly diminished	Virtual and erect

Note:

If the medium on either side of the lens is not same, a given lens, made of a single material, will have two focal lengths, one for each medium.

CONCEPT STRANDS

Concept Strand 26

A thin converging lens of focal length 25 cm forms an image on a screen at 5 m from the lens. The screen is brought closer to the lens by 18 cm. By what distance should the object be shifted away from/towards the lens so that its image on the screen is sharp again?

Solution

The object should be moved away. The object is always between f and 2f ($\because$ image is between 2f and ∞ in both cases). Let us apply the formula and verify.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow u = \frac{fv}{f-v} = \frac{(+25)(+500)}{(25-500)}$$

$$\Rightarrow \quad u = \frac{-500}{19} = -26.32 \text{ cm}$$

$$u' = \frac{(+25)(+482)}{(25-482)} = -26.37 \text{ cm}$$

$\therefore$ $\Delta u = u' - u \approx -0.5$ mm (should be moved away by $\approx$ 0.5 mm)

Δu being small, we could have applied approximation as below.

Considering Δu to be small, differentiate $u = \dfrac{fv}{f-v}$

$$\Rightarrow \quad du = \frac{f^2}{(f-v)^2}dv$$

$$= \frac{25^2.}{(-475)^2}(-18) \approx -0.5 \text{ mm}$$

Concept Strand 27

A source and a screen are fixed at a distance ℓ from each other. A thin lens is placed between them such that the image is focussed on the screen . Determine

(i) the focal length of the lens
(ii) the position of the lens.

Solution

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Writing $v = \left|\ell - |u|\right| \Rightarrow \dfrac{1}{+\ell - |u|} - \dfrac{1}{-|u|} = \dfrac{1}{f}$

$$\Rightarrow \quad \frac{1}{\ell - |u|} + \frac{1}{|u|} = \frac{1}{f}$$

$$\Rightarrow \quad |u|^2 - \ell|u| + f\ell = 0$$

$$\Rightarrow \quad |u| = \frac{\ell}{2}\left[1 \pm \sqrt{1 - \frac{4f}{\ell}}\right]$$

Case (i)

If $1 - \dfrac{4f}{\ell} < 0 \Rightarrow f > \dfrac{\ell}{4}$,

then $|u|$ is not real which is not possible $\therefore$ $f \le \dfrac{\ell}{4}$

Case (ii)

If $f = \dfrac{\ell}{4}$, then $|u| = \dfrac{\ell}{2}$

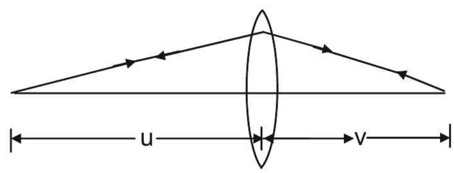
Case (iii)

If $f < \dfrac{\ell}{4}$, then $|u|$ has two values

$$\frac{\ell}{2}\left[1 + \sqrt{1 - \frac{4f}{\ell}}\right]$$

$$\text{and} \quad \frac{\ell}{2}\left[1 - \sqrt{1 - \frac{4f}{\ell}}\right]$$

you will find that if $|u|$ is one of them, v is the other value i.e., the values of $|u|$ and v are interchangable.



If $u = = \dfrac{\ell}{2}, v = \dfrac{\ell}{2}$,

the equation $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$ gives the position of the focus, $f = \dfrac{\ell}{4}$

Concept Strand 28

An object and a screen are fixed and a lens is being moved from one point along the axis to another resulting in sharp images of heights h_1 and h_2. What is the object height?

Solution

Let height of object = h

$$h_1 = h\frac{v}{|u|}$$

$$h_2 = h\frac{|u|}{v}$$

$$\Rightarrow \quad h^2 = h_1h_2 \Rightarrow h = \sqrt{h_1h_2}$$

Concept Strand 29

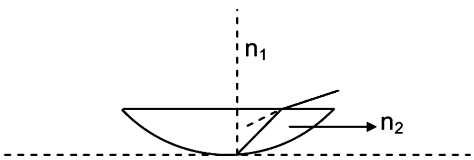
A plano convex lens of thickness 5 cm is kept on a flat table with its curved surface in contact with the table. The apparent depth of the bottom point is 4 cm. When the lens is reversed, the depth is $\frac{100}{21}$ cm.

What is its focal length?

Solution

Let the refractive index of lens be n_2, that of medium n_1 (need not be air). We cannot use lens formula because thickness of lens is very large.

Case (i)



You can use either apparent depth formula or spherical surface formula.

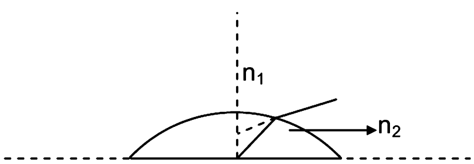
Application of depth formula:

$$4 = \frac{5}{n_{21}} \Rightarrow n_{21} = \frac{5}{4}$$

(or) spherical surface formula: (R = ∞, for plane)

$$\frac{1}{-4} - \frac{n_{21}}{-5} = \frac{1-n_{21}}{\infty} \Rightarrow n_{21} = \frac{5}{4}$$

Case (ii)



(Here we cannot use apparent depth formula because interface is not plane)

$$\frac{1}{-100/21} - \frac{n_{21}}{-5} = \frac{1-n_{21}}{R}$$

$$\Rightarrow \quad \frac{-21}{100} + \frac{1}{4} = \frac{-1}{4R} \Rightarrow R = \frac{-100}{16}\text{ cm}$$

$$\frac{1}{f} = (n_{21}-1)\left(\frac{1}{\infty} - \frac{1}{R}\right)$$

$$= \left(\frac{5}{4} - 1\right)\left(\frac{16}{100}\right) \Rightarrow f = 25\text{ cm}$$

Concept Strand 30

Given a convex lens of focal length f, find the minimum distance between object and screen, below which a real image on the screen cannot be obtained, wherever the lens is kept?

Solution

If the distance between object and screen is ℓ, we saw that formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ leads to the equation

$$|u|^2 - \ell|u| + \ell f = 0$$

$\Rightarrow$ Discriminant $\ell^2 - 4f\ell$ should be $\geq 0 \Rightarrow \ell \geq 4f$

$\Rightarrow$ $\ell_{min} = 4f$.

Concept Strand 31

The power of a lens (n = 1.5) is + 5.0 D. Find its power when immersed in water $\left(n = \frac{4}{3}\right)$.

Solution

$$P = \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad +5 = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{— (1)}$$

$$\left[f = \frac{1}{+5} = +0.20\text{ metre}\right]$$

$$P_w = \frac{n_w}{f_w} = (n-n_w)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{— (2)}$$

$$\frac{(2)}{(1)} \Rightarrow \frac{P_w}{+5} = \frac{1.5 - \frac{4}{3}}{1.5 - 1} \Rightarrow P_w = \frac{+5}{3} D$$

$$\left[f_w = \frac{n_w}{P_w} = \frac{4/3}{5/3} = 0.80 \text{ m} \right]$$

Concept Strand 32

In the above problem, if the two surfaces of the lens are of equal radius (i) what is the radius of curvature? (ii) what is its power with the lens kept with air on one side and water on the other side?

Solution

The formula is

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

By putting $u = -\infty$ and $v = f$,

$$\frac{n_3}{f} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$$

and if $n_3 = n_1 = 1$,

$$\frac{1}{f} = (n_2 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = (n_2 - 1)\frac{2}{R}$$

(i) Now take the data for $n_3 = n_1 =$ air

$$+5 = \frac{1}{f} = \frac{2(1.5 - 1)}{R} = \frac{1}{R} \Rightarrow R = f = 0.2 \text{ m}$$

(ii) Using data for one side air and other side water:

$$P' = \frac{4/3}{f'} = \frac{1.5 - 1}{0.2} + \frac{\frac{4}{3} - 1.5}{-0.2}$$

$$\Rightarrow P' = +\frac{10}{3} D \text{ focal length } f' = 0.4 \text{ m}$$

Displacement method to determine the focal length of a convex lens

The relation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ shows that the positions u and v are interchangeable (due to principle of reversibility of light). These positions of object and image are called conjugate positions.

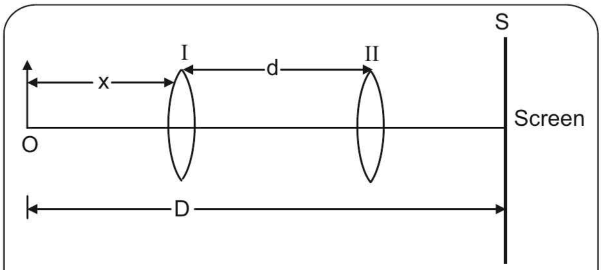


Fig. 1.47

Consider a linear object O, of height h_0, fixed at a position and a screen S, separated from the object by a distance D as shown in Fig. 1.47. A convex lens of focal length f, placed at location I, produces a real image of the object on the screen and the height of image is h_1. Now the lens is moved along the principal axis by a distance 'd' to position II, where again a real image of height h_2 is formed at the same location of screen. The positions I and II of the lens are called the conjugate positions.

Linear magnification for position I,

$$m_1 = \frac{h_1}{h_0} = \frac{(D - x)}{x} \quad \text{----- (i) (only magnitude)}$$

Linear magnification for position II,

$$m_2 = \frac{h_2}{h_0} = \frac{D - (d + x)}{(d + x)} \quad \text{----- (ii) (only magnitude)}$$

We have $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ (for only magnitudes for a lens)

$$\Rightarrow \frac{1}{f} = \frac{1}{(D - x)} + \frac{1}{x} = \frac{1}{D - (x + d)} + \frac{1}{(x + d)} \quad \text{----- (iii)}$$

$$\Rightarrow \frac{x + (D - x)}{(D - x)x} = \frac{(x + d) + [D - (x + d)]}{[D - (x + d)](x + d)}$$

$$\Rightarrow \frac{D}{(D - x)x} = \frac{D}{[D - (d + x)](d + x)}$$

$\Rightarrow$ $(D - x)\,x = [D - (d + x)]\,(d + x)$

$\Rightarrow$ $Dx - x^2 = Dd + Dx - d^2 - x^2 - 2xd$

$\Rightarrow$ $0 = Dd - d^2 - 2xd$

$\Rightarrow \quad 2xd = d[D - d]$

$$\Rightarrow \quad x = \frac{(D - d)}{2} \qquad \text{------ (iv)}$$

Substituting the value of x from (iv) in (iii), we get

$$\frac{1}{f} = \frac{1}{\left[D - \left(\frac{D-d}{2}\right)\right]} + \frac{1}{\left(\frac{D-d}{2}\right)}$$

$$= \frac{1}{\left(\frac{D+d}{2}\right)} + \frac{1}{\left(\frac{D-d}{2}\right)} = 2\left[\frac{1}{(D+d)} + \frac{1}{(D-d)}\right]$$

$$= 2\left[\frac{D - d + D + d}{D^2 - d^2}\right] = \frac{4D}{D^2 - d^2}$$

$$\Rightarrow \quad f = \frac{D^2 - d^2}{4D}$$

using the value of x from (iv) and (i), we get

$$m = \frac{D - \left(\frac{D-d}{2}\right)}{\left(\frac{D-d}{2}\right)} = \frac{(D+d)}{(D-d)}$$

$$\therefore \quad m_1 = \frac{(D+d)}{(D-d)}$$

Using the value of x from (iv) in (ii), we get

$$m_2 = \frac{(D-d)}{(D+d)}$$

$$\therefore \quad m_1 m_2 = \frac{(D+d)}{(D-d)} \cdot \frac{(D-d)}{(D+d)} = 1$$

$$\therefore \quad m_1 m_2 = 1$$

$$\therefore \quad m_1 m_2 = \frac{h_1}{h_0} \times \frac{h_2}{h_0} = \frac{h_1 h_2}{h_0^2} = 1$$

$$\therefore \quad h_0 = \sqrt{h_1 h_2}$$

Also

$$\frac{d}{m_1 - m_2} = \frac{d}{\left(\frac{D+d}{D-d}\right) - \frac{(D-d)}{(D+d)}} = \frac{d(D+d)(D-d)}{(D+d)^2 - (D-d)^2}$$

$$= \frac{d(D^2 - d^2)}{4Dd} = \frac{D^2 - d^2}{4D} = f$$

$$\therefore \qquad f = \frac{d}{m_1 - m_2} = \frac{D^2 - d^2}{4D}$$

It must be noted that throughout the above derivation, we have used only the magnitudes (and no signs as per Cartesian sign convention)

Combination of lenses

Consider two thin convex lenses of focal lengths f_1 and f_2, placed in juxtaposition as shown in Fig. 1.48. Refraction at the first lens gives

$$\frac{1}{v'} - \frac{1}{u} = \frac{1}{f_1} \qquad \text{—(1)}$$

The optic centres of the two lenses are treated to be at the same point. Then, refraction at the second lens gives

$$\frac{1}{v} - \frac{1}{+v'} = \frac{1}{f_2} \qquad \text{—(2)}$$

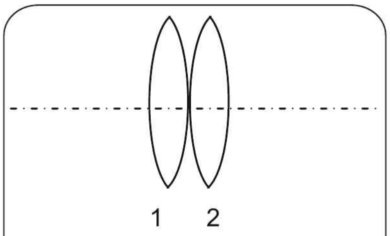


Fig. 1.48

(v' calculated from equation (1) can be anything. If positive, it will serve as virtual object for lens 2. If negative, it is real object for lens 2. Equation (2) is correct for either case)

$$\therefore \quad (1) + (2) \Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

If equivalent focal length (of the system) is f, then,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$\Rightarrow$

$$P = P_1 + P_2$$

Whatever be the type of each of the lenses,

$$m = m_1 m_2$$

If several lenses are in contact

$$P = P_1 + P_2 + \ldots.$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \ldots..$$

$$m = m_1 m_2 m_3 \ldots$$

CONCEPT STRANDS

Concept Strand 33

A convex lens of focal length 40 cm is placed 20 cm away from another convex lens of focal length 50 cm. An object 2 cm tall is 60 cm in front of the first lens. Determine its image characteristics.

Solution

Refraction by the first lens:

$$\frac{1}{v'} - \frac{1}{-60} = \frac{1}{+40} \Rightarrow v' = +120 \text{ cm}$$

$$m_1 = \frac{+120}{-60} = -2$$

Refraction by the second lens:

$$u = 120 - 20 = +100$$

$$\frac{1}{v} - \frac{1}{+100} = \frac{1}{+50} \Rightarrow v = +\frac{100}{3} \text{ cm}$$

$$m_2 = \frac{+100/3}{+100} = +\frac{1}{3},$$

Total magnification is $m_1 m_2 = -\frac{2}{3}$

Final image is $\frac{100}{3}$ cm to the right of the second lens, is real and is inverted, $\frac{4}{3}$ cm tall.

Concept Strand 34

A convex lens of focal length 40 cm and a concave lens of focal length 30 cm are separated by a gap of 20 cm; object is 1 cm tall at 20 cm in front of the first (convex) lens. Describe image details.

Solution

At lens 1:

$$\frac{1}{v'} - \frac{1}{-20} = \frac{1}{+40}$$

$$\Rightarrow v' = -40 \text{ cm}$$

$$m_1 = \frac{-40}{-20} = +2$$

At lens 2:

$$u = -40 - 20 = -60$$

$$\frac{1}{v} - \frac{1}{-60} = \frac{1}{-30}$$

$$\Rightarrow \quad v = -20 \text{ cm}$$

$$m_2 = \frac{-20}{-60} = +\frac{1}{3}$$

$$m_1 m_2 = +\frac{2}{3}$$

Image: 20 cm to left of the second lens $\Rightarrow$ at the optic centre of the first lens, virtual, erect and $\frac{2}{3}$ cm tall.

Concept Strand 35

Two lenses one of which is crown glass with $\omega_1 = \frac{1}{30}$ and the other flint glass with $\omega_2 = \frac{1}{20}$. How can an achromatic combination, (No dispersion), of focal length 40 cm, be formed?

Solution

Let n_1, R_{11}, R_{12}, ω_1 and n_2, R_{21}, R_{22}, ω_2 represent the parameters of three lenses.

$$\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{R_{11}} - \frac{1}{R_{12}}\right) = (n_1 - 1)K_1$$

$$\frac{1}{f_2} = (n_2 - 1)K_2$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = (n_1 - 1)K_1 + (n_2 - 1)K_2$$

For no dispersion, $\frac{df}{d\lambda} = 0$

$$\Rightarrow \quad -\frac{1}{f^2}\cdot\frac{df}{d\lambda} = 0 = K_1\frac{dn_1}{d\lambda} + K_2\frac{dn_2}{d\lambda}$$

$$0 = \frac{1}{f_1(n_1 - 1)}\cdot\frac{dn_1}{d\lambda} + \frac{1}{f_2(n_2 - 1)}\frac{dn_2}{d\lambda}$$

$$\Rightarrow \quad \frac{dn_1}{f_1(n_1 - 1)} + \frac{dn_2}{f_2(n_2 - 1)} = 0$$

But $\omega_1 = \dfrac{dn_1}{n_1 - 1}$

and $\omega_2 = \dfrac{dn_2}{(n_2 - 1)}$

$$\therefore \quad \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

$$\Rightarrow \quad \frac{1}{30f_1} + \frac{1}{20f_2} = 0 \qquad \text{— (1)}$$

If the lenses are placed in juxtaposition, $\dfrac{1}{f} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

$$\Rightarrow \quad \frac{1}{40} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{— (2)}$$

Solving equations (1) and (2)

$f_1 = +\dfrac{40}{3}$ cm (convex)

$f_2 = -20$ cm (concave)

Combination of mirrors and lenses

One surface of a lens is silvered as shown in Fig. 1.49. Let the radii of the two surfaces be R_1, R_2. Let us determine the focal length of this silvered lens. Consider Fig. 1.50. A ray from infinity undergoes

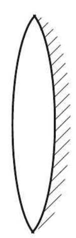


Fig. 1.49

(i) refraction at the first surface, giving image I_1,
(ii) reflection at the second surface, giving image I_2 (second surface is a spherical mirror) and then
(iii) refraction at the first surface again, giving image I, which is the focus for the system (silvered lens)

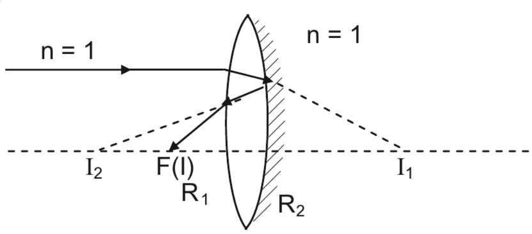


Fig. 1.50

Step (i) Refraction on surface 1: (Air to glass)

$$\frac{n}{v_1} - \frac{1}{\infty} = \frac{n-1}{R_1} \qquad \text{— (1)}$$

Step (ii) Reflection on a spherical mirror: (glass to glass)

$$\frac{n}{v_2} + \frac{n}{v_1} = \frac{2n}{R_2} \qquad \text{— (2)}$$

Step (iii) refraction on surface 1: (glass to air)

Now incoming direction is reversed. Sign convention is to remain, $(\xrightarrow[+]{})$

$$\frac{1}{-v} - \frac{n}{-v_2} = \frac{1-n}{-R_1} \qquad \text{— (3)}$$

Now eliminate v_1 and v_2 from the three equations by (1) – (2) + (3)

$$\Rightarrow \frac{1}{-v} = \frac{2(n-1)}{R_1} - \frac{2n}{R_2}$$

Now, since the parallel beam is finally focused at F, v = f = focal length of the silvered lens system.

$$\Rightarrow \quad \frac{1}{-f} = \frac{2(n-1)}{R_1} - \frac{2n}{R_2}$$

$$\Rightarrow \quad \frac{1}{f} = \frac{2n}{R_2} - \frac{2(n-1)}{R_1}$$

$$= \frac{2}{R_2} + \frac{2(n-1)}{R_2} - \frac{2(n-1)}{R_1}$$

$$= \frac{2}{R_2} - 2(n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{R_2/2} - 2\left[(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right]$$

$$\frac{1}{f_{system}} = \frac{1}{f_{mirror}} - \frac{2}{f_{lens}}$$

with sign convention $\xrightarrow{+}$

If f_{system} turns out to be negative, we say that the silvered lens acts as if it is a concave mirror of focal length = f_{system} If it turns out positive, it acts like a convex mirror of focal length = f_{system}

CONCEPT STRANDS

Concept Strand 36

A convex lens (n = 1.5) has radii 40 cm and 60 cm. The 40 cm surface is silvered. What is the focal length of this lens?

Solution

$$\frac{1}{f_{lens}} = (1.5-1)\left(\frac{1}{+60} - \frac{1}{-40}\right) \Rightarrow \frac{1}{f_{lens}} = +\frac{5}{240}$$

$$\frac{1}{f_{mirr}} = \frac{1}{-40/2} = -\frac{1}{20}$$

$$\therefore \quad \frac{1}{f_{silv.lens}} = \frac{1}{f_{mir}} - \frac{2}{f_{lens}} = -\frac{1}{20} - \frac{10}{240} = -\frac{22}{240}$$

$$\Rightarrow \quad f_{silv.lens} = -\frac{120}{11} \text{ cm}$$

$\Rightarrow$ behaves like a concave mirror of $f = -\dfrac{120}{11}$ cm

Concept Strand 37

The convex side of a plano-convex lens (n = 1.5) with R = 60 cm is silvered. An object 2 cm tall is located 25 cm in front of this. Describe image details.

Solution

Obviously, if the silvered side faces this object, it does not produce an image.

$$\frac{1}{f_{lens}} = (1.5-1)\left(\frac{1}{\infty} - \frac{1}{-60}\right) = \frac{1}{120}$$

$$\frac{1}{f_{mirr}} = \frac{1}{-60/2} = -\frac{1}{30}$$

$$\therefore \quad \frac{1}{f_{sys}} = -\frac{1}{30} - \frac{2}{120} = -\frac{1}{20}$$

$$\Rightarrow \quad f_{sys} = -20 \text{ cm}$$

$\therefore$ The system behaves like a concave mirror with f = −20 cm

$$\therefore \quad \frac{1}{v} + \frac{1}{-25} = \frac{1}{-20}$$

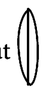
$$v = -100 \text{ cm}$$

negative sign for a mirror indicates real image.

$$m = -\frac{v}{u} = \frac{+100}{-25} = -4$$

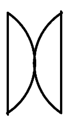
The image is inverted, with height = 8 cm

Modified lenses

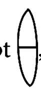
(i) A lens  of focal length f is cut  into two halves and kept in contact as shown: 

The focal length now has the same value, f.

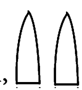
(ii) The focal length of each half is not $\dfrac{f}{2}$ but 2f because each part has a focal length f' given by $\dfrac{1}{f'} = (n-1)\left(\dfrac{1}{R} - \dfrac{1}{\infty}\right) = \dfrac{(n-1)}{R} = \dfrac{1}{2f}$.

(iii) If kept in contact, but the other way  the focal length is f because $\dfrac{1}{f'} = \dfrac{1}{2f} + \dfrac{1}{2f} = \dfrac{1}{f}$

(iv) If kept as shown  the focal length is again f.

(v) If cut as shown and kept , the focal length f is that of the uncut lens; the focal length of each half  is f and the intensity of image due to only one part will be half of the intensity of the image of original lens, because the area exposed to incident rays is one half of that of the full lens.

(vi) If the two halves are kept with a small gap  the focal length is f, but the contrast of the image will be reduced.

(vii) If the two halves are kept as shown,  the resultant focal length will be $\left(\dfrac{1}{f'} = \dfrac{1}{f} + \dfrac{1}{f}\right) \Rightarrow f' = \dfrac{f}{2}$.

OPTICAL INSTRUMENTS (FOR ADDITIONAL READING)

The eye

The normal eye is almost spherical in shape. The retina acts like the screen. Light enters the eye through a transparent fibrous medium called cornea and falls on the eye lens. The lens of the eye is made of a fibrous jelly of average refractive index 1.437. Behind the lens the eye is filled with a jelly called vitreous humour of refractive index about 1.336 nearly equal to that of water. Distance between the lens and the retina is approximately 2.5 cm. (Fig. 1.51)

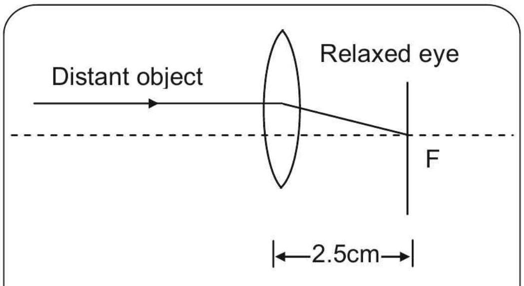


Fig. 1.51

An object at infinity is focussed on the retina. *Ciliary muscles* which help adjust the focal length of the eye lens is relaxed when the object is at infinity. Focal length for relaxed eye = 2.5 cm. When the object is brought closer to the eye the ciliary muscles adjust the focal length of the eye lens (*accommodation*) so that the lens forms a distinct image on the retina.

But the power of accommodation is limited. The closest distance for clear vision is called the *least distance of distinct vision* (D) which is taken as 25 cm for persons with normal eye. If the object is closer than this, the image is blurred. We can calculate f_{min} (strained eye) by writing

$$\frac{1}{f_{min}} = \frac{1}{2.5} - \frac{1}{-D} = \frac{1}{2.5} + \frac{1}{25}$$

$$f_{min} = \frac{25}{11} = 2.27 \text{ cm}$$

A normal eye cannot adjust its focal length to a value less than f_{min}.

Defects of vision

Due to age, the ciliary muscles of the eye are no longer efficient in accommodation and the eye may develop defects of vision. *Myopia* is a defect of vision due to which objects near the eye are clearly visible while distant objects appear blurred. The blurring occurs because the eye lens produces the image somewhat in front of the retina. To rectify the defect, a concave lens of appropriate focal length is interposed between the object and the eye. The image is now properly focused on the retina.

The opposite case is when the eye can see objects clearly at a distance but not those close to the eye. This defect is known as *hypermetropia*. In this case, the image is produced behind the retina. For properly focusing the object on the retina, a convex lens is used.

CONCEPT STRANDS

Concept Strand 38

What focal length should the reading spectacles have for a person whose D value is 40 cm?

Solution

D value > 25 cm

$\Rightarrow$ hypermetropia

$\because$ at u = −40 cm, his v = +2.5 cm

$\Rightarrow$ his F_{min} is given by

$$\frac{1}{2.5} - \frac{1}{-40} = \frac{1}{f_{min}} \Rightarrow f_{min} = \frac{40}{17}\text{cm}$$

$= 2.35$ cm as against $\frac{25}{11} = 2.27$ cm for a normal person. $\Rightarrow$ This requires a convex lens so that

$$\frac{1}{40/17} + \frac{1}{f_{lens}} = \frac{1}{25/11} \Rightarrow f_{lens} = +\frac{200}{3}\text{cm.}$$

The same question can be alternatively posed by asking what kind of lens should give a virtual image at 40 cm, for an object at 25 cm?

$$\frac{1}{-40} - \frac{1}{-25} = \frac{1}{f_{lens}} \Rightarrow f_{lens} = +\frac{200}{3}\text{ cm}$$

Concept Strand 39

A person cannot see beyond 5 m. How can his vision be corrected?

Solution

His far point is now 5 m. It should be made ∞. Object at infinity should give a virtual image at 5 m. i.e., for him the object should appear to be at 5 m. Therefore, $\frac{-1}{5} - \frac{1}{-\infty} = \frac{1}{f}$.

A concave lens is required and it should have a focal length $= -5$ m.

$\Rightarrow$ i.e., $P_{lens} = -0.2$ D

Concept Strand 40

The near point and far point of a person are 40 cm and 400 cm, respectively. He wants to read a book kept at 25 cm. Find the focal length of the lens required and the distance upto which objects can be viewed.

Solution

Put $u = -25$,

$v = -40$

$$\frac{1}{f_{lens}} = \frac{1}{-40} - \frac{1}{-25} \Rightarrow f_{lens} = +\frac{200}{3} \text{ cm}$$

With this lens at what distance an object will give virtual image at his far point of 400 cm?

$$\frac{1}{-400} - \frac{1}{u} = \frac{1}{+200/3}$$

$$\Rightarrow \quad u = -\frac{400}{7} \text{ cm}$$

Visual angle

The size of an object sensed by us is related to the size of the image formed on the retina. The size of the image on the retina is roughly proportional to angle subtended by object at the eye, θ, called visual angle. (Fig. 1.52) Optical instruments increase this angle.

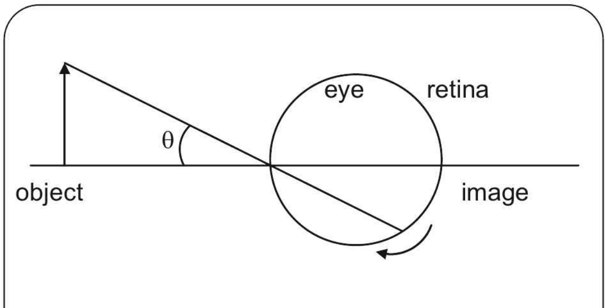


Fig. 1.52

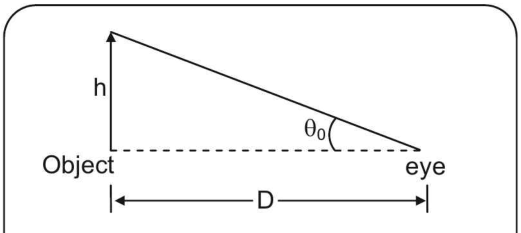


Fig. 1.53

Simple microscope (magnifier)

When we view an object at the near point D with the naked eye, the angle subtended by the object at the eye is the maximum, (Fig. 1.53). $\theta_0 = \frac{h}{D}$

Suppose we place a converging lens (convex lens) of short focal length (f less than D), immediately in front of the eye and place the object at a point between the lens and the focus F, the lens produces a virtual image at a large distance as shown in Fig. 1.54.

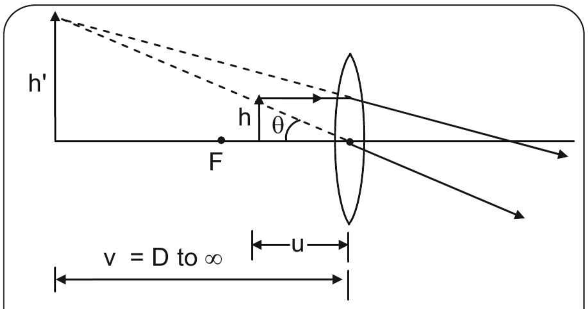


Fig. 1.54

The size of the image is large, and it is comfortable to view. The angle subtended by the image at the eye (which is at the lens) = $\theta' = \frac{h}{u}$ which is $>\theta_0$ because u < D.

If the object is moved to the focus, the image is at infinity having infinite size. This is called normal adjustment. At this point, the angle subtended by the image is $\theta = \frac{h}{f}$.

We define the *magnifying power* of a simple microscope as

$$m = \frac{h/f}{h/D} = \frac{D}{f} = \frac{\theta}{\theta_0}$$

which is also known as *angular magnification*. This magnifying power is 'at normal adjustment' i.e., when image is at ∞ and it is comfortable to view. Therefore, the eye is relaxed.

The angular magnification of a magnifying lens is the ratio of the angle subtended by the image produced by the lens with the angle subtended by the object at the near point of the observer.

Suppose you place the object between F and the pole as in Fig. 1.55, such that you can still view it clearly, but with a strained eye. θ will be larger and $\theta = \frac{h}{u}$. Magnification m will be larger. ($\because$ image is at near point; you cannot make image closer than this because you cannot view it)

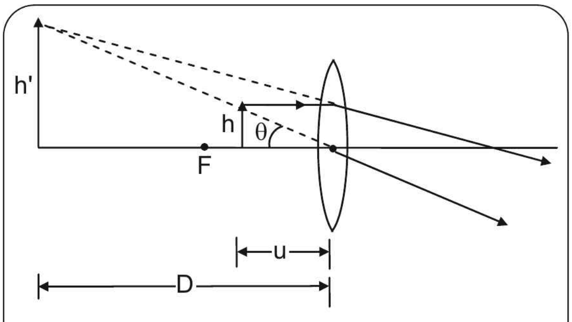


Fig. 1.55

$$\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f} \Rightarrow \frac{D}{u} = \frac{D}{f} + 1$$

$$\therefore \quad \text{Maximum } m = \frac{\theta_{max}}{\theta_0} = \frac{h/u}{h/D} = \frac{D}{u} = \frac{D}{f} + 1 \Rightarrow$$

$$\text{At normal adjustment, } m = \frac{D}{f}$$

At maximum adjustment (image at near point)

$$m = 1 + \frac{D}{f}$$

The equations also show that if f of the magnifier is small, m is large.

Compound microscope

The compound microscope consists of two converging lenses as shown in Fig. 1.56. The lens of focal length f_o facing the object is called *objective* or *field lens* while the lens of focal length f_e facing the eye is called the *eye piece* or *ocular*. Objective has a smaller focal length than the eye piece.

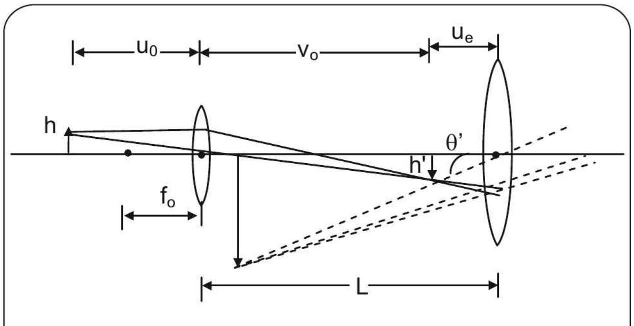


Fig. 1.56

Case (i)

Object is placed at u_o from objective (u_o is slightly > f_o) so that real inverted magnified image is formed at v_o. This serves as object for the eye piece.

For normal adjustment, the position of eye piece is adjusted so that the image of the object is at focal plane of the eye piece. The final image is formed at infinity for comfortable viewing.

Case (ii)

The eye piece is positioned so as to get final image at near point.

Ray diagram is shown for this near point. (Fig. 1.54)

Magnifying power:

θ_0 = maximum visual angle for unaided eye = $\frac{h}{D}$ as per usual definition.

θ' = Angle subtended by final image at eye piece (hence eye) = $\frac{h'}{u_e}$

$$m = \frac{\theta'}{\theta_0} = \frac{h'}{u_e} \times \frac{D}{h} = \frac{h'}{h} \times \frac{D}{u_e}$$

Note that $\frac{h'}{h} = \frac{v_o}{u_o}$ and also $\frac{D}{u_e}$ = magnifying power of eye piece treated as simple microscope and is equal to $\frac{D}{f_e}$ for normal adjustment and is equal to $1 + \frac{D}{f_e}$ for maximum.

$\therefore$ m of compound microscope

$$m = \frac{v_o}{u_o}\left[\frac{D}{f_e}\right] \text{ (normal) and}$$
$$= \frac{v_o}{u_o}\left[1 + \frac{D}{f_e}\right] \text{ (maximum)}$$

Now to calculate $\frac{v_o}{u_o}$:

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

$$\Rightarrow \quad \frac{v_o}{u_o} = 1 - \frac{v_o}{f_o} \approx \frac{-v_o}{f_o},$$

because the focal length of the objective is kept very small so that $\frac{v_o}{f_o} >> 1$. L is the distance between objective and eye piece and is equal to the length of the tube, f_o is very small and f_e also is small (but > f_o) compared to L

$\Rightarrow \quad v_o \approx L$

$$\therefore \quad \frac{v_o}{u_o} \approx -\frac{v_o}{f_o} = \frac{-L}{f_o} \Rightarrow$$

$$m = -\frac{L}{f_o}.\frac{D}{f_e} \text{ (normal adjustment)}$$

$$m = \frac{-L}{f_o}\left[1 + \frac{D}{f_e}\right] \text{ (maximum magnifying power)}$$

The negative sign appears because the final image is inverted.

Astronomical telescope

In the astronomical telescope, the objective is of large focal length and, large aperture. The eye piece is of small focal length and small aperture. Fig. 1.57

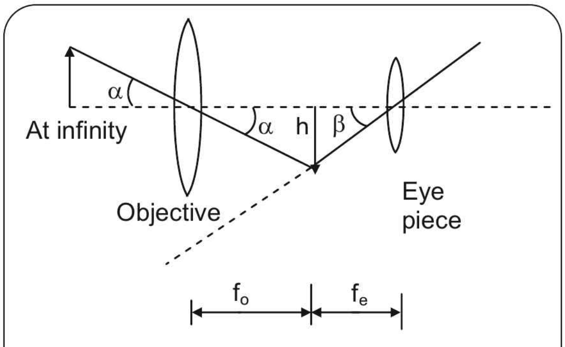


Fig. 1.57

$$m = \frac{\beta}{\alpha} = \frac{f_o}{f_e}$$

where, β, α, f_e and f_o are as shown in Fig. 1.56

Resolving Power

By the term *resolving power*, we mean the ability of an optical instrument to produce separate images of objects very close together. But whenever a parallel beam of light passes through a small aperture, it produces a *diffraction* pattern which consists of a central maximum and subsidiary maxima around the central maximum of much smaller intensity. (In a telescope or a microscope, the objective behaves like a small circular aperture. In principle, we could calculate the intensity at any point P in the diffraction pattern by dividing the area of the aperture into small (imaginary) circular elements and finding the resultant intensity by integration. This is obviously quite tedious.) If the separation between two objects is less than the width of the central maximum, the two objects cannot be resolved.

We quote the results here obtained by Sir George Airy, the English astronomer. The angle θ (see Fig. 1.58) which represents the angular radius of each dark ring is given by

$$\sin\theta_1 = 1.22\,\frac{\lambda}{D}$$
$$\sin\theta_3 = 2.33\,\frac{\lambda}{D}$$
$$\sin\theta_5 = 3.24\,\frac{\lambda}{D}$$

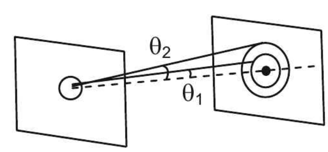


Fig. 1.58

Where, λ the wavelength and D is the aperture diameter. The bright rings are given by the relations

$$\sin\theta_2 = 1.63\,\frac{\lambda}{D}$$
$$\sin\theta_4 = 2.68\,\frac{\lambda}{D}$$
$$\sin\theta_6 = 3.70\,\frac{\lambda}{D}$$

The central bright spot is called the *Airy disc*. The intensities in the bright rings drop off very fast with angle θ.

The peak intensity of the first ring is 1.7% of its value at the centre of the Airy disc; the peak intensity of the second ring is only 0.4%.

Images formed by the objective lens of a telescope are not just a single spot, but a diffraction pattern as described above. The images of two close objects are two diffraction patterns close together which superpose, one on the other, making the images indistinguishable.

Two point objects are distinguishable if the centre of one diffraction pattern coincides with the first minimum of the other. This is known as *Rayleigh's criterion*, after the English Physicist Lord Rayleigh . The condition demands that two point objects are resolvable only if the angular separation of the centres of the Airy discs is given by

$$\sin\theta = 1.22\frac{\lambda}{D}$$

According to the above formula, the resolving power improves with shorter wavelength and larger diameter of the aperture.

CONCEPT STRAND

Concept Strand 41

A very large array radio telescope of NASA has a special arrangement of reflectors which gives an effective aperture of 36 km diameter. What is the limit of resolution? ($\lambda = 10^{-2}$m)

Solution

$$\sin\theta \approx \theta = \frac{1.22 \times 10^{-2}}{36 \times 10^{3}}$$
$$= 3.4 \times 10^{-7} \text{ rad.}$$

SUMMARY

For Mirrors

$d = 180 - 2i$
$= 360 - 2\theta$

$d \rightarrow$ angle of deviation when a ray is reflected from a plane mirror
$i \rightarrow$ angle of incidence
$\theta \rightarrow$ glancing angle (Angle between mirror and incident ray)

$$f = \frac{R}{2}$$

$f \rightarrow$ focal length of a spherical mirror
$R \rightarrow$ radius of curvature of the spherical mirror

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Mirror formula
$u \rightarrow$ object distance
$v \rightarrow$ image distance
$f \rightarrow$ focal length

$$m = \frac{h_i}{h_0} = \frac{-v}{u}$$

$m \rightarrow$ magnification in a spherical mirror
$h_i \rightarrow$ height of image
$h_0 \rightarrow$ height of object
All values to be used with signs as per New Cartesian sign convention.

$$m = \frac{f}{f-u} = \frac{f-v}{f}$$

$m \rightarrow$ magnification in a spherical mirror
All values to be used with signs as per New Cartesian sign convention.

$$m_A = \frac{A_i}{A_0} = m^2$$

$m_A \rightarrow$ areal magnification
$A_i \rightarrow$ area of image
$A_0 \rightarrow$ area of object

Refraction (General)

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

$i \rightarrow$ angle of incidence
$r \rightarrow$ angle of refraction
$\mu_2 \rightarrow$ refractive index of medium on which light falls
$\mu_1 \rightarrow$ refractive index of medium from which light falls

$$\mu = \frac{c}{v} = \frac{\lambda_{vac}}{\lambda_{med}}$$

$\mu \rightarrow$ refractive indeed of a medium
$c \rightarrow$ velocity of light in vacuum
$v \rightarrow$ velocity of light in medium
$\lambda_{med} \rightarrow$ wavelength of light in medium
$\lambda_{vac} \rightarrow$ wavelength of light in vacuum

$$S = t\left(1 - \frac{1}{\mu}\right)$$

$S \rightarrow$ shift produced when object is in the denser medium and observer is in the rarer medium.
$t \rightarrow$ actual depth

$$S = t\,(\mu - 1)$$

$S \rightarrow$ shift when object in rarer medium and observer in denser medium

$$\mu = \frac{t}{t'}$$

When object is in denser medium

$$\mu = \frac{t'}{t}$$

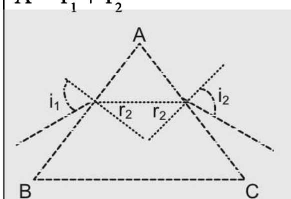
When object is in rarer medium
$t \rightarrow$ actual depth
$t' \rightarrow$ apparent depth

$$\sin C = \frac{\mu_1}{\mu_2}$$

$C \rightarrow$ critical angle
$\mu_1 \rightarrow$ refractive index of rarer medium
$\mu_2 \rightarrow$ refractive index of denser medium

For Prism

$$d = i_1 + i_2 - A$$
$$A = r_1 + r_2$$



$i_1 \rightarrow$ angle of incidence to face AB
$i_2 \rightarrow$ angle of emergence from face AC
$d \rightarrow$ angle of deviation
$r_1, r_2 \rightarrow$ angles of refraction
$A \rightarrow$ angle of this prism

$$\mu = \frac{\sin\left(\frac{A + D}{2}\right)}{\sin\frac{A}{2}}$$

$D \rightarrow$ angle of minimum deviation

$$d = (\mu - 1)A$$

$d \rightarrow$ deviation produced in a small angled prism

$$\omega = \frac{d_v - d_r}{d} = \frac{\mu_v - \mu_r}{\mu - 1}$$

$$d = \frac{d_v + d_r}{2}$$

$$\mu = \frac{\mu_v + \mu_r}{2}$$

$\omega \rightarrow$ dispersive power of a prism
$d_V \rightarrow$ deviation for violet ray
$d_r \rightarrow$ deviation for red ray
$d \rightarrow$ mean deviation
$\mu_V \rightarrow$ refractive index for violet ray
$\mu_r \rightarrow$ refractive index for red ray
$\mu \rightarrow$ mean refractive index

For Lenses

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

(Use New Cartesian sign convention)

$\mu_1 \rightarrow$ refractive index of the medium from which light comes.
$\mu_2 \rightarrow$ refractive index of the medium to which light goes
$v \rightarrow$ image distance
$u \rightarrow$ object distance
$R \rightarrow$ radius of curvature

$$\frac{1}{f} = \left(\frac{\mu}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

(Use New Cartesian sign convention)

Lens maker's formula
$f \rightarrow$ focal length
$\mu \rightarrow$ refractive index of the material of the lens
$\mu_m \rightarrow$ refractive index of surrounding medium
R_1 and $R_2 \rightarrow$ radii of curvature of the two faces.

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

(Use New Cartesian sign convention)

Law of distances for lens
$f \rightarrow$ focal length
$v \rightarrow$ image distance
$u \rightarrow$ object distance

$$m = \frac{h_i}{h_0} = \frac{v}{u}$$

$m \rightarrow$ magnification of the lens
(Use New Cartesian sign convention)

$$m = \frac{f}{f + u}$$

$$m = \frac{f - v}{f}$$

$M_\ell = m$

$M_\ell \rightarrow$ longitudinal magnification

$M_A = m^2$

$M_A \rightarrow$ areal magnification

$$P = \frac{1}{f}$$

$P \rightarrow$ power of lens
$f \rightarrow$ focal length in metre

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$F \rightarrow$ effective focal length when two thin lenses of focal lengths f_1 and f_2 are separated by a distance d

$$P = P_1 + P_2 - P_1 P_2 d$$

$P \rightarrow$ effective power

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$F \rightarrow$ effective focal length when $d = 0$

$P = P_1 + P_2$

$P \rightarrow$ effective power when $d = 0$

Optical Instruments

$$m_N = \frac{D}{f}$$

$m_N \rightarrow$ magnification of simple microscope for normal vision
$D \rightarrow$ least distance of distinct vision
$f \rightarrow$ focal length of lens

$$m_d = 1 + \frac{D}{f}$$

$m_d \rightarrow$ magnification of simple microscope for distinct vision

$m = m_0 m_e$	$m \rightarrow$ magnification of compound microscope $m_0 \rightarrow$ magnification of objective $m_e \rightarrow$ magnification of eye piece
$M_N = \frac{v_0}{u_0}\left(\frac{D}{f_e}\right)$	$M_N \rightarrow$ magnification of compound microscope for normal vision
$m_d = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$	$v_0, u_0 \rightarrow$ distance of object and its image from objective $f_e \rightarrow$ focal length of eye piece $m_d \rightarrow$ magnification for distinct vision
$m_N = \frac{f_0}{f_e}$	$m_N \rightarrow$ magnification of an astronomical telescope for normal vision
$m_d = \frac{f_0}{f_e}\left(1 + \frac{f_e}{D}\right)$	$m_d \rightarrow$ magnification at an astronomical telescope for distinct vision
$L = f_0 + f_e$	$L \rightarrow$ length of the tube of telescope

WAVE OPTICS

Certain properties of light cannot be explained by ray optics. Wave optics deals with the phenomena resulting from the wave nature of light. The wave theory of light was put forward by the Dutch mathematician Christian Huygens in the year 1678. He used the concept of 'wavefront' to explain the phenomenon of reflection, refraction, total internal reflection etc.

Huygens' principle

A wavefront is defined as the locus of all the points in the field of an optical disturbance having the same phase at a given instant. The shape of a wavefront depends upon the source of disturbance. A point source of disturbance produces spherical wavefronts. A line (linear) source of disturbance produces cylindrical wavefronts. The speed with which a wavefront advances in a medium is called phase velocity.

A spherical wavefront appears as plane wavefront after travelling a large distance from the point source. The phase difference between any two points on the same wavefront is zero.

A line drawn perpendicular to the wavefront gives the direction of propagation of the wave and it is called a ray of light. i.e., a line drawn in the direction of propagation of light is a ray and the wavefront is perpendicular to the ray. The energy of light flows along the rays (or normal to the wavefront). Converging rays and diverging rays produce spherical wavefronts. In a homogeneous medium, the distance between any two wavefronts measured along any ray is the same.

The time taken by the light to travel from a position in one wavefront to a corresponding position in another wavefront, measured along any ray will be the same, whether the wavefronts are in the same medium or not.

According to Huygens's principle,

(i) A source of light is a centre of disturbance from which light waves spread in all directions. All points equidistant from the source and vibrating in phase lie on a surface known as wavefront.

(ii) Every point on a wavefront is a source of new disturbance which produces secondary wavelets. These wavelets are spherical and travel with the speed of light in all directions in that medium.

(iii) The forward envelope of the secondary wavelets at any instant (i.e., surface tangent to all secondary wavefront) gives the new position of wavefront.

(iv) In a homogeneous medium, the wavefront is always normal to the direction of wave propagation.

Huygens' construction for wave fronts

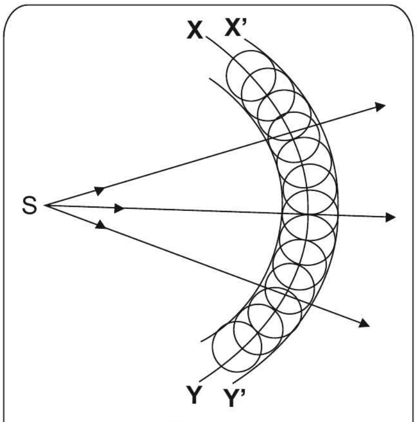


Fig. 1.59

A point source of light S sends spherical wave fronts in space surrounding it. In Fig. 1.59, XY is such a wave front at the instant of time measurement (say t = 0). The wave front at another instant of time t can be predicted using Huygen's construction.

Every point on wave front XY is a source of secondary wavelets. Hence with every point on arc XY as center, draw circles of radius = ct (c = speed of light and t = time). The forward envelope of the secondary wavelets, represented by arc X'Y' represents the wave front at time t.

Deriving law of reflection by Huygens' principle

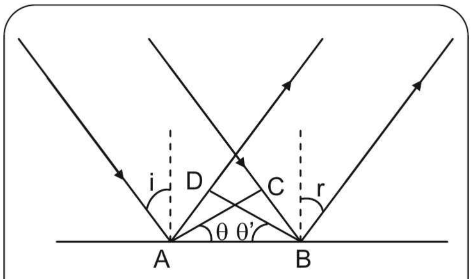


Fig. 1.60

Recalling that a wavefront is a tangent surface of the spherical waves, the wavefronts incident and reflected are AC and BD, respectively, as marked in Fig. 1.60.
Take triangles ACB and ADB.

$\angle C = \angle D = 90°$

AB is common

$\therefore$ triangles ACB and ADB are congruent

$\therefore$ $\theta' = \theta$

θ = angle of incidence i

θ' = angle of reflection r

$\therefore$ i = r

Deriving law of refraction by Huygens' principle

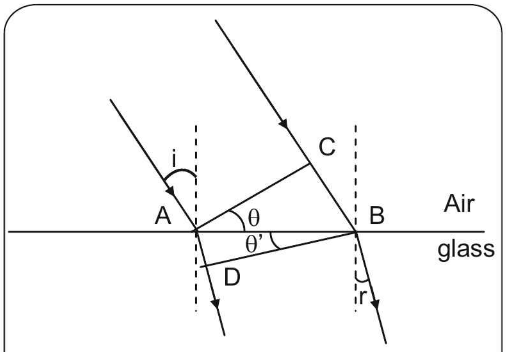


Fig. 1.61

The incident wavefront is AC and the refracted wavefront is BD as shown in Fig. 1.61.

Consider triangles ACB and ADB

$$\sin\theta = \frac{BC}{AB}$$

$$\sin\theta' = \frac{AD}{AB} \Rightarrow \frac{\sin\theta}{\sin\theta'} = \frac{BC}{AD} = \frac{v_1}{v_2} = n_{21} = \frac{n_2}{n_1}$$

$\Rightarrow$ $n_1 \sin\theta = n_2 \sin\theta'$

$\theta = i$

$\theta' = r$

$\Rightarrow$ $n_1 \sin i = n_2 \sin r$

Optical Path

If light travels a distance d in vacuum in a time interval t, then $t = \frac{d}{c}$. To travel this same distance d in any other medium, light takes time t_1 ($t_1 = \frac{d}{v}$, where v = speed of light in the medium). Hence t_1 is always greater than t (i.e., $t_1 > t$). This is because both the wavelength of light and speed are different in different media. Hence to compare the light waves traveling in different media, we imagine the following construction. $t_1 = \frac{d}{v}$ is the time of travel of light in the medium. If it were vacuum, light would have travelled a distance ct' in the time interval t'. Obviously, ct' > vt'. We define the equivalent optical path of the medium as ct'. But $ct' = c.\frac{d}{v} = \frac{c}{v}.d = nd$ where n is the refractive index of the medium. When a monochromatic light wave travels a distance 'd' in vacuum and the same distance 'd' in a medium of refractive index 'n', the optical path difference,

$$\Delta d = nd - d = (n - 1)\,d.$$

We know that if Δd is the path difference and λ is the wavelength (in vacuum), then the time difference $\Delta t = (t_1 - t) = \frac{\Delta d}{\lambda} \times T$. i.e.,

$$\Delta t = t_1 - t = \frac{(n-1)d}{\lambda}T$$

where, $T = \frac{1}{f}$ = time period of light (f = frequency of light)

INTERFERENCE

Interference is the phenomenon of redistribution of energy in a medium due to the super position of two or more waves in the same medium.

It occurs due to the simultaneous propagation of two or more waves through a medium and under certain conditions produce regions in the medium varying from a very high intensity (greater than the sum of the intensities of the individual waves) to regions of very low intensity (less than the average intensity of the waves). In the regions where the resultant intensity is maximum, constructive interference is said to take place and in the regions where the resultant intensity is minimum, destructive interference is said to take place. Interference phenomena can occur in any form of wave (i.e., electric wave or electromagnetic wave). Since light is a form of electromagnetic wave, the interference of light waves can also occur under certain conditions. However, this is not easily seen in our daily lives due to the very small wavelength of light, which is one of the factors which influence interference phenomena. If you hold the data side of a compact disc (CD) in the path of sunlight, you will see some very bright colours and some dark regions caused by interference phenomena. Similar effects can be seen on thin oil film floating on water or on observing light passing through thin soap bubbles etc. The first experimental demonstration of interference of light was carried out by Thomas Young with his famous double slit experiment. The interference of light is a direct consequence of superposition principle of waves.

Coherent sources

If the two sources of waves are of same frequency and their phase difference remains constant or stable, such sources are called coherent sources.

If the two sources of waves are:

(i) of same frequency but their phase difference does not remain constant or stable or
(ii) of different frequencies, such sources are called incoherent sources.

Production of coherent sources

There are two methods of producing coherent sources of light:

(a) By division of the wave front.
(b) By division of the amplitude of the wave.

Theory of Interference-Analytical Treatment

Consider waves from two coherent sources crossing a point P. Let the waves have the same angular frequency ω and constant phase difference ϕ. The S.H.M at P due to the first wave can be represented as $y_1 = a \sin \omega t$ and that due to the second wave can be represented as $y_2 = b \sin (\omega t + \phi)$. When both the waves pass through the point P the combined effect is given by $y = y_1 + y_2$ (according to principle of superposition of waves).

i.e., $y = a \sin \omega t + b \sin (\omega t + \phi)$.

This can be simplified as $y = a \sin \omega t + b \sin \omega t \cos \phi + b \cos \omega t \sin \phi$

i.e., $y = (a + b \cos \phi) \sin \omega t + (b \sin\phi) \cos \omega t$

Put $R \cos\theta = a + b \cos \phi$ ------ (i)

and $R \sin\theta = b \sin \phi$ ------ (ii)

Now $y = R \cos\theta \sin\omega t + R \sin\theta \cos \omega t$

i.e., $y = R \sin (\omega t + \theta)$

The resultant motion is simple harmonic with amplitude R. Squaring and adding (i) and (ii)

$$R^2 = (a + b\cos\phi)^2 + b^2\sin^2\phi$$

$$= a^2 + b^2\cos^2\phi + 2ab\cos\phi + b^2\sin^2\phi$$

$$= a^2 + b^2 + 2ab\cos\phi$$

$$R = \sqrt{a^2 + b^2 + 2ab\cos\phi}$$

The intensity (I) at the point will be proportional to R^2.

i.e., $I \propto (a^2 + b^2 + 2ab\cos\phi)$

Also $I_1 = Ka^2$ and $I_2 = Kb^2$ and $I_R = K(a + b)^2$

$$\therefore I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

This expression shows that intensity I varies from point to point depending on the value of ϕ.

If $I_1 = I_2 = I$, then $I_R = I + I + 2I\cos\phi \Rightarrow$

$$I_R = 4I\cos^2\left(\frac{\phi}{2}\right)$$

Case I

$\phi = 2n\pi$ where n is an integer. Now $\cos\phi$ attains its maximum value so that I becomes maximum. Points of maxima are called regions of constructive interference or reinforcing interference. In these regions, crest of one wave meets with the crest of other wave or trough of one wave meets with the trough of the other wave so that both reinforce each other to get greater amplitude.

$$I_{max} \propto a^2 + b^2 + 2ab$$

$$\Rightarrow \quad I_{max} \propto (a + b)^2$$

Again $\phi = 2n\pi \Rightarrow$

Path difference $= n\lambda$

Hence condition for **reinforcing interference or constructive interference** can be stated as follows.

Phase difference $= 2n\pi$ = integral multiple of 2π = even multiple of π

Path difference $= n\lambda$ = integral multiple of λ = even multiple of $\lambda/2$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

Case II

$\phi = (2n + 1)\pi$ where n is an integer.

Now $\cos\phi$ attains its minimum value -1 so that I becomes minimum. Points of minima are called regions of destructive interference or annulling interference. In these regions crest of one wave meets with trough of the other, so that the resultant amplitude becomes less. If the two sources are of equal amplitude points of minimum will be of perfect darkness.

$$I_{min} \propto a^2 + b^2 - 2ab$$

$$\Rightarrow \quad I_{min} \propto (a - b)^2$$

Again $\phi = (2n + 1)\pi$

$\Rightarrow$ Path difference $= (2n + 1)\frac{\lambda}{2}$

Hence condition for destructive interference can be stated as follows.

Phase difference $= (2n + 1)\pi$ = odd multiple of π

or

Path difference $= (2n + 1)\frac{\lambda}{2}$ = odd multiple of $\frac{\lambda}{2}$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

Note:

In regions other than maxima and minima, intensity I will have intermediate values as decided by $\cos\phi$.

Special case

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \frac{(a + b)^2}{(a - b)^2} = \frac{(r + 1)^2}{(r - 1)^2}, \text{ where}$$

$r = \frac{a}{b}$, the amplitude ratio

When the two coherent sources are of the same amplitude $a = b$ so that $I_1 = I_2 = I$, then

$$R^2 = a^2 + a^2 + 2a^2\cos\phi$$

$$\Rightarrow \quad R^2 = 2a^2(1 + \cos\phi)$$

When $\cos\phi = 1$, $R^2_{max} = 4a^2$ and when $\cos\phi = -1$, $R_{min}^2 = 0$.

$\therefore$ $I_{max} \propto 4a^2$ and $I_{min} = 0$ i.e., $I_{max} = 4I$ and $I_{min} = 0$

The resultant intensity at any location, where the two light waves meet with a phase difference of ϕ is

$$I_R = 4I\cos^2\left(\frac{\phi}{2}\right) = I_0\cos^2\left(\frac{\phi}{2}\right)$$

where, $I_0 = 4I$ = intensity of central maximum.

Note:

Since the minimum is zero, there will be perfect darkness at the minimum and good contrast between the dark and bright bands.

Also $I \propto a^2$ where a = amplitude of wave

$I \propto w$, where w = width of source

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{w_1} + \sqrt{w_2}\right)^2}{\left(\sqrt{w_1} - \sqrt{w_2}\right)^2}$$

Law of conservation of energy and interference

We see that during interference, some regions have intensity greater than $(I_1 + I_2)$ and some regions have intensity less than $(I_1 + I_2)$, whereas the total average intensity of the interfering waves is only $(I_1 + I_2)$. This only means that there is redistribution of energy during interference and no additional energy is created, which will amount to violation of law of conservation of energy.

$$I_{AV} = \frac{I_{max} + I_{min}}{2} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2 + \left(\sqrt{I_1} - \sqrt{I_2}\right)^2}{2}$$

$$= \frac{2\left(I_1 + I_2\right)}{2} = \left(I_1 + I_2\right)$$

Thus the average intensity of the interfering waves remains the same as before.

If two incoherent waves of intensities I_1 and I_2 interfere in a region, the resultant intensity at that point will be always $I_1 + I_2$ only.

CONCEPT STRANDS

Concept Strand 42

Light waves from two coherent sources reach a point with a phase difference ϕ. I_1 is the intensity at the point due to the first source alone and I_2 that due to the second source alone. Find the resultant intensity I when both are switched on.

Solution

Let a be the amplitude of vibration at the point due to I wave and b that due to the II wave. Then

$I_1 \propto a^2$ and $I_2 \propto b^2$.

i.e., $a \propto \sqrt{I_1}$ and $b \propto \sqrt{I_2}$

Now $I = a^2 + b^2 + 2\,ab\cos\phi$ gives

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

When $\cos\phi = +1$ one gets I_{max} when

$\cos\phi = -1$ one gets I_{min}.

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

Note:

When $I_1 = I_2$ the above three formulae get simplified as

$$I = 2\,I_1\,(1 + \cos\phi)$$

$$I_{max} = 4\,I_1$$

$$I_{min} = 0$$

Note that maximum intensity is 4 times that due to a single source.

Concept Strand 43

There are two coherent sources of intensities I and 4 I. At a point P the light waves from the two reach with a phase difference $\frac{\pi}{3}$. Find the resultant intensity at the point.

Solution

$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi = I + 4I + 2\sqrt{I \times 4I}\cos\frac{\pi}{3}$$

$$= 5\,I + 2\,I = 7\,I$$

Concept Strand 44

In the above problem, intensity at another point is found to be 5 I. If λ is the wavelength of light used what is the minimum path difference to this point from the two sources?

Solution

$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

$$5I = I + 4I + 2\sqrt{I \times 4I}\cos\phi$$

$\Rightarrow$ $\cos\phi = 0$

$$\phi = \left(2n + 1\right)\frac{\pi}{2} \text{ where, n is an integer.}$$

Corresponding path difference

$$\Delta x = \frac{\varphi}{2\pi}\lambda = \frac{(2n+1)\frac{\pi}{2}}{2\pi} \times \lambda = (2n+1)\frac{\lambda}{4}$$

Minimum path difference is there when n = 0

Minimum path difference = $\frac{\lambda}{4}$

Concept Strand 45

Intensity of two coherent sources are in the ratio 1 : 4. Find the ratio of $I_{max} : I_{min}$

Solution

$I_1 : I_2 = 1 : 4$

$\Rightarrow \quad a^2 : b^2 = 1 : 4$

$\Rightarrow \quad a : b = 1 : 2$

$$\frac{I_{max}}{I_{min}} = \frac{(a+b)^2}{(a-b)^2} = \frac{(1+2)^2}{(1-2)^2} = \frac{9}{1} = 9 : 1$$

Concept Strand 46

In an interference pattern due to two coherent sources $I_{max} : I_{min}$ is found to be 25 : 1. Find ratio of intensities of the two sources.

Solution

$I_{max} : I_{min} = 25 : 1$

$(a + b)^2 : (a - b)^2 = 25 : 1$

$a + b : a - b = 5 : 1$

$\frac{a+b}{a-b} = \frac{5}{1} \Rightarrow$

$a + b = 5a - 5b$

$6b = 4a$

$\frac{a}{b} = \frac{6}{4} = \frac{3}{2}$

$\frac{a^2}{b^2} = \frac{9}{4}$

$\frac{I_1}{I_2} = \frac{9}{4}$

Conditions for sustained and good interference pattern

1. The two sources should be monochromatic. Otherwise the interference pattern due to different wavelengths will overlap. If white light is used instead of monochromatic light, interference pattern will be coloured since different colours produce different interference pattern. However at points corresponding to zero path difference all colours reinforce and a white maximum will be produced. Other points will be coloured. Perfect darkness cannot be achieved at any point.
2. The two sources should be coherent which implies that the sources should have the same wavelength and constant phase difference.
3. The two sources should emit waves of the same amplitude or nearly the same amplitude. This will give good contrast between bright and dark fringes.
4. The two sources should be narrow. Otherwise single slit diffraction of each slit will spoil the interference pattern.
5. In case polarized light is used, the two waves should have same plane of polarization.
6. The distance between the two sources should be sufficiently small. Otherwise distance between maxima and minima becomes too small to distinguish.
7. The direction of motion of the two interfering waves should be the same or at most differ only slightly.

Young's double slit experiment

This is a simple experiment to demonstrate interference of light.

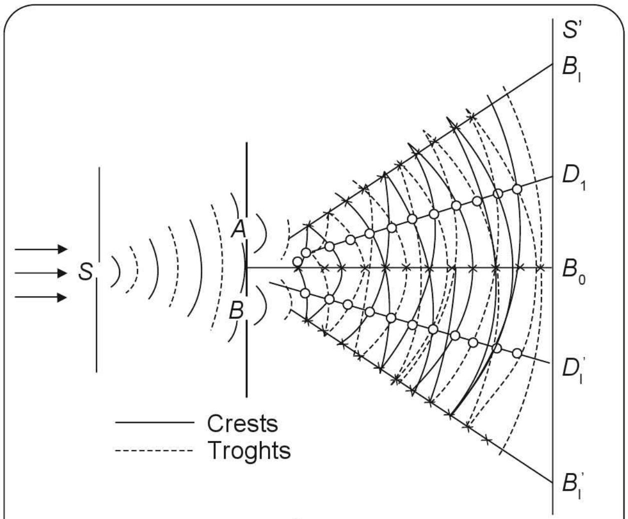


Fig.1.62

A narrow slit S illuminated with monochromatic light is the source. Two other narrow slits A and B of same sizes are placed parallel to S and equidistant from it. A screen S' is placed in front of the slits A and B parallel to the plane of the slits.

The slit S sends cylindrical wave fronts towards A and B and these wave fronts reach A and B at the same time. From A and B, waves of same frequency, same amplitude and zero phase difference emerge. Hence slits A and B act as coherent sources. The waves from A and B interfere in the region between the slits and the screen.

Arcs shown by continuous lines represent the crests of the waves and arcs shown by dotted lines represent the troughs of the waves. Where a crest of wave from A superimposes on a crest of wave from B (or a trough of wave from A superimposes on a trough of wave from B), the resultant intensity will be maximum i.e., constructive interference takes place. These regions are marked as crosses (x) in the sketch. Lines joining the crosses indicate the rays, which reach the screen to produce maxima at various locations on the screen.

Where a crest of wave from A superimposes on a trough of wave from B (or a trough of wave from A superimposes on a crest of wave from B), the resultant intensity will be zero i.e., destructive interference takes place. These regions are marked as circles (o) in the sketch. Lines joining the circles indicate the rays which reach the screen to produce minima at various locations on the screen.

The interference pattern obtained on the screen consists of alternate dark and bright bands. The shape of the interference pattern depends on the shape of the slits. If the slits are long and narrow, the pattern would be alternate bright and dark lines. If the slits are pin holes, the pattern will be hyperboloids.

Expression for fringe width

The distance between two consecutive bright fringes or two consecutive dark fringes is called the fringe width.

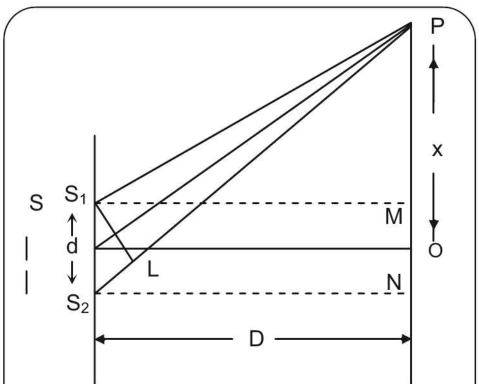


Fig. 1.63

The geometry of formation of fringes is shown in Fig. 1.63.

S is a narrow cylindrical monochromatic source of light with its length normal to the plane of the figure. S_1 and S_2 are narrow identical slits arranged at a small distance d. They are parallel to the cylindrical source and equidistant from it. Now S_1 and S_2 behave as two coherent sources.

A screen is placed at a distance D >> d parallel to $S_1 S_2$. Interference pattern is obtained on this screen. O is the foot of the perpendicular from the middle of $S_1 S_2$ on the screen. O is the centre of the fringe system.

At O waves from S_1 and S_2 reach with zero path difference so that there is reinforcing interference to give a maximum (i.e., a bright band). This is called central maximum.

Now consider a point P on the screen at a distance OP = x. The path difference of the waves reaching the point P is $S_2P - S_1P$. This is calculated as follows.

Draw S_1M and S_2N perpendicular to the screen.

$$S_1P^2 = S_1M^2 + MP^2 = D^2 + \left(x - \frac{d}{2}\right)^2 \qquad \text{—(1)}$$

$$S_2P^2 = S_1M^2 + NP^2 = D^2 + \left(x + \frac{d}{2}\right)^2 \qquad \text{—(2)}$$

(2) – (1) gives

$$S_2P^2 - S_1P^2 = \left(x + \frac{d}{2}\right)^2 - \left(x - \frac{d}{2}\right)^2$$

$\Rightarrow$ $(S_2P - S_1P)\,(S_2P + S_1P) = 2\,xd$

$\Rightarrow$ $(S_2P - S_1P)\,(D + D) = 2\,xd$

$\Rightarrow$ [Here $S_2P \approx D$ and $S_1P \approx D$]

$$\Rightarrow \quad S_2P - S_1P = \frac{2xd}{2D}$$

$$\text{path difference} = \frac{xd}{D}$$

If the point P coincides with nth bright band from the centre this path difference will be nλ. If the corresponding x is denoted by x_n one get

$$\frac{x_n d}{D} = n\lambda \Rightarrow$$

$$x_n = \frac{nD\lambda}{d}$$

n = 0, 1, 2, 3, ...

The distance of (n + 1)th bright band from O is

$$x_{n+1} = \frac{(n+1)D\lambda}{d}$$

Distance between two consecutive bright bands is the band width β.

$$\beta = x_{n+1} - x_n = \frac{(n+1)D\lambda}{d} - \frac{nD\lambda}{d} = \frac{D\lambda}{d}$$

$$\beta = \frac{D\lambda}{d}$$

Thus bandwidth β is independent of the order n.

In a similar manner, one can calculate the distance of perfectly dark bands from the centre of the fringe system.

For the nth dark fringe (where, n = 1, 2, 3 ……) the path difference $S_2P - S_1P = \frac{xd}{D}$ is equal to $(2n-1)\frac{\lambda}{2}$. Denoting this value of x by x_n' one gets

$$\frac{x_n' d}{D} = (2n-1)\frac{\lambda}{2} \Rightarrow$$

$$x'_n = (2n-1)\frac{D\lambda}{2d}$$

$$n = 1, 2, 3, \ldots\ldots.$$

Note:

n = 0 does not exist for dark fringes.

$$x'_{n+1} = (2n+1)\frac{D\lambda}{2d}$$

$$\beta' = x'_{n+1} - x'_n = \frac{(2n+1)D\lambda}{2d} - \frac{(2n-1)D\lambda}{2d} = \frac{D\lambda}{d}$$

This is the distance between consecutive dark bands and is known as bandwidth represented by β'.

$$\beta' = \frac{D\lambda}{d}$$

It is seen that β' is also independent of 'n'. Again β' = β.

Hence in this case one may define bandwidth as distance between consecutive bright bands or distance between consecutive dark bands. In both cases, the symbol β may be used.

$$\beta = \frac{D\lambda}{d}$$

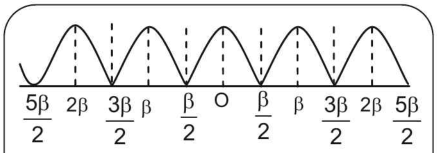


Fig.1. 64

Notes:

(i) The intensity of the bright bands are same throughout

(ii) Bright and dark bands occur alternately and are equidistant. β is same for bright bands as well as for dark bands.

(iii) If white light is used instead of monochromatic light, all bands except central maximum will be coloured. Dark bands will be absent. Central band will be white.

(iv) If the experiment is arranged in a transparent medium of refractive index n, then $\beta = \frac{D\lambda'}{d}$ where $\lambda' = \frac{\lambda}{n}$. This method can be used for determining 'n' of transparent gases and liquids.

(v) If a glass plate of thickness t and refractive index n is introduced in the path of the upper ray the fringe system shifts down by $\frac{(n-1)t}{\lambda}\beta$ or $\frac{(n-1)tD}{d}$. This method can be used for determining thickness 't' or refractive index 'n' of a transparent material.

Angular distance and angular bandwidth

If distance (x) along the screen is divided by D one gets angular distances in radians. Hence the following results.

Angular distance of nth bright band

$$\theta_n = \frac{n\lambda}{d}$$

Angular distance of nth dark band

$$\theta_n' = \frac{(2n-1)\lambda}{2d}$$

$$\text{Angular band width} = \varphi = \frac{\lambda}{d}$$

This is independent of the distance of screen 'D'.

Note:

In Young's double slit experiment, the condition for destructive interference is path difference, δ = (2n + 1) λ/2, where n = 0 for first dark band

n = 1 for second dark band and so on

It can also be stated as

δ = (2n −1) λ/2

where

n = 1, for first dark band

n = 2, for second dark band and so on.

For constructive interference

δ = nλ

where,

n = 0 for central bright band

n = 1 for first bright band and so on.

CONCEPT STRANDS

Concept Strand 47

In a Young's double slit experiment the two slits were separated by a distance of 0.4 mm and the wavelength of light used was 7000Å. The screen was placed at a distance of 1 m from the slits. Find

(i) bandwidth of the interference pattern
(ii) distance of 6th bright band from the central bright band
(iii) distance of 3rd dark band from the central maximum.

Solution

$d = 0.4\text{ mm} = 0.4 \times 10^{-3}\text{ m}$

$\lambda = 7000\text{ Å} = 7000 \times 10^{-10}\text{ m}$

$D = 1\text{ m}$

(i) $\beta = \frac{\lambda D}{d} = \frac{7000 \times 10^{-10} \times 1}{0.4 \times 10^{-3}}$
$= 1.75 \times 10^{-3}\text{ m} = 1.75\text{ mm}$

(ii) Distance of nth bright band from central maximum is

$$x_n = \frac{nD\lambda}{d}$$

$$x_6 = \frac{6D\lambda}{d} = 6\beta = 6 \times 1.75$$
$$= 10.5\text{ mm}$$

(iii) Distance of nth dark band from the central maximum

$$x_n' = \frac{(2n-1)D\lambda}{2d}$$

$$x_3' = \frac{5}{2}\frac{D\lambda}{d} = \frac{5}{2}\beta = \frac{5}{2} \times 1.75 = 4.375\text{ mm}$$

Concept Strand 48

In a Young's double slit experiment, the slit separation is 0.20 mm and the wavelength of light used is 5600Å Find

(i) the angular band width
(ii) angular distance of 5th bright band from centre
(iii) angular distance of 1st dark band from centre.

Solution

(i) Angular band width

$$\phi = \frac{\beta}{D} = \frac{\lambda}{d} = \frac{5600 \times 10^{-10}}{0.2 \times 10^{-3}}$$
$$= 0.0028\text{ radian} = 9' 38''$$

(ii) Angular distance of nth bright band from centre is

$$\theta_n = \frac{n\lambda}{d}$$

$$\theta_5 = \frac{5\lambda}{d} = 5(0.0028)\text{ radian}$$
$$= 0.014\text{ radian} = 48' 8''$$

(iii) Angular distance of nth dark band from centre is

$$\theta_n' = \frac{(2n-1)\lambda}{2d}$$

$$\theta_1' = \frac{\lambda}{2d}$$

$$= \frac{1}{2}(0.0028)\text{ radian}$$

$$= 0.0014\text{ radian}$$

$$= 4' 49''$$

Concept Strand 49

In a Young's double slit experiment, the bandwidth obtained is 3 mm. Without disturbing other arrangements imagine that the whole apparatus is immersed in a transparent liquid of refractive index $\frac{4}{3}$. Find the new bandwidth.

Solution

$$\beta = \frac{\lambda D}{d}\text{ (in air)}$$

$$\beta' = \frac{\lambda' D}{d}\text{ (in liquid)}$$

$$\frac{\beta'}{\beta} = \frac{\lambda'}{\lambda}$$

$$\beta' = \beta.\frac{\lambda'}{\lambda} = \beta.\frac{\frac{\lambda}{n}}{\lambda} = \frac{\beta}{n}$$

$$\beta' = \frac{\beta}{n}$$

$$\beta' = \frac{3\text{mm}}{\frac{4}{3}} = 2.25\text{mm}$$

Concept Strand 50

The source of light used in a Young's double slit experiment contained two wavelengths 4000 Å and 6000 Å. The separation between the slits is 0.3 mm and distance to the screen from the slit is 1 m. Find the minimum distance from the central maximum where bright bands of the two coincide.

Solution

$d = 0.3 \times 10^{-3}$ m D = 1 m

Let x be the distance from the central maximum where nth bright of $\lambda_1 = 4000$ Å coincide with mth bright of $\lambda_2 = 6000$ Å.

$$x = \frac{n\lambda_1 D}{d} = \frac{m\lambda_2 D}{d} \Rightarrow \frac{n}{m} = \frac{\lambda_2}{\lambda_1} = \frac{6000}{4000} = \frac{3}{2}$$

Minimum values for integers n and m are n = 3 and m = 2

$$x = \frac{n\lambda_1 D}{d} = \frac{3 \times 4000 \times 10^{-10} \times 1}{0.3 \times 10^{-3}} = 0.004 \text{ m} = 4 \text{ mm}$$

Extra path difference caused because of reflection at the boundary of a denser medium when incident medium is rarer

It can be shown that when light of wavelength λ, coming from a rarer medium, gets reflected at the boundary of a denser medium, its path shows an extra increase of $\frac{\lambda}{2}$ in addition to the geometrical path. Because of this increase $\frac{\lambda}{2}$ the phase increases by π. This phenomenon is applicable to any wave. However, no path difference occurs, if incident medium was denser.

Modifications of Young's double slit experiment

We now consider the effect of introducing a thin transparent sheet in front of one slit.

Consider a typical set up (as shown in Fig. 1.65.) Path length S_1C = path length S_2C. Now introduce a thin mica sheet of thickness t and of refractive index n, in front of S_1.

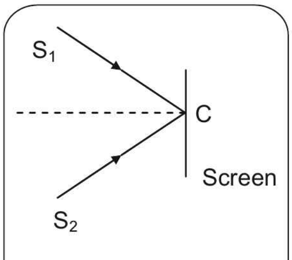


Fig. 1.65

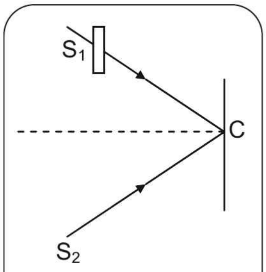


Fig. 1.66

The optical path length S_1C is now more than optical path length S_2C.

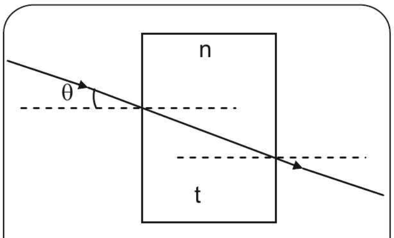


Fig. 1.67

Ignoring the effect of deviation of ray in the mica sheet, geometric length t is equivalent to the optical path length nt.

∴ Increase in path length is nt − t = (n − 1)t.

So consider a point C' on the screen above C such that geometric length S_1C' + (n − 1)t equals geometric length S_2C'.

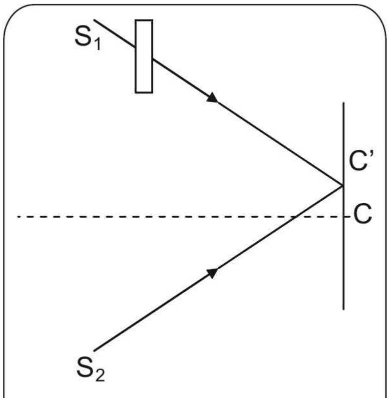


Fig. 1.68

C' will be a bright fringe. Fringes of all orders correspondingly shift.

$$\therefore \text{ The shift in the fringe pattern} = \Delta y = (n-1)t\frac{D}{d}$$

FOR ADDITIONAL READING

Thin film interference

A transparent film of thickness of the order of the wavelength of light is known as a thin film. When light illuminates a thin film such as a soap film or a film of oil on water surface, bright colours are seen. This is due to the interference of light waves reflected from the front and back surfaces of the film.

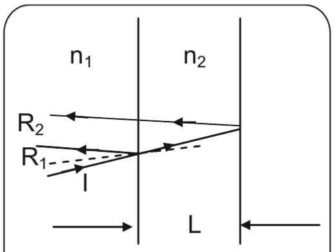


Fig. 1.69

Consider a bright source of light of wavelength λ illuminating a, thin film of refractive index n_2 as shown in Fig. 1.69. The ray I is partly reflected and partly refracted at the front surface of the film. The refracted ray is subsequently reflected at the back surface of the film which finally comes out through the front surface after refraction. A viewer receives rays R_1 and R_2 which come back. If the rays are exactly in phase at the viewer's eye, they interfere constructively producing maximum amplitude while if they are out of phase, the interference leads to minimum amplitude. Phase difference occurs due to the following factors.

(i) Change in phase due to reflection

When light travelling in a medium of refractive index n_1 gets reflected at the interface with a medium of refractive index n_2, the electric field amplitude of the reflected wave is given by

$$E_r = \frac{(n_1 - n_2)}{(n_1 + n_2)} E_i$$

where, E_i is the incident wave electric field amplitude. Therefore, when $n_1 < n_2$ (for example light ray traveling in air striking a soap film), the reflected electric field has a sign opposite to that of the incident ray. Hence, the reflected ray undergoes a change in phase of π rad or half the wavelength. If, on the other hand, $n_1 > n_2$, the reflected wave will have no phase change.

(ii) Change in phase due to path length

The reflected ray R_2 arrives at the eye after crossing te width of the film twice. Taking the angle of incidence to be very small, the length of path traversed by the ray inside the film can be taken as 2L where, L is the thickness of the film. This introduces an additional phase difference $\frac{2L}{\lambda_2}.2\pi$, where, λ_2 is the wavelength of the ray in medium 2. If λ_1 is the wavelength in medium 1 (say, air), then $\lambda_2 = \frac{\lambda_1}{n_2}$.

Interference condition

The waves R_1 and R_2 will interfere constructively if the total phase difference is an integral multiple of 2π and they will interfere destructively if the phase difference is an odd integral multiple of π. Thus, we have the interference condition given by

$$\left.\begin{aligned} &\frac{2L}{\lambda_2}.2\pi - \pi = 2m\pi \text{ (Constructive interference)} \\ &\frac{2L}{\lambda_2}.2\pi - \pi = (2m+1)\pi \text{ (Destructive interference)} \end{aligned}\right\} m = 0, 1, 2, \ldots\ldots$$

On simplification, these conditions are

$$\left.\begin{aligned} &2L = \left(m + \frac{1}{2}\right)\frac{\lambda_1}{n_2} \text{ (Constructive interference)} \\ &2L = m\frac{\lambda_1}{n_2} \text{ (Destructive interference)} \end{aligned}\right\} m = 0, 1, 2, \ldots\ldots$$

These equations represent the conditions for bright and dark colours of the film. The colour of the film depends on the thickness of the film.

If $L << \lambda_2$, the phase difference $\frac{2L}{\lambda_2}$ is so small that it can be neglected. Then, the incident and reflected rays will be exactly out of phase and the film will appear dark.

CONCEPT STRAND

Concept Strand 51

A MgF_2 film of refractive index 1.378 is coated on a glass window of refractive index 1.5. What should be the thickness of the film to avoid reflection of red light?

Solution

Assuming that light falls normal to the film surface, the condition for destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}$$

$$L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n}$$

$$\Rightarrow \quad = \frac{1}{2}\frac{650}{2 \times 1.378} = 117.9 \text{ nm}$$

Diffraction

When a beam of light falls on a narrow slit, the beam spreads out and emerges to form a pattern of alternate bright and dark fringes on a screen as shown in Fig. 1.70. This phenomenon is known as *diffraction*. The central fringe appears broad and carries about 85% of the intensity. The secondary fringes on either side are narrower. The width of the central fringe is usually inversely proportional to the slit width.

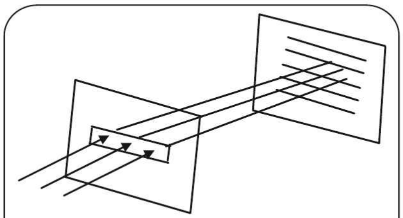


Fig. 1.70

The phenomenon of diffraction is essentially due to the interference of light waves passing through the slit. There is no fundamental distinction between interference and diffraction. Both are based on the principle of interference of waves, that is, the superposition of Huygens' wavelets. In the interference phenomena that we usually discuss, we consider only secondary wavelets coming from two point apertures on a screen. The term diffraction, on the other hand, is used for the interference of Huygens' wavelets coming from a continuous set of sources, for example, the wavelets coming from all points in a single aperture.

When the point source and the screen are close to the aperture, the diffraction pattern is called *Fresnel diffraction*. If the source, the aperture and the screen are far apart, light rays can be considered almost parallel. In this situation, the diffraction pattern obtained on the screen is called *Fraunhofer diffraction* (Fig. 1.70).

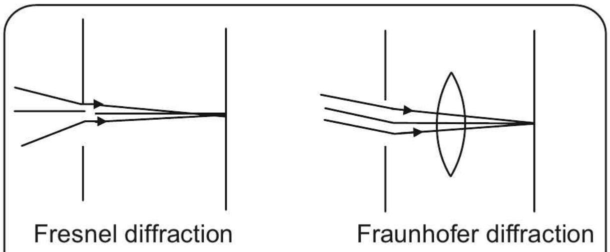


Fig. 1.71

Fraunhofer diffraction

Fraunhofer diffraction is easier to analyse. To obtain the Fraunhofer diffraction pattern on a screen, we may use a lens, which focuses the pattern on a screen without any phase change between wavelets and, therefore, reproducing the same pattern as would have been produced by the parallel beam.

Imagine the slit to be divided into two halves (Fig. 1.71); consider two narrow portions of the slit, one at the top and the other at the centre of the slit as shown. The difference in path length of wavelets coming from the top point and the central point of the slit to the point P is $\frac{a}{2}\sin\theta \approx \frac{a}{2}\theta$ (since D is very large, θ is extremely small). If the path difference is equal to $\frac{\lambda}{2}$, where λ is the wavelength of the monochromatic beam, wavelets from these two source points arrive at P with a half cycle phase difference, canceling each other.

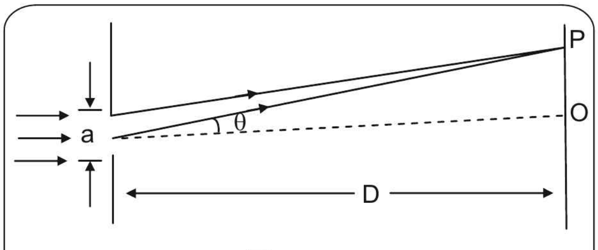


Fig. 1.72

We may consider such pairs of beams coming from points below the two previously considered, which also arrive at P with $\frac{\lambda}{2}$ path difference. Therefore, taking source point in pairs, there is complete cancellation of wavelets arriving at P, because, for every source point in the upper half of the slit, there is a corresponding one in the lower half. The condition for this to happen is,

$$\frac{a}{2}\theta = \pm\frac{\lambda}{2} \quad \Rightarrow \theta = \pm\frac{\lambda}{a}$$

We may also consider the slit to be divided into four, six, etc., equal portions, the corresponding condition for diffraction (for dark fringes), being

$$\theta = m\frac{\lambda}{a} \quad (m = \pm 1, \pm 2, \ldots\ldots)$$

m = 0 would correspond to the central bright fringe where there is no wavelet cancellation. Other bright fringes appear between the dark fringes. Symmetry requires fringes to appear above and below the central fringe, hence the ± sign for m.

width of the central maximum (β_{max}) = $D.2q = \frac{D2\lambda}{a}$

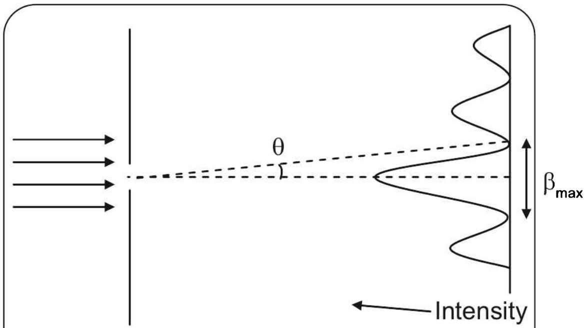


Fig. 1.73

Figure 1.73 shows a schematic sketch of the intensity pattern in a Fraunhofer diffraction pattern.

Polarization

Light from a source is due to the emission by numerous excited atoms of the source. These light waves are characterized by mutually perpendicular electric and magnetic fields.

The electric field of the light waves will have, in general, random directions. Such waves are said to be unpolarized. If the electric field vector of all the waves are oscillating in the same direction, we say that the light is linearly polarized in that direction. It is also called plane polarized light, the plane of polarization being that containing the direction of propagation and the direction of oscillation of the electric vector.

The light waves emitted by the atoms of the filament of an electric bulb are random and hence it is unpolarized. The elements of the roof top television antenna are aligned horizontally because the waves received by the antenna are polarized in that direction.

The electric vector of a y-polarized light wave propagating in the x-direction can be written as

$$\overline{E}(x, t) = \hat{j}\, E_m \cos(kx - \omega t)$$

Polarizer and analyser

Unpolarized light can be polarized by certain materials exhibiting the property called dichroism. These materials selectively transmit waves with their electric vectors in a particular direction and absorb all other waves.

The polarization direction of linearly polarized wave can be found out by an analyser. The analyser also is a polarizer with its own polarizing axis. Suppose the polarizing axis of the analyser is at an angle ϕ with respect to the polarising axis of the polarizer. Then the intensity of light transmitted through the analyser is given by

$$I = I_m \cos^2\phi$$

This is known as Malus's law. It shows that the transmitted intensity is maximum when the polarizing axes of the polarizer and analyser are parallel ($\phi = 0$). When $\phi = \frac{\pi}{2}$, the transmitted intensity is zero. This is a method of determining the polarization direction of light from the polarized.

Brewster's law

Unpolarized light can also be polarized by reflection. Consider unpolarized light incident on a dielectric such as a glass plate. The light rays will be partly reflected and partly refracted.

Brewster found that at a particular angle of incidence called the polarizing angle ϕ_P, light wave with $\overline{E}$ in the plane of incidence is not reflected at all but is completely refracted. At this same angle, waves with $\overline{E}$ parallel to the reflecting plane are partly reflected and partly refracted. The reflected wave, therefore, is plane polarized with its $\overline{E}$ parallel to the reflecting plane.

It is also found that at the Brewster angle ϕ_P, the reflected and refracted rays are perpendicular to each other. Thus, the angle of refraction ϕ_B is the complement of ϕ_P, so that

$$\phi_B = 90° - \phi_P$$

Snell's law can be written as

$$n_a \sin \phi_p = n_b \sin (90° - \phi_p) = n_b \cos\phi_p \Rightarrow$$

$$\tan \phi_p = \frac{n_b}{n_a}$$

This is known as Brewster's law

CONCEPT STRAND

Concept Strand 52

Unpolarized light traveling through air ($n_1 = 1.0$) falls on a glass plate ($n_2 = 1.54$). Calculate Brewster's angle for the pair of media.

Solution

$$\text{Tan } \phi_p = \frac{n_b}{n_a} = \frac{1.54}{1} = 1.54$$

$$\therefore \theta_p = \tan^{-1} (1.54) = 57°$$

SUMMARY

$I \propto A^2$	$I \rightarrow$ intensity of a wave $A \rightarrow$ amplitude of the wave
$I \propto w$	$w \rightarrow$ slit width
$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\varphi$ $I_R = 4I \cos^2\left(\frac{\phi}{2}\right)$ $= I_0 \cos^2\left(\frac{\phi}{2}\right)$, when $I_1 = I_2 = I$; $I_0 = 4I$	$I_R \rightarrow$ resultant intensity at a point in the region of superposition $\phi \rightarrow$ phase difference between the waves at that point I_1 and $I_2 \rightarrow$ intensities of the two waves
$\beta = \frac{\lambda D}{d}$	$\beta \rightarrow$ fringe width of interference fringes $\lambda \rightarrow$ wavelength of light used $D \rightarrow$ distance between screen and double slit. $d \rightarrow$ distance between slits
$\beta_m = \frac{\beta}{\mu}$	$\beta_m \rightarrow$ fringe width in a medium $\beta \rightarrow$ fringe width in air $\mu \rightarrow$ refractive index of the medium
$\beta_{max} = \frac{2\lambda D}{d}$	$\beta_{max} \rightarrow$ width of central maximum in a diffraction pattern $d \rightarrow$ width of slit $D \rightarrow$ distance between slit and screen $\lambda \rightarrow$ wavelength of light

$(2\theta) = 2\sin^{-1}\left(\frac{\lambda}{d}\right) \approx \frac{2\lambda}{d}$	$2\theta \rightarrow$ angular width of central maximum
$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$	$\Delta\phi \rightarrow$ phase difference between two waves $\Delta x \rightarrow$ path difference between the two waves $\lambda \rightarrow$ wavelength
$I = I_0 \cos^2\theta$	$I \rightarrow$ intensity of light coming from a rotating Polaroid $I_0 \rightarrow$ intensity of plane polarized light falling on a rotating Polaroid $\theta \rightarrow$ angle between polarizer and analyser
$\mu = \tan i_p$	$\mu \rightarrow$ refractive index of the medium $i_p \rightarrow$ angle of polarization, Brewster angle
$i_p + r = 90^0$	$r \rightarrow$ angle of refraction
$\theta = \frac{1.22\lambda}{D}$	$\theta \rightarrow$ limit of resolution of a telescope $\lambda \rightarrow$ wavelength of light used $D \rightarrow$ diameter of the aperture of objective
$P = \frac{D}{1.22\lambda}$	$P \rightarrow$ resolving power of a telescope
$P = \frac{1}{\theta}$	$\theta \rightarrow$ limit of resolution of a telescope
$\theta = \frac{\lambda}{2\mu\sin\phi}$	$\theta \rightarrow$ limit of resolution of a microscope $\phi \rightarrow \phi$-the angle subtended by a radius of the objective on one of the objects.
$P = \frac{2\mu\sin\phi}{\lambda}$	$P \rightarrow$ resolving power of a microscope $\mu \rightarrow$ refractive index of the material of the lens.

CONCEPT CONNECTORS

Connector 1: An object of length 5 cm is placed perpendicular to the principal axis of a concave mirror of focal length f at a distance of 1.5f. Find the length of the image formed.

Solution: Here focal length = – f and u = –1.5f

$$\text{from } \frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

$$\frac{1}{-1.5f}+\frac{1}{v}=\frac{1}{-f}$$

$$\therefore \quad v=-3f$$

$$m=-\frac{v}{u}=\frac{3f}{-1.5f}=-2=\frac{h_2}{h_1}$$

$$\therefore \quad h_2=-2h_1=-2\times 5=-10\text{ cm}$$

Minus sign shows that the image is inverted.

Connector 2: Find the distance from a convex mirror of a rod, which has to be kept perpendicular to the principal axis, so that the image formed has half the object length (focal length of mirror = 2.5 cm).

Solution: For a real object, a convex mirror forms a virtual image

$$m=-\frac{v}{u}=\frac{1}{2} \text{ or } v=-\frac{u}{2}$$

Also we have $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

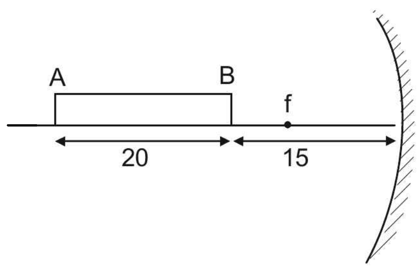
$$\therefore \quad \frac{1}{u}+\frac{1}{-\frac{u}{2}}=\frac{1}{2.5}$$

$$\therefore \quad -\frac{1}{u}=\frac{1}{2.5}$$

$$\therefore \quad u=-2.5\text{ cm}$$

Connector 3: A rod of length 20 cm is placed in front of a concave mirror of focal length 10 cm along its principal axis. If the end of the rod closer to the pole is 15 cm from the mirror, find the length of the image.

Solution: $f=-10$ cm; $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$



For end B,

u = –15 cm

$$\frac{1}{-15}+\frac{1}{v_B}=\frac{1}{-10};$$

$$v_B=-30\text{ cm}$$

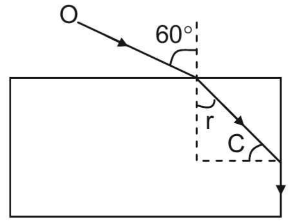
For end A, u = –35 cm

$$\frac{1}{-35}+\frac{1}{v_A}=\frac{1}{-10};$$

$$v_A=-14\text{ cm}$$

Length of image of rod = 30 – 14 = 16 cm

Connector 4: A ray of light incident on the horizontal surface of a glass slab at 60° grazes the adjacent vertical surface after refraction. Find the refractive index of glass slab.



Solution: From figure

$r + C = 90°; r = 90° - C$

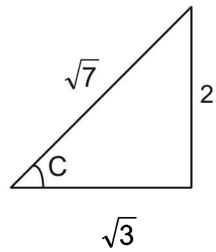
By Snells law,

$$n = \frac{\sin i}{\sin r} = \frac{\sin 60}{\sin(90 - C)}$$

$$n = \frac{\sin 60}{\cos C} \text{ also } n = \frac{1}{\sin C}$$

$$\frac{\sin 60}{\cos C} = \frac{1}{\sin C}; \tan C = \frac{1}{\sin 60} = \frac{2}{\sqrt{3}}$$

$$n = \frac{1}{\sin C} = \frac{\sqrt{7}}{2} = 1.32$$



Connector 5: Optical path of a monochromatic beam of light is the same if it goes through 2 cm of glass or 2.25 cm of water. If refractive index of water is 1.33, find the refractive index of glass.

Solution: $n \times 2 \text{ cm} = 1.33 \times 2.25 \text{ cm}$

$$\therefore \quad n = \frac{1.33 \times 2.25}{2} = 1.5$$

Connector 6: The angle of minimum deviation for a prism is 37°. If the angle of the prism is 53°, find the refractive index of the material of the prism.

Solution: We have $n = \dfrac{\sin\left(\dfrac{A + D}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} = \dfrac{\sin\dfrac{53° + 37°}{2}}{\sin\dfrac{53°}{2}}$; We know $\sin\theta = \sqrt{\dfrac{1 - \cos 2\theta}{2}}$

$$= \frac{\sin 45°}{\sqrt{\dfrac{1 - \cos 53}{2}}} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{\dfrac{1 - 0.6}{2}}} = \sqrt{\frac{5}{2}} = 1.58$$

Connector 7: Dispersive powers of two lenses A and B are in the ratio 1 : 2. Find the focal length of each lens so that a converging achromatic combination of the two lenses has mean focal length of 30 cm.

Solution: Let f_1 and f_2 be the focal lengths of these lenses

Then $$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$$

or $$\frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2} = \frac{1}{2}$$

$\therefore$ $$f_2 = -2f_1$$

$$\frac{1}{30} = \frac{1}{f_1} - \frac{1}{2f_1}$$

i.e., $$f_1 = 15 \text{ cm and } f_2 = -30 \text{ cm}$$

Connector 8: The angle of a prism is 60° and angle of minimum deviation is 30°. Find the velocity of light through the material of the prism.

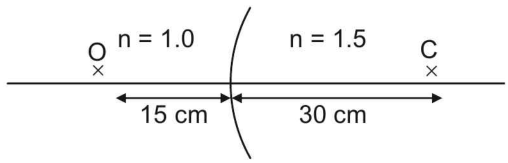
Solution: A = 60°; D = 30°

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin 45}{\sin 30} = \sqrt{2}$$

$$n = \frac{c}{v} \text{ or } v = \frac{c}{n} = \frac{3\times 10^8}{\sqrt{2}} = 2.12\times 10^8 \text{ m s}^{-1}$$

Connector 9: A spherical surface of radius of curvature 30 cm separates two media of refractive indices 1.0 and 1.5. Locate the image formed due to the point object 'O' placed at a distance 15 cm away, from the surface in the medium of refractive index 1.0, surface is conic x as seen from O.

Solution: We have $\frac{n_v}{v} - \frac{n_u}{u} = \frac{n_v - n_u}{R}$



Here u = –15 cm; R = 30 cm, n_u = 1 and

n_v = 1.5

$$\therefore \quad \frac{1.5}{v} - \frac{1.0}{-15} = \frac{1.5-1}{30}$$

Solving v = –30 cm

Connector 10: A beaker of height d is kept on a horizontal table and a plane mirror is kept horizontal above the beaker at a height 'h' above the bottom of the beaker. If the beaker is filled with water (refractive index n) to a height d. Locate the position of the image of the bottom of the beaker.

Solution: The bottom of the beaker appears to be shifted by a distance given by $\Delta t = \left[1 - \frac{1}{n}\right]d$

∴ The apparent distance between the bottom of the beaker and the mirror will be

$$h - \Delta t = h - \left[1 - \frac{1}{n}\right]d = \left(h - d + \frac{d}{n}\right)$$

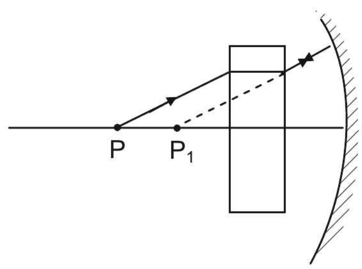
∴ The image will be formed $\left(h - d + \frac{d}{n}\right)$ away (behind the mirror)

Connector 11: An object is placed 26 cm in front of a concave mirror of focal length 12 cm. A glass slab of refractive index 1.5 and thickness 6 cm is placed close to the mirror between object and mirror. Find the position of the final image formed.

Solution: Because of the refraction at the two surfaces of the slab, the object

P will appear to be at P_1, shifting towards the mirror by a distance

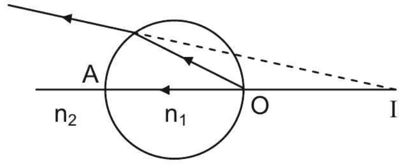
$$t\left(1 - \frac{1}{n}\right) = 6\left(1 - \frac{1}{1.5}\right) = 2 \text{ cm}$$



Thus the ray falling on the concave mirror will appear to diverge from P_1 which is at 26 – 2 = 24 cm from the mirror. Given focal length f = 12 cm or radius of curvature R = 2f = 24 cm. Therefore, object is apparently at the centre of curvature. And so rays fall normally on the mirror and retrace their path and the image will be formed at P itself.

Connector 12: A black spot on the surface of a glass sphere is viewed from a diametrically opposite position. Refractive index of glass is 1.5 and radius of the sphere is 10 cm. Find the position of the image.

Solution: For refraction through a spherical surface,



$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Here $n_2 = 1$, $n_1 = 1.5$, $u = -20$ cm, $R = -10$ cm
(Considering distances to the right of A)

$$\frac{1}{v} - \frac{1.5}{-20} = \frac{1-1.5}{-10}$$

$$\frac{1}{v} = \frac{-0.5}{-10} + \frac{-1.5}{20}; v = -40 \text{ cm}$$

The spot appears to be at a distance 40 cm to the right of A.

Connector 13: An object of length 2 cm is kept perpendicular to the principal axis of a convex lens of focal length 12 cm. Find the size of the image if the object is at a distance of 8 cm from the lens.

Solution: Here $u = -8$ cm and $f = 12$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

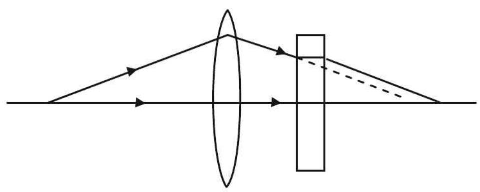
$$\frac{1}{v} = \frac{1}{12} + \frac{1}{-8}$$

$v = -24$ cm

Also $m = \frac{v}{u} = \frac{-24}{-8} = 3 \therefore m = 3 = \frac{h_2}{h_1}$; $h_2 = 3h_1 = 6$ cm

Connector 14: A point object is placed on the principal axis of a convex lens (f = 15 cm) at a distance of 30 cm from it. A glass plate ($\mu = 1.5$) of thickness 1 cm is placed on the other side of the lens perpendicular to the axis. Locate the image of the point object.

Solution: For refraction through lens



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}; \; u = -30 \text{ cm}, f = 15 \text{ cm}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{15} - \frac{1}{30} = \frac{1}{30};$$

$v = 30$ cm

Additional shift produced by glass plate,

$$\Delta t = \left(1 - \frac{1}{n}\right) \times t = \left(1 - \frac{1}{1.5}\right) \times 1 = 0.33 \text{ cm}$$

Distance of final image from the lens = 30 + 0.33 = 30.33 cm

Connector 15: A point object is placed at a distance of 15 cm from a convex lens. The image is formed on the other side at a distance of 30 cm from the lens. When a concave lens is placed in contact with the convex lens, the image shifts away further by 30 cm. Calculate the focal length of the two lenses.

Solution: For the convex lens, $u = -15$ cm, $v = 30$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}; \frac{1}{30} + \frac{1}{15} = \frac{1}{f} \Rightarrow f = 10 \text{ cm}$$

Now the problem can be approached by two methods.

Method 1: The image produced by the convex lens will act as the object for concave lens in that case, u = +30 cm and v given as +60 cm.

Method 2: Let F be the focal length of the combination. Then for the combination,
u = −15 cm, v = +60 cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F};\ \frac{1}{60} + \frac{1}{15} = \frac{1}{F};\ F = 12 \text{ cm}$$

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}.$$

$$\frac{1}{12} = \frac{1}{10} + \frac{1}{f_2};\ f_2 = -60 \text{ cm} \Rightarrow \frac{1}{f_2} = \frac{1}{60} - \frac{1}{30} \Rightarrow f_2 = -60 \text{ cm}$$

Thus focal length of convex lens is 10 cm and that of concave lens is 60 cm

Connector 16: An object is seen through a simple microscope of focal length 12 cm. Find the angular magnification produced if the image formed is at the near point of eye which is at 25 cm away from it

Solution: Angular magnification is given by

$$M = 1 + \frac{D}{f} = 1 + \frac{25}{12} = 3.08$$

Connector 17: If the focal length of the objective and eyepiece of a microscope are 2 cm and 5 cm respectively and the distance between them is 20 cm, what is the distance of the object from the objective when the image seen by the eye is 25 cm from eyepiece? What is the magnifying power?

Solution: Given $f_o = 2$ cm, $f_e = 5$ cm

$|v_o| + |u_e| = 20$ cm, $v_e = -25$ cm

For the eyepiece, $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$;

$$\frac{1}{-25} - \frac{1}{u_e} = \frac{1}{5}$$

$$u_e = -4.167 \text{ cm}$$

$$v_o = 20 - 4.167 = 15.833 \text{ cm}$$

For the objective,

$$\frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o};\ \frac{1}{15.833} - \frac{1}{u_o} = \frac{1}{2}$$

$$u_o = -2.29 \text{ cm}$$

$$M = \frac{v_o}{u_o}\left[1 + \frac{D}{f_e}\right] = \frac{15.833}{-2.29}\left[1 + \frac{25}{5}\right] = -41.5$$

Connector 18: Two men, one 160 cm tall and the other 170 cm tall are standing 4 m and 5 m away respectively from an observer. Which man will appear taller to the observer?

Solution: The angle subtended on the observer's eye by the first man, $\alpha_1 = \frac{1.6}{4} = 0.4$

And that by second man, $\alpha_2 = \frac{1.7}{5} = 0.34$

since $\alpha_1 > \alpha_2$, the first man will appear taller than the second.

Connector 19: In a double slit experiment, separation between the slits is 0.1 mm, wavelength of the light used = 600 nm and interference pattern is observed on a screen 1 m away. Find the band width.

Solution: We have $\beta = \dfrac{\lambda D}{d} = \dfrac{1 \times 600 \times 10^{-9}\ \text{m}}{0.10 \times 10^{-3}\ \text{m}} = 6\times10^{-3}\,\text{m}$

Connector 20: In a Young's double slit experiment, the distance between two sources is 0.1 mm and distance of the screen is 20 m. If wavelength used is 5460 Å, find the angular position of first dark fringe.

Solution: For first dark fringe n = 1

$$x = (2n-1)\frac{\lambda}{2}\frac{D}{d} = \frac{D\lambda}{2d}$$

$$\text{Angular position } \theta = \frac{x}{D} = \frac{\lambda}{2d}$$

$$= \frac{5460 \times 10^{-10}}{2 \times 10^{-4}} = 0.00273 \text{ rad}$$

Connector 21: In Young's double slit experiment, lights of $\lambda = 5.52\times10^{-7}\,\text{m}$ and $\lambda = 6.9\times10^{-8}\,\text{m}$ are used in turn, keeping the same geometry. Compare fringe widths in the two cases.

Solution:

$$\beta_1 = \frac{D}{d}\lambda_1 = \frac{D}{d} \times 5.52\times10^{-7}\,\text{m}$$

$$\beta_2 = \frac{D}{d}\lambda_2 = \frac{D}{d} \times 6.9 \times10^{-8}\,\text{m}$$

$$\frac{\beta_1}{\beta_2} = \frac{5.52 \times 10^{-7}}{6.9 \times 10^{-8}} = 8$$

Connector 22: A source of red light (λ = 7000 Å) produces interference through two slits placed at a distance of 0.01 cm. At what distance should a screen be placed from the slits so that interference bands are spaced 0.01 m apart?

Solution:

$\lambda = 7000\text{Å} = 7000 \times 10^{-10}\,\text{m}$

$d = 0.01 \text{ cm} = 0.01 \times 10^{-2}\,\text{m}$

$\beta = 0.01$ m

$\beta = \dfrac{D\lambda}{d}$ or

$$D = \frac{\beta d}{\lambda} = \frac{0.01 \times 0.01 \times 10^{-2}}{7000 \times 10^{-10}}$$

$= 1.429$ m

Connector 23: In a Young's double slit experiment, the width of the fringes obtained with light of wavelength 6000 Å is 2.0 mm. What will be the fringe width, if the entire apparatus is immersed in a liquid of refractive index 1.33?

Solution:

$$\beta = \frac{D\lambda}{d} \propto \lambda$$

$$\frac{\beta_a}{\beta_w} = \frac{\lambda_a}{\lambda_w} = n_w$$

$$\beta_w = \frac{\beta_a}{n_w} = \frac{2}{1.33} = 1.5 \text{ mm}$$

Connector 24: Light waves from two coherent sources of intensity ratio 25 : 1 produce interference. Calculate the ratio of maxima and minima of the intensity in the interference pattern.

Solution:

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{25}{1}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{5}{1} \text{ or } a_1 = 5a_2$$

In the interference pattern,

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(5a_2 + a_2)^2}{(5a_2 - a_2)^2} = \frac{6^2 a_2^2}{4^2 a_2^2} = 2.25$$

Connector 25: In Young's double slit experiment, two coherent sources of wavelength 585 nm are separated by 1.5 mm. The fringes are obtained at a distance of 2.5 m from the sources. Find the number of fringes in the interference pattern 4.9×10^{-3} wide.

Solution: $\lambda = 585 \times 10^{-9}$ m; $d = 1.5 \times 10^{-3}$ m; $D = 2.5$ m

$$\text{Fringe width } \beta = \frac{\lambda D}{d} = \frac{585 \times 10^{-9} \times 2.5}{1.5 \times 10^{-3}} = 9.75 \times 10^{-4} \text{ m}$$

$$\text{Number of fringes} = \frac{x}{\beta} = \frac{4.9 \times 10^{-3}}{9.75 \times 10^{-4}} = 5.02 \approx 5$$

Connector 26: Two slits 0.125×10^{-3} m apart are illuminated by light of wavelength 4500 Å. The screen is one metre away from the plane of the slits. Find the separation between the second bright fringes on both sides of the central maximum.

Solution: Separation between second bright fringes on both sides of the central maximum is equal to the total width of four fringes.

$$Y = \frac{4D\lambda}{d} = \frac{4 \times 1 \times 4500 \times 10^{-10}}{0.125 \times 10^{-3}}$$

$$= 14.4 \times 10^{-3} \text{ m} = 14.4 \text{ mm}$$

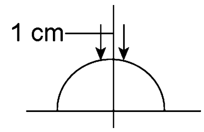
TOPIC GRIP

Subjective Questions

1. Two prisms, one of crown glass, the other of flint glass have dispersive powers of 0.036 and 0.048 respectively. Their refractive indices for yellow light are 1.5 and 1.6 respectively. Determine the refracting angle of the prisms required to form an achromatic combination capable of deviation 1°.

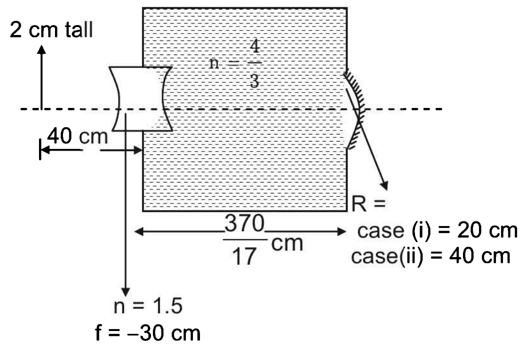
2. A solid sphere of glass (n = 1.5) and radius R is silvered over one hemisphere. A small object is located on the axis of the sphere at a distance R from the pole of the unsilvered hemisphere. Find the position of the final image.

3. A solid glass hemisphere having a radius of 8 cm and a refractive index of 1.50 is placed with its flat face downward on a table. A parallel beam of light of circular cross section of 1 cm diameter travels vertically downward and enters the hemisphere symmetical around its vertical radius. Determine the diameter of the circle of light formed on the table.

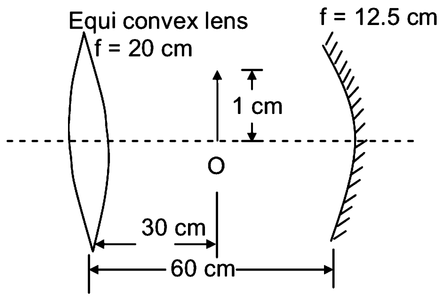


4. A tank contains water $\left(n = \frac{4}{3}\right)$. An equiconcave lens (n = 1.5, focal length = 30 cm) is fixed on one wall. A concave mirror of radius R is fixed coaxially on the opposite wall. Distance between them (i.e., width of the water tank) is $\frac{370}{17}$ cm. An object 2 cm tall is outside the tank on the common axis of the lens and mirror and 40 cm in front of the lens. Where should a screen be placed to get an image for

Case (i) R = 20 cm   Case (ii) R = 40 cm



5.



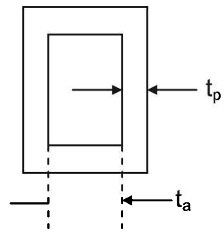
Give the details of all possible screen-catchable images of the object given in the figure. Also draw, an approximate ray diagram confirming your calculations

6. A convex lens and a concave lens have a ratio of their focal lengths $f_1/f_2 = -\frac{5}{4}$ in air. If they are kept in contact in a liquid medium the power of the combination is – 0.6 D. Refractive index of the liquid is 1.2 and that of the glass is 1.5
   (i) Determine their individual powers in air.
   (ii) What are their radii of curvature if they are equi-concave and equi-convex?
   (iii) Find the separation between the lenses in the liquid, for which they converge a parallel beam.

7. In a standard Young's double-slit experiment set up, two slits are spaced 0.4 mm apart and are placed 80 cm from a screen.
   (i) What is the distance between the fourth dark fringe on one side to the third bright fringe on the other side of centre if the slits are illuminated by light of wavelength 500 nm?
   (ii) Suppose the entire apparatus is immersed in a liquid of refractive index 1.2, what is the answer?

8. In the above problem, (with the apparatus immersed in the liquid) a thin film of refractive index 1.1 is introduced in front of the upper slit S_1, the intensity at the centre of the screen is now $\frac{1}{4}$ of what it was earlier.

(i) Has the fringe pattern shifted towards S_1 or S_2?
(ii) What is the minimum possible thickness of the film introduced?



9. (i) In a Young's double-slit experiment, a thin layer of air of thickness t_a is enclosed in a hollow squarish plastic container of wall thickness t_p, and it is placed in front of one of the slits. The refractive of index of the plastic $\mu_p = 1.4$.
Now what was originally the central maximum is shifted up by 4 fringe widths, when the wavelength used is λ = 500 nm. What is t_p?

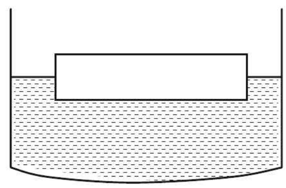
(ii) The entire setup is immersed in a liquid of refractive index $\mu_\ell = 1.3$ and it is observed that central maximum is restored to the original position. What is t_a?

10. At points of constructive interference between waves of equal amplitude, the intensity is 4 times that of either individual wave. Prove that this does not violate energy conservation.

Straight Objective Type Questions

Dirctions: **This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

11. A glass plate (n = 1.5) is placed on a container filled with water $\left(n = \frac{4}{3}\right)$. At what angle should a ray of light from air fall on the plate for total internal reflection to occur at the interface of water and glass?



(a) 45°
(b) 53°
(c) 90°
(d) TIR at glass/water interface not possible for any ray entering from air

12. The refractive index of the material of a prism is $\frac{5}{3}$ for a certain monochromatic ray. What should be the minimum angle of incidence of this ray on the prism so that no total internal reflection occurs when the ray leaves the prism? Refraction angle of prism = 45°.

(a) 20° (b) 18° (c) 16° (d) 13°

13. In a prism, if the incident ray is incident at an angle 90°, at what minimum refraction angle of the prism, will the refracted ray be subjected to total internal reflection on the second face, if $n = \frac{5}{3}$?

(a) 30° (b) 60° (c) 74° (d) 85°

14. In Young's double slit experiment, if red light (λ =650 nm) is used instead of green light (λ = 500 nm), the ratio of fringe width (red) to fringe width (green) is

(a) 1.1 (b) 1.3 (c) 1.5 (d) 1.71

15. In Young's double slit experiment, a thin glass plate is placed in the path of one slit, causing central bright band to shift to a position initially occupied by fifth bright band. n = 1.5 for glass, λ = 600 nm. The thickness of the plate is

(a) 10 μm (b) 9 μm (c) 6 μm (d) 5.1 μm

Assertion–Reason Type Questions

Directions: **Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

16. Statement 1

If a given pair of colours sent through a combination of two small angled prisms has dispersion without deviation, then the two prisms should be of different materials.

and

Statement 2

Deviation produced by one prism is cancelled by the other only if they are of different materials.

17. Statement 1

In a typical Young's double slit experiment set up, if the amplitudes of the two waves are unequal, though perfectly bright fringes can be seen, perfectly dark fringes cannot be seen.

and

Statement 2

By the superposition principle, if I_1 and I_2 are the intensities of individual waves and I the resultant intensity at a point

$I = I_1 + I_2$ for a bright fringe,

and $I = I_1 - I_2$ for a fringe of minimum intensity, which will not vanish if $I_1 \neq I_2$.

18. Statement 1

In a typical Young's double slit experiment, interference pattern cannot be observed if the separation between the slits is less than the wavelength of the light.

and

Statement 2

For interference patterns to be observed, there should be no phase difference between the two constituent waves.

19. Statement 1

If a light ray from air entering into a medium travels along a curved path in the medium (due to the fact that the refractive index varies continuously) and emerges out of a parallel face (parallel to the incident face), then the emergent ray need not be parallel to the incident ray.

and

Statement 2

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ ….. (symbols have usual meaning)

20. Statement 1

A lens of negative power in air can be made converging type by immersing it in a suitably chosen medium.

and

Statement 2

The focal length of a lens is related to the radii of curvatures R_1 and R_2 of the two faces by the formula $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

where n is the relative refractive ixndex of the lens with respect to the surrounding medium which can have positive values less than one as well

Linked Comprehension Type Questions

Directions: **This section contains a paragraph. Based upon the paragraph, multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

Passage I

Interference in thin films

The brilliant colours that are often seen when light is reflected from a soap bubble or from a thin layer of oil floating on water are produced by interference effects between two light waves reflected at opposite surfaces of the thin film of soap solution or of oil.

See figure A. ab is a ray in a beam of monochromatic light incident on the upper surface of a thin film. A part of the incident light is reflected at the first surface, as indicated by ray bc; and a part represented by bd is transmitted. At second surface, a part is again reflected, and of this, a part emerges as represented by ray ef. When multiple rays are present, the rays bc and similar to ef come together and interfere. Depending upon phase relationship they may interfere constructively or destructively. Because different colours have different wavelengths, the interference may be constructive for some colours and destructive for some others. Hence the appearance of coloured rings or fringes.

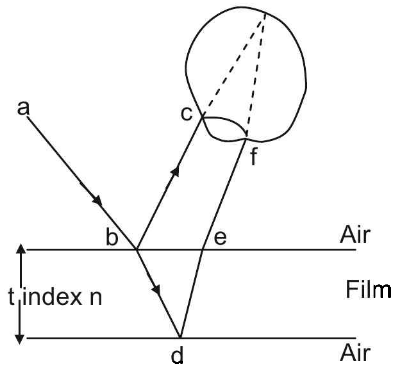


Fig. A

To discuss these phenomena in an experimental context, let us consider interference of monochromatic light reflected from two nearly parallel surfaces. Fig. B shows two plates of glass separated by a wedge of air;

We want to consider interference between the two light waves reflected from the surfaces adjacent to the air wedge as shown. If the observer is at a great distance compared to the other dimensions, and the rays are nearly perpendicular, then the path difference between the two waves is just twice the thickness d of the air wedge at each point. At points for which this path difference is an integer number of wavelengths, we expect to see constructive interference. Along the line where the plates are in contact, there is no path difference and we expect a bright area.

When we actually carry out the experiment, the bright and dark fringes appear as expected, but they are interchanged! Along the line of contact, a dark fringe, not a bright one, is found.

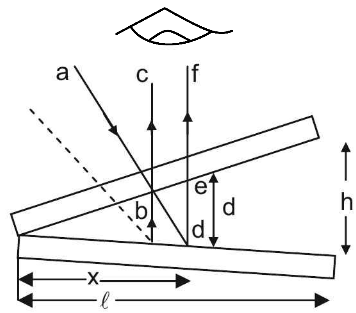


Fig. B

The inescapable conclusion is that one of the waves has undergone a π phase shift during its reflection.

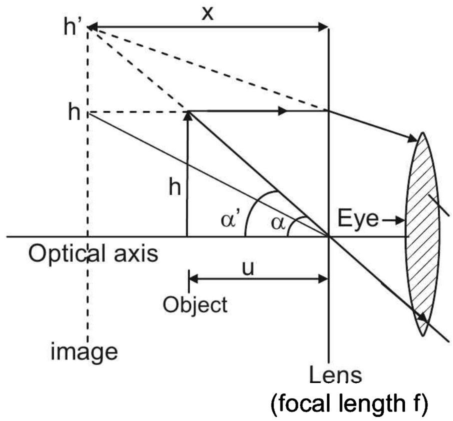
Further experiments show that the π phase change occurs whenever the material in which the wave is initially travelling before reflection has a smaller refractive index.

21. If the arrangement in Fig. B is illuminated first by blue and then by red, then
(a) the spacing of red fringes is greater than that of blue
(b) the spacing of red fringes is smaller than that of blue
(c) the spacing of red fringes is equal to that of blue
(d) It can be any of the above depending on the value of i

22. In the arrangement in Fig. B with a monochromatic light, the spacing between successive fringes.
(a) increases with x
(b) decreases with x
(c) constant over x
(d) any of the above depending upon value of i

23. Suppose the two glass plates in Fig.2 are $\ell = 20$ cm, at one end they are in contact; At the other end they are separated by a thin piece of tissue paper 0.04 mm thick. $\lambda = 500$ nm. Then the fringe adjacent to their line of contact is
(a) dark
(b) bright
(c) of average intensity
(d) can be any one of the above depending on value of h

Passage II



A magnifying glass consists of a converging lens forming a virtual, erect and enlarged image of the object. The image must be formed beyond a certain minimum distance from the eye for clarity, called the least distance (near point) of distinct vision D. The schematic diagram of a magnifier is shown.

We assume that the rays make small angles with the optic axis.

The magnification M is defined as the angle subtended by the image divided by the angle the object would subtend if it were observed, unaided, at the near point.

Observation 1

We have alleady stated that image is at D, i.e., x = D hence a and d an correct error!!

24. The value of the magnification M, when image is at D is:

(a) $M = \frac{2x}{u}$ (b) $M = \frac{u}{D}$

(c) $M = \frac{D}{u}$ (d) $M = \frac{x}{D}$

25. The eye will be able to focus on the image if $x \geq D$. The correct lens equation is:

(a) $\frac{1}{u} = \frac{1}{f} + \frac{1}{x}$ (b) $\frac{1}{u} = \frac{1}{f} - \frac{1}{x}$ (c) $\frac{1}{u} = \frac{1}{x} - \frac{1}{f}$ (d) $\frac{1}{u} = -\frac{1}{x} - \frac{1}{f}$

26. We intend to get a maximum magnification by placing the eye near the lens and increasing the angle α as far as possible. The maximum value of M possible is

(a) $\frac{D}{f}$ (b) $1 + \frac{D}{f}$ (c) $\frac{f}{D}$ (d) $1 + \frac{f}{D}$

Multiple Correct Objective Type Questions

Directions: **Each question in this section has four suggested answers out of which ONE OR MORE answers is/are correct.**

27. An object is placed 15 cm in front of a concave mirror, which forms a real image of the object. When the object is moved 0.1 cm away from mirror, image moves by 0.4 cm.

(a) The image moves towards the mirror
(b) The image moves away from the mirror
(c) Magnification in original position is −2
(d) Focal length of the mirror is 10 cm (modulus value)

28. An object and a screen are separated by 20 cm. A convex lens of focal length 4.2 cm is placed between them.

(a) When the lens is 6 cm from the object, a magnified image is formed on the screen
(b) When the lens is 8 cm from the object, a magnified image is formed on the screen
(c) When the lens is 14 cm from the object, an inverted image is formed on the screen
(d) When the lens is 14 cm from the object, no image is formed on the screen.

29. In a Young double slit experiment, λ = 6000 Å, D = 1 m, d = 0.8 mm (all standard notations), when a film of μ = 1.6 and thickness t_1 is covering the top slit, the central bright band shifts to the position of original 3rd bright band on the top. The above film is removed and another film of thickness t_2 and μ = 1.6 covers the bottom slit. Now the central bright fringe shifts to a position between the original 2nd and 3rd bright fringe and at the position of the original central bright fringe a dark fringe appears. Then,

(a) $t_1 = 3\ \mu m$
(b) $t_2 = 2.5\ \mu m$
(c) when both films are used together the original central fringe position is occupied by a dark band.
(d) when both films are used together the original central fringe position is a white band

Matrix-Match Type Question

Directions: **Match the elements of Column I to elements of Column II. There can be single or multiple matches.**

30. A lens of focal length f is interposed between a real object and a screen separated by a distance D. Then

Column I	Column II
(a) $D < 4f$	(p) Image position at $\frac{D}{2}$ for a convex lens
(b) $D > 4f$	(q) No screen catchable- image for a concave lens
(c) $D = 4f$	(r) $uv > 4f^2$: for convex lens
(d) $D = \infty$	(s) No screen-catchable image for a convex lens

IIT ASSIGNMENT EXERCISE

Straight Objective Type Questions

Directions: **This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

31. An object is placed 5 cm in front of a concave mirror of radius of curvature 15 cm. Then u, v, f (in cm) and m are respectively

(a) –5, –15, –7.5, –3 (b) 5, –15, –7.5, 3 (c) –5, 15, 7.5, –3 (d) –5, 15, –7.5, 3

32. Which of the following is not true for the image formed in the above problem?

(a) Virtual (b) Erect (c) Behind the mirror (d) diminished

33. Which of the following is not true?

(a) Focal length of a plane mirror is zero
(b) Power of a plane mirror is zero
(c) Linear magnification produced by plane mirror is unity
(d) Deviation produced on one reflection from a plane mirror = (180 – 2i)

34. A man 1.8 m tall wishes to see his full-length image in a plane mirror hanging vertically on a wall. The length of the shortest mirror in which he can see his full-length image is

(a) 0.45 m (b) 0.9 m (c) 1.8 m (d) 1 m

35. The height of the mirror from the ground in the above question is (the eyes of the man are at 1.6 m above the ground)

(a) 0.6 m (b) 0.8 m (c) 1 m (d) 1.6 m

36. The number of images formed between two parallel mirrors is

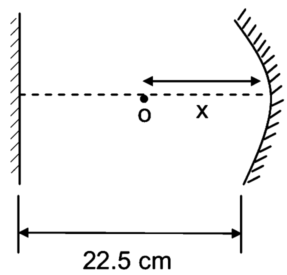
(a) 2 (b) 4 (c) 6 (d) Infinite

37. A concave mirror with a radius of curvature R is illuminated by a candle at point P_1, 4 m from the mirror. A real image of the candle is formed at the point P_2. If the candle is placed at P_2, then the mirror will produce the image at

(a) 6 m from the mirror (b) 3 m from the mirror

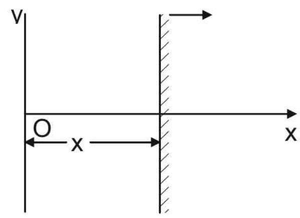
(c) 4 m from the mirror (d) A distance $\frac{R\ell}{2}$ (where, ℓ = distance between P_1 and P_2)

38. A concave mirror of focal length 10 cm is separated from a plane mirror by 22.5 cm. The value of x, so that the first image of object 'o' due to the concave mirror and plane mirror coincide is:



(a) 20 cm (b) 30 cm (c) 15 cm (d) 10 cm

39. A point object is placed at O. A plane mirror placed parallel to y-axis is at rest at $x = x_0$ at $t = 0$ has acceleration $\vec{a} = a\hat{i}$. The velocity of the image of the object O, in the plane mirror at time $t = \frac{x_0}{a}$ is:



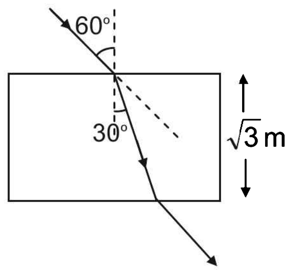
(a) $2x_0$ (b) x_0

(c) $\frac{x_0}{2}$ (d) $\frac{x_0}{4}$

40. A ray of light is incident on a rarer medium (μ_1) from the denser medium (μ_2) at an angle of incidence i. The angle between the reflected ray and the refracted ray is 90°. The critical angle of incidence is

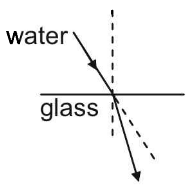
(a) $\sin^{-1}(\cot i)$ (b) $\cot^{-1}(\sin i)$ (c) $\sin^{-1}(\tan i)$ (d) $\tan^{-1}(\sin i)$

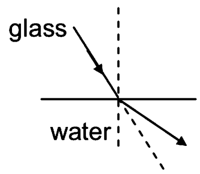
41. The lateral shift produced by the slab is:

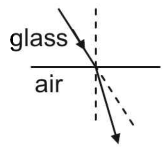


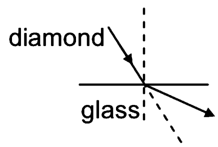
(a) 0.5 m (b) 1 m (c) 1.5 m (d) 2 m

42. The incorrect ray diagram is

(a) 

(b) 

(c) 

(d) 

43. A ray travels from a medium of refractive index 1.52 to a medium of refractive index 1.33. The critical angle for the boundary separating the two media is:

(a) $\sin^{-1}\frac{1}{1.33}$ (b) $\sin^{-1}\frac{1}{1.52}$ (c) $\sin^{-1} 0.5$ (d) $\sin^{-1} 0.875$

44. To obtain a parallel reflected beam from a torch, reflector in the torch should be

(a) convex (b) plane (c) concave (d) either (a) or (c)

45. A fish looking up through water sees the outside world contained in a circular horizon of radius 'r'. If the refractive index of water is 4/3 and the fish is 12 cm below the surface of water, the radius 'r' of the circle in cm is

(a) $36\sqrt{7}$ (b) $\frac{36}{\sqrt{7}}$ (c) $4\sqrt{5}$ (d) $2\frac{5}{\sqrt{3}}$

46. A beaker is filled with water to a height of 10 cm. A microscope is focused on a mark at the bottom of the beaker. Water is now replaced by a liquid of refractive index 1.60 to the same height. By what distance should the microscope be moved to focus on the mark again? (Refractive index of water is 1.33)

(a) 1.27 cm upwards (b) 2.67 cm upwards (c) 1.67 upwards (d) 1.5 cm downwards

47. A small pin on a flat table is viewed from 30 cm directly above. If the pin is viewed from the same point through a glass slab of thickness 10 cm and refractive index 1.6, how far does it appear to be raised?

(a) 5 cm (b) 3.75 cm (c) 6.75 cm (d) 9.5 cm

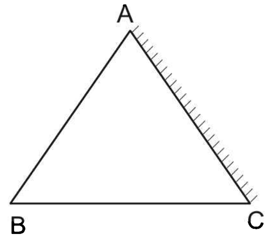
48. A thin prism P_1 of angle 4° and made of glass of refractive index 1.54, is combined with another thin prism P_2 of angle 3° to produce dispersion without deviation. The refractive index of P_2 is

(a) 1.54 (b) 1.72 (c) 2.16 (d) 1.33

49. A prism has refracting angle of 60°. When a ray is incident at 50° on one face, it suffers minimum deviation (δ_m). The value of δ_m is

(a) 45° (b) 60° (c) 55° (d) 40°

50. ABC is an equilateral prism with side AC (silvered). A ray incident normally on face AB emerges out through face BC after reflection at AC. The angle made by the emergent ray with face BC is ($\mu = \sqrt{3}$)

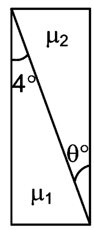


(a) 30°
(b) 60°
(c) 45°
(d) 90°

51. A prism of angle 75° has refractive index $\sqrt{2}$. The maximum angle of incidence so that total internal reflection does not occur is

(a) 25° (b) 22.5° (c) 60° (d) 45°

52. The figure shows a combination of two thin prisms producing dispersion without deviation ($\mu_1 = 1.54$, $\mu_2 = 1.72$). The value of θ is

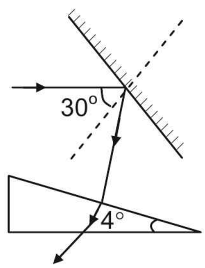


(a) 1°
(b) 2°
(c) 3°
(d) 4°

53. The deviation produced on yellow light by a small angled prism of angle 5° and refractive index 1.5 is

(a) 1° (b) 1.5° (c) 2° (d) 2.5°

54. A ray of light strikes a plane mirror at an angle of 30° as shown in the figure. After reflection, the ray passes through a prism of refractive index 1.5 whose apex angle is 4°. The total angle through which the ray deviates is



(a) 90°
(b) 270°
(c) 145°
(d) 122°

55. A solid hemisphere of glass of refractive index $3/2$ has radius 20 cm. For a parallel beam falling normally on the plane surface, the focal point from the plane surface is at

(a) 60 cm (b) 50 cm (c) 40 cm (d) 25 cm

56. An object is placed at a distance of 4 cm in front of a concave lens of focal length 12 cm. The nature and position of the image from the lens is

(a) real and 4 cm (b) virtual and 3 cm (c) real and 3 cm (d) virtual and 4 cm

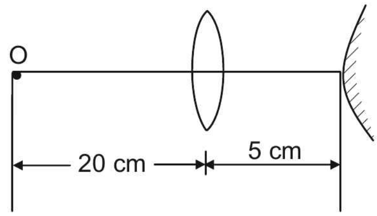
57. When an object is moved along the axis of a lens, images three times the size of the object are obtained when the object is at 16 cm and at 8 cm from the lens. The focal length and nature of the lens are

(a) 12 cm, concave (b) 4 cm, concave (c) 4 cm, convex (d) 12 cm, convex

58. A thin converging lens of refractive index 1.5 has a power of +5 D. When this lens is immersed in a liquid, it acts as a diverging lens of focal length 100 cm. The refractive index of the liquid is

(a) 4/3 (b) 3/2 (c) 5/3 (d) 1/3

59. A convex lens of focal length 15 cm is placed in front of a convex mirror. When the object is placed at O, the image coincides with it. The radius of curvature of mirror is



(a) 20 cm
(b) 40 cm
(c) 15 cm
(d) 55 cm

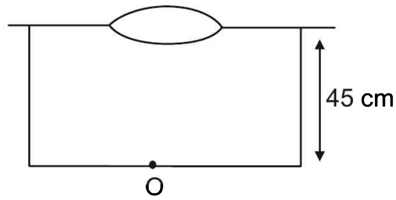
60. A source and a screen are separated by a distance of 'd'. A convex lens of focal length 16 cm produces a sharp image of an object in two positions which are 60 cm apart when placed between source and screen. The value of d is

(a) 160 cm (b) 120 cm (c) 100 cm (d) 140 cm

61. From a convex lens of focal length f, a luminous point object is placed at a distance of 3f. A glass slab of thickness 't' is placed between object and lens. A real image of the object is formed at the shortest possible distance from the object. The refractive index of material of the slab is

(a) $\frac{t}{t-f}$ (b) $\frac{t}{f}$ (c) $\frac{f}{f-t}$ (d) $\frac{f}{t}$

62. A convex lens is placed over a beaker of depth 45 cm. The image of an object at the base is seen at a height of 36 cm over the lens. When the beaker is filled with a transparent liquid upto a height 40 cm the image of the object is at a height of 48 cm above the lens. The refractive index of material of the liquid is

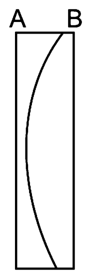


(a) 1.52 (b) 1.37
(c) 1.44 (d) 1.33

63. A bi-convex lens of equal radius of curvature R for each refracting surface, is placed at a point P in the path of a beam converging to a point M, such that PM = $\frac{R}{3}$. If the refractive index of the material of lens is 1.5, the magnification produced by the lens is

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$ (c) $\frac{4}{3}$ (d) 1

64. The figure shows a combination of two thin lenses A and B having refractive indices μ and $\frac{4\mu}{3}$ respectively. The radius of curvature of the interface is R. The focal length of the combination is



(a) $\frac{R}{\mu}$ (b) $\frac{R}{3\mu}$
(c) $\frac{3R}{\mu}$ (d) $\frac{4R}{3\mu}$

65. The focal length of a plano convex lens of radius of curvature R is 3R. The refractive index of the lens is

(a) 1.33 (b) 1.50 (c) 1.66 (d) 3

66. The power of a convex lens (μ = 1.5) is P in air. When surrounded by a medium of refractive index μ', its power becomes $-\frac{P}{2}$. The refractive index of the medium μ' is

(a) 2 (b) 1.75 (c) 1.25 (d) 2.5

67. If the focal length of a convex lens is reduced to half its original value, its focal power in dioptre becomes x times the original focal power, where

(a) $x = \frac{1}{2}$ (b) $x = 1$ (c) $x = 2$ (d) $x = \frac{3}{2}$

68. A square object is magnified 16 times by a convex lens at 30 cm from the object. The image distance from the lens in cm is

(a) 40 (b) 80 (c) 120 (d) 480

69. When a ray of light enters from one medium to another, the quantity that remains constant is

(a) amplitude (b) wavelength (c) speed (d) frequency

70. Light of wavelength 6000 Å travelling in vacuum enters a medium of refractive index 1.5. The refracted beam has a wavelength of

(a) 300 nm (b) 400 nm (c) 500 nm (d) 600 nm

71. Some relations are given below (v – velocity of light in the medium, n – refractive index of the medium, λ – wavelength of light in air, λ' – wavelength in the medium)

(i) $v \propto \frac{1}{n}$ (ii) $v \propto \frac{1}{\lambda}$

(iii) $\lambda' = \frac{\lambda}{n}$ (iv) $v \propto \lambda'$

(v) $v \propto n$

The correct relations are

(a) (i), (ii), (iii) (b) (ii), (iii), (v) (c) (i), (iii), (iv) (d) (i), (v), (iii)

72. If c and v represent velocity of light in air and water, then $\frac{c}{v}$ represents

(a) frequency of light (b) wavelength of light
(c) refractive index of water (d) reciprocal of refractive index of water

73. The refractive index of glass is 1.5. The speed of light in glass is

(a) 1×10^8 m s^{-1} (b) 2×10^8 m s^{-1} (c) 3×10^8 m s^{-1} (d) 6×10^8 m s^{-1}

74. If v_1 and v_2 are the velocities of light in media of refractive indices n_1 and n_2 respectively, such that $v_1 > v_2$ then

(a) $n_1 < n_2$ (b) $n_1 > n_2$ (c) $n_1 = n_2$ (d) $\frac{n_1}{n_2} \le 1$

75. A monochromatic beam of light of λ = 6000Åin vacuum enters a medium of refractive index 1.5. Inside the medium its frequency is

(a) 0.5×10^{14} Hz (b) 5×10^{14} Hz (c) 5×10^{10} Hz (d) 0.5×10^{12} Hz

76. In Young's double slit experiment, the central fringe shifts to the position occupied by the fifth bright fringe when a thin film of refractive index $\frac{3}{2}$ is placed in front of one of the slits. The wavelength of light is 4000 Å. The thickness of the film is

(a) 3 μm (b) 5 μm (c) 4 μm (d) 2 μm

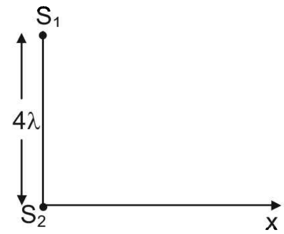
77. Two light waves represented by $E_1 = E_0 \sin\left(\omega t + \frac{\pi}{3}\right)$ and $E_2 = E_0 \cos\left(\omega t + \frac{\pi}{3}\right)$ interfere to produce a wave of amplitude

(a) $2 E_0$ (b) $\sqrt{2} E_0$ (c) $\sqrt{3} E_0$ (d) $\frac{\sqrt{3}}{2} E_0$

78. Two waves of intensities I_1 and I_2 interfere. The ratio of the maximum to minimum intensity of the resultant is n. Then, $\frac{I_1}{I_2}$ is

(a) $\frac{\sqrt{n}+1}{\sqrt{n}-1}$ (b) $\frac{n^2+1}{n^2-1}$ (c) $\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)^2$ (d) $\left(\frac{n+1}{n-1}\right)^2$

79. S_1 and S_2 are two coherent sources placed along the y-axis with S_2 at the origin. Distance between S_1 and S_2 is 4λ. The minimum distance from S_2 on x-axis where the intensity of light is maximum is



(a) $7.5\,\lambda$
(b) $3.5\,\lambda$
(c) $2.5\,\lambda$
(d) $1.5\,\lambda$

80. A thin film ($\mu = 1.6$) of thickness 10^{-5} mm is introduced in the path of one of the two interfering beams. The central fringe moves to a position occupied by the tenth bright fringe earlier. The wavelength of the light is

(a) 4800 Å (b) 6000 Å (c) 7200 Å (d) 5500 Å

81. A concave mirror of focal length f produces a real image $\frac{1}{n}$ th of the size of the object placed transverse to the principal axis. The distance of the object is

(a) nf (b) $\frac{f}{n}$ (c) $(1 + n)\,f$ (d) $\left(1+\frac{1}{n}\right)f$

82. The possible position of the object in front of a concave mirror (focal length 30 cm) so as to produce an image thrice the object size is

(a) 30 cm (b) 40 cm (c) 20 cm (d) (b) and (c)

83. A thin rod of length ℓ is placed along the principal axis of a concave mirror of focal length 5ℓ. The image of the rod is real and diminished and just touches the rod. The magnitude of the magnification is

(a) $\frac{2}{3}$ (b) $\frac{5}{6}$ (c) $\frac{5}{8}$ (d) $\frac{3}{8}$

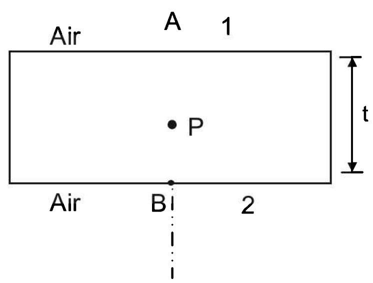
84. A concave mirror has a focal length f_0 in air. The focal length of the mirror in water $\left(\mu = \frac{4}{3}\right)$ is

(a) f_0 (b) $\frac{4f_0}{3}$ (c) $\frac{3f_0}{4}$ (d) $\frac{f_0}{3}$

85. A convex mirror of radius of curvature 40 cm is placed with its pole at a point P in the path of a beam converging to a point M on the axes of the mirror, such that PM = 30 cm. The image is

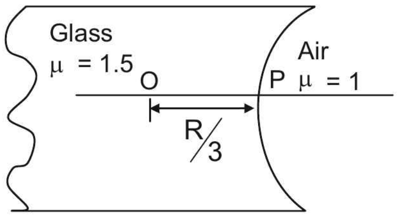
(a) real, inverted and diminished
(b) virtual, erect and enlarged
(c) virtual, erect and diminished
(d) virtual, inverted and enlarged

86. A black spot P inside a glass slab (refractive index μ) appears to be at a distance d from A when viewed from side 1 and at a distance 3d from point B when viewed from the opposite side 2. (Assume near normal viewing). The thickness (t) of the slab must be



(a) 4d (b) $4\mu d$ (c) $(4\mu - 1)\,d$ (d) $4(\mu - 1)\,d$

87.



The image of the point O by refraction through the spherical surface of radius R as shown in fig. is

(a) $\frac{R}{4}$ to the right of P (b) $\frac{R}{5}$ to the right of P (c) $\frac{R}{4}$ to the left of P (d) $\frac{R}{5}$ to the left of P

88. The angle of minimum deviation of an equilateral prism of refractive index $\sqrt{2}$ is

(a) 60° (b) 30° (c) 45° (d) 53°

89. A prism of refracting angle 60° produces a minimum deviation of 30°. The angle of incidence is

(a) 45° (b) 90° (c) 30° (d) 60°

90. A convex lens of refractive index μ has a focal length f_0 in air. The focal length, when it is immersed in a liquid of refractive index $\frac{5\mu}{6}$, will be

(a) $\frac{5f_0}{6}$ (b) $\frac{5\mu f_0}{6}$ (c) $5(\mu - 1) f_0$ (d) $\frac{(\mu - 1)f_0}{6}$

91. A combination of two thin lenses in contact has a power P diopter. If the focal length of one lens is f_0 cm, the focal length of the other lens in cm is

(a) $\frac{f_0}{Pf_0 - 100}$ (b) $\frac{100f_0}{\frac{Pf_0}{100} - 1}$ (c) $\frac{100f_0}{Pf_0 - 100}$ (d) $\frac{f_0}{Pf_0 - 1}$

92. A bi-concave lens (refractive index $\frac{3}{2}$) has radius of curvature R for each surface. If it is immersed in a medium of refractive index 2, it will behave as

(a) a converging lens of focal length 2R (b) a converging lens of focal length $\frac{2R}{3}$

(c) a diverging lens of focal length 2R (d) a diverging lens of focal length $\frac{R}{3}$

93. A plano concave lens has a radius of curvature 30 cm and refractive index 1.5. The power of the lens (kept in air) is

(a) 5 D (b) $\frac{5}{3}$D (c) $-\frac{5}{3}$D (d) −3 D

94. When a monochromatic beam of light passes from glass to air, then

(a) the frequency of light increases (b) the speed of light decreases

(c) the wavelength of light decreases (d) the speed and wavelength of light both increase.

95. The refractive index μ of a medium is $\mu = a + \frac{b}{\lambda^2}$, where a, b are positive constants and λ the wavelength of the incident light, then (v = violet, r = red, g = green)

(a) $\mu_R > \mu_V$ (b) $\mu_V > \mu_R$ (c) $\mu_G > \mu_V$ (d) (a) and (b)

96. In Young's double slit experiment using blue and red lights, the nth blue bright fringe coincides with the (n – 1)th red bright fringe. If the wavelength of blue and red light are 5200 Å and 7800 Å respectively, the value of n is

(a) 1 (b) 2 (c) 3 (d) 4

97. In Young's double slit experiment with monochromatic light, the fringe width changes by 30 μm when the screen is moved by 5 cm towards the slits. The wavelength of the light is (distance between the slits = 1 mm)

(a) 4000 Å (b) 6000 Å (c) 3000 Å (d) 5000 Å

98. In Young's double slit experiment with white light, the point P directly opposite to one of the slits is observed on the screen. If the separation of the slits is d and the distance of the screen D >> d, the missing wavelengths at P are

(a) $\frac{d^2}{D}, \frac{4d^2}{3D}$ (b) $\frac{d^2}{3D}, \frac{4d^2}{5D}$ (c) $\frac{d^2}{D}, \frac{d^2}{3D}$ (d) (b) and (c)

99. In Young's double slit experiment the distance between the slits is doubled and the wavelength of light is doubled, then the fringe width will be

(a) four times (b) doubled (c) the same (d) half

100. In the Young's double slit experiment, the two slits are identical (each having intensity I_0). The intensity of the central maximum is

(a) $2I_0$ (b) $4I_0$ (c) I_0 (d) $3I_0$

101. A sustained interference pattern is obtained from two sources and the bandwidth is 2 mm. If λ_a and λ_b are the wavelengths of the two sources, then

(a) $\frac{\lambda_a}{\lambda_b} = 1$ (b) $\frac{\lambda_a}{\lambda_b} = 2$

(c) $\frac{\lambda_a}{\lambda_b} = 4$ (d) There is no relation between λ_a and λ_b

102. A thin transparent sheet is placed in front of a slit in Young's double slit experiment. The fringe width will

(a) increase (b) decrease (c) remain the same (d) become nonuniform

103. Which of the following is conserved when light waves interfere?

(a) Intensity (b) Energy (c) Amplitude (d) None of these

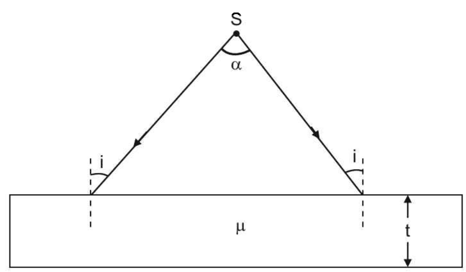
104. In a Young's double slit experiment, if the separation between coherent sources is halved and distance of screen from coherent sources is tripled, then bandwidth becomes

(a) tripled (b) halved (c) one-sixth (d) six times

105. If 16 : 4, is the intensity ratio of the two coherent sources producing the interference fringes, the ratio of minimum intensity to the maximum intensity in the interference pattern is

(a) 1:12 (b) 1:4 (c) 1:3 (d) 1:9

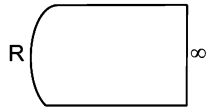
106. A diverging beam of light from a point source S having divergence angle α, falls symmetrically on a glass slab as shown. The angles of incidence of two extreme rays are equal. If the thickness of the glass slab is t and its refractive index is μ, then the divergence angle of the emergent beam is



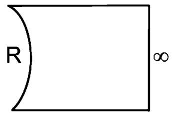
(a) zero (b) α (c) $\sin^{-1}\left(\frac{1}{\mu}\right)$ (d) $2\sin^{-1}\left(\frac{1}{\mu}\right)$

107. Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams.

(a) 

(b) 

(c) 

(d) 

108. Signal lights are usually red in colour because
(a) it is more sensitive to eye
(b) its wavelength is minimum in the visible range of spectrum
(c) its velocity is more than that of other colours
(d) scattering is least compared to other colours

109. If wavelength is doubled, resolving power of a microscope becomes
(a) double (b) half (c) 3 times (d) one fourth

110. For normal incidence of white light on air film, a few colours in the reflected light are absent. The condition for path difference of the interfering beam for missing colours is
(a) $n\lambda$ (b) $(2n + 1)\dfrac{\lambda}{2}$ (c) $(n - 1)\dfrac{\lambda}{2}$ (d) $(2n + 1)\lambda$

Assertion–Reason Type Questions

Directions: **Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

111. Statement 1
If an object becomes invisible when seen through a coloured glass, the object and the background can be of different colours.
and

Statement 2
A coloured glass absorbs light of that colour.

112. Statement 1
A lens of positive power can be made more converging by immersing it in a suitably chosen medium.
and

Statement 2
In the formula $\dfrac{1}{f} = (n - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$, n is the relative refractive index of the lens with respect to the surrounding medium, other symbols having usual meanings.

113. Statement 1

If a diver A under the surface of water in a swimming pool and a diver B on a diving board, normally above the water surface, are separated by an actual normal distance 'd', the apparent separation seems to be more than 'd' for A and less than 'd' for B.

and

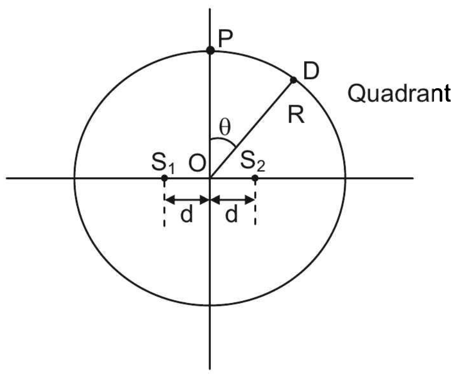
Statement 2

Water is optically denser to air and normal shift of the image is away from the viewer when viewed from a denser medium into a rarer medium, and towards the viewer from a rarer medium into a denser medium.

Linked Comprehension Type Questions

Directions: **This section contains a paragraph. Based upon the paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

Passage I



Two coherent sources are placed symmetrically about the centre O of a circle of radius R. A detector D moves around the circle. Its location is specified by the angle θ. The wavelength of the light is λ. (d<<R)

114. Consider the case where the source S_1 is described by $E_1 = E_0 \sin \omega t$ and S_2 by $E_2 = E_0 \cos \omega t$. The values of θ for maxima at the detector D are given by (m = 1, 2, 3,...)

(a) $\cos\theta = \dfrac{m\lambda}{4d}$

(b) $\cos\theta = \dfrac{\left(2m + \frac{1}{2}\right)\lambda}{4d}$

(c) $\cos\theta = \dfrac{m\lambda}{2d}$

(d) $\cos\theta = \dfrac{\left(2m - \frac{1}{2}\right)\lambda}{4d}$

115. The number of points on the circle having maxima at the detector in a quadrant $(0 \le \theta \le \pi/2)$, if $d = 5\lambda$, is

(a) 11 (b) 19 (c) 10 (d) 7

116. At the position P (see figure above) where the perpendicular bisector of S_1, S_2 meets the circle, the intensity I is (I_0 is the maximum intensity)

(a) zero (b) I_0 (c) $\dfrac{I_0}{2}$ (d) $\dfrac{I_0}{4}$

Multiple Correct Objective Type Questions

***Directions:* Each question in this section has four suggested answers out of which ONE OR MORE answers is/are correct.**

117. In a Young's double slit experiment, I_0 is the maximum intensity and β is the fringe width. If δ is the phase difference between the interfering beams at any point P at a distance y from the centre band, then

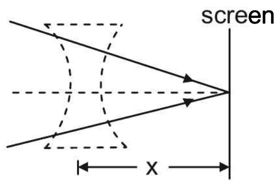
(a) intensity I' at $P = I_0 \cos^2 \dfrac{\delta}{2}$

(b) intensity I' at $P = I_0 \cos^2 \delta$

(c) intensity I' at $P = I_0 \cos^2 \left(\dfrac{\pi y}{\beta}\right)$

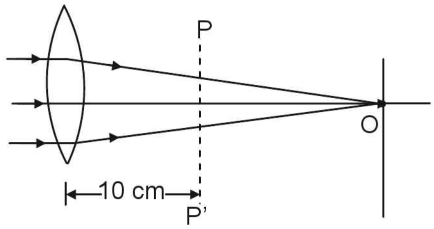
(d) If y = 0.25 mm & β = 1 mm, intensity I' at $P = \dfrac{I_0}{4}$

118. A converging beam of light produces an image on the screen. Now a concave lens of focal length 10 cm is placed on the path of the beam at a distance x from the position of the screen. The new position of image will be:



(a) If x = 8 cm image is formed on RHS
(b) If x = 9 cm image is formed on RHS
(c) If x = 10 cm image is formed on RHS
(d) If x = 20 cm image is formed 20 cm from lens

119. A parallel beam of light is converged to a point O by a lens of power +5 dioptre. PP' may be a lens or mirror as referred in each option and is positioned as shown.



(a) If PP' is a convex mirror of focal length 5 cm, the beam as such is reflected back from the system so that it retraces its path.
(b) If PP' is a concave mirror of focal length 5 cm, the beam as such is reflected back from the system so that it retraces its path.
(c) If PP' is a convex lens of focal length 10 cm, a parallel beam progresses towards right side
(d) If PP' is a concave lens of focal length 10 cm, a parallel beam progresses towards right side

Matrix-Match Type Questions

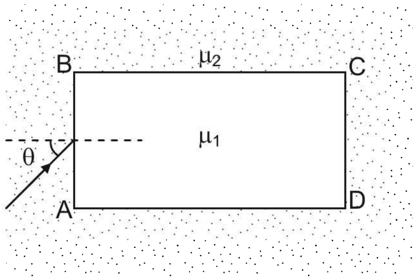
***Directions:* Match the elements of Column I to elements of Column II. There can be single or multiple matches.**

120. A ray of light is incident on a prism of angle A and refractive index μ at an angle i_1 and emerges at an angle i_2. The refracted angles in the prism are r_1 and r_2, respectively. The deviation of the incident ray is δ. Then

Column I	Column II
(a) δ	(p) $(\mu - 1)A$
(b) δ($\angle A$ and $\angle i_1$ are small)	(q) $(2i - A)$
(c) δ_{min}	(r) $(i_1 + i_2) - A$
(d) δ_{max}	(s) $\left(\dfrac{\pi}{2} + i_2 - A\right)$

ADDITIONAL PRACTICE EXERCISE

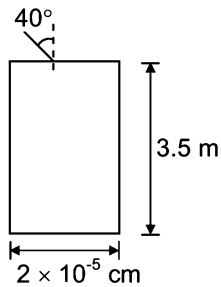
Subjective Questions



121. A rectangular slab ABCD of refractive index μ_1 is immersed in water of refractive index μ_2 ($\mu_2 < \mu_1$)

(a) Find the range of the angle of incidence θ inside water, such that the ray reaching BC gets totally internally reflected and comes out of the slab from the surface CD.

(b) For $\mu_2 = 1$, find the value of μ_1 so that at any angle of incidence, if the ray reaches BC, it will have TIR at BC and the ray emerges through CD.



122. An optical fibre ($\mu = 1.3$) is 3.5 m long and of 2×10^{-5} m diameter. A ray of light is incident on one end of the fibre at an angle $\theta = 40°$. Show that the condition for total internal reflection inside the fibre is fulfilled. Find the number of reflections the ray makes before emerging from the other end. (Take $\sin 40° = 0.65$)

123. Find the position and nature of the image of the fish in the transparent spherical the tank containing water $\left(\mu = \frac{4}{3}\right)$

Fish is positioned in the spherical container as shown in Fig. A.

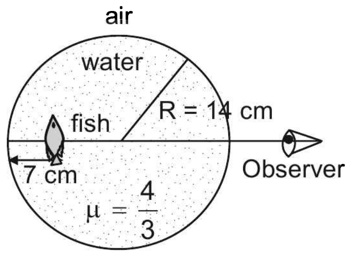
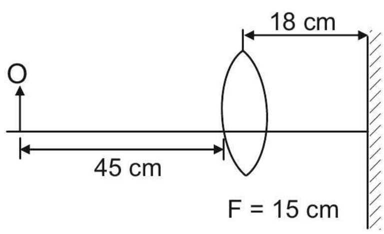


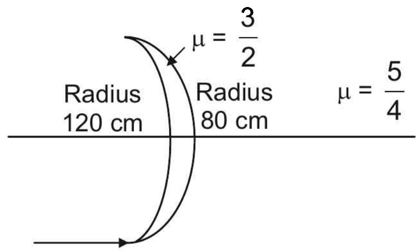
Fig. A

124. A glass sphere of refractive index $\frac{3}{2}$ and radius 10 cm has a spherical cavity of radius 5 cm concentric with the sphere. A narrow beam of parallel light rays is incident radially on the sphere. Find the location and nature of the final image.

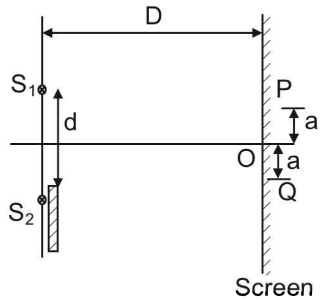
125. Describe the screen-catchable image formed by the lens – mirror combination, for the object O.



126. Determine the focal length of the thin lens $\left(\mu = \frac{3}{2}\right)$ immersed in a transparent medium $\left(\mu = \frac{5}{4}\right)$. Is it a converging or diverging lens? If the light is incident from right side, what difference will it make?



127. Young's double slit experiment is done in a medium of refractive index μ_1. Light of wavelength λ (in vacuum) falls on the slits S_1 and S_2. Slit S_2 is covered by a thin transparent sheet of thickness t and refractive index μ_2.



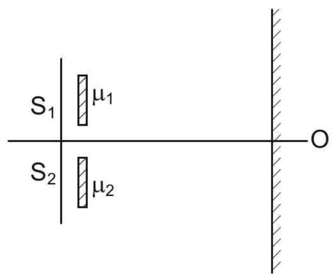
(i) Determine the optical path difference between the waves coming from two slits at points P and Q, distant a from O, on the screen

(ii) What is the condition that the central point O is (1) dark fringe (2) bright fringe?

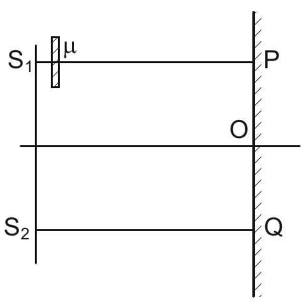
128. In the Young's double slit experiment, the sources S_1 and S_2 are coherent, represented by $E_1 = E_0 \sin \omega t$ and $E_2 = 2\,E_0 \sin\left(\omega t + \frac{\pi}{3}\right)$ respectively. The slits are covered by thin sheets of refractive index μ_1, μ_2 and thickness t_1 and t_2 respectively. The intensity of the source S_1 is I_0.

(i) Derive an expression for the intensity at the central point O.

(ii) What is the condition for O to be the central bright fringe?



129. The figure shows coherent sources S_1 and S_2 that are in phase. Slit S_1 is covered with a transparent sheet of refractive index μ and thickness t. The intensity of each source is I_0.



(i) Determine the intensities at points P and Q that are just opposite to the slits.

(ii) Find the condition for the intensities at P and Q to be equal.

(iii) What is the intensity at the central point O?

130. In Young's double slit experiment, the slits are 2 mm apart and illuminated by a mixture of two wavelengths $\lambda_1 = 750$ nm, $\lambda_2 = 900$ nm. Find the minimum distance from the common central bright fringe on a screen, 2 m from the slits, where the bright fringes from the two wavelengths coincide.

Straight Objective Type Questions

Directions: **This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

131. An object is placed symmetrically between two plane mirrors set at 37° to each other. The number of images is

(a) 2 (b) 8 (c) 9 (d) 10

132. A light ray, reflected successively by two plane mirrors subtending an angle θ between them, suffers a deviation of 220°. The number of images produced of an object placed symmetrically between them is

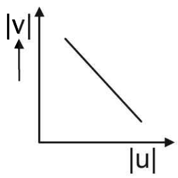
(a) 4 (b) 5 (c) 6 (d) 7

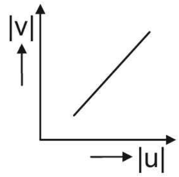
133. Which among the following can produce a parallel beam of light from a point source of light?

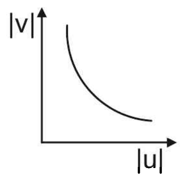
(i) concave mirror
(ii) convex mirror
(iii) plane mirror

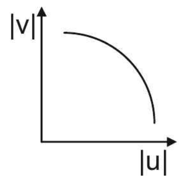
(a) (i) only (b) (i) or (ii) (c) (i) or (ii) or (iii) (d) none

134. For a spherical mirror graph of |v| vs |u| is

(a)


(b)


(c)


(d)


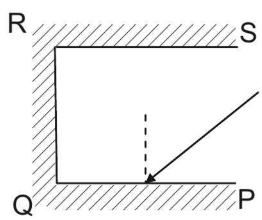
135. If a plane mirror reflecting an incident ray is rotated in the plane of incidence through an angle θ, then the reflected ray rotates through

(a) zero (b) $\frac{\theta}{2}$ (c) θ (d) 2θ

136. If an incident ray is successively reflected by two plane mirrors kept perpendicular to each other, the deviation suffered by the ray is

(a) 0° (b) 90°
(c) 180° (d) depends on the angle of incidence

137. If an incident ray is successively reflected by three plane mirrors kept mutually perpendicular as shown in figure, the deviation suffered by the ray is



(a) 0°
(b) 90°
(c) 180°
(d) depends on the angle of incidence

138. A ray incident at an angle θ on a face of a transparent cube just fails to emerge from the adjacent face. The refractive index of the material of the cube is

(a) $1 + \sin^2\theta$ (b) $\sqrt{1 + \sin^2\theta}$ (c) $1 + \cos^2\theta$ (d) $\sqrt{1 + \cos^2\theta}$

139. A fish in a lake at depth 1 m sees outside world through a circular aperture at the water surface, n of water = $\frac{4}{3}$. Radius of the aperture is

(a) $\frac{\sqrt{7}}{3}$ m (b) $\frac{3}{\sqrt{7}}$ m (c) $\frac{\sqrt{7}}{4}$ m (d) $\frac{4}{\sqrt{7}}$ m

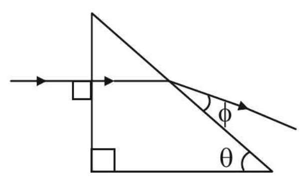
140. The minimum angle of deviation for a prism of refracting angle A and of refractive index 2 cos $\frac{A}{2}$ is one of the following; which is it?

(a) $\frac{A}{2}$ (b) 90° – A (c) 180° – 2 A (d) 360° – 3 A

141. In continuation of the above question, how many among the following values namely 35°, 45°, 55° are acceptable values for the angle of incidence giving the minimum angle of deviation?

(a) None (b) one (c) two (d) three

142. In the figure shown, $\phi = \frac{\theta}{2}$ if surrounding medium is water ($n = \frac{4}{3}$) and $\phi = 0$ if surrounding medium is air. Then θ is



(a) $\sin^{-1}\frac{3}{4}$ (b) $2\sin^{-1}\frac{3}{4}$

(c) $\cos^{-1}\frac{3}{4}$ (d) $2\cos^{-1}\frac{3}{4}$

143. Angle of deviation for a ray incident at small angle on a thin prism of angle θ° of refractive index n is (in figures):

(a) zero (b) n θ (c) (n – 1) θ (d) (n + 1) θ

144. Refractive index of glass for violet and red lights are 1.59 and 1.55 respectively. The angular dispersion produced by a 8° prism made of this glass is

(a) 0.16° (b) 0.64° (c) 0.32° (d) 0.48°

145. The power of a lens which produces an inverted image x cm tall of an 1 cm tall object, with a distance y cm separating the object and image is (in dioptre)

(a) $\frac{100(x+1)}{xy}$ (b) $\frac{100(x+1)^2}{xy}$ (c) $\frac{100x}{(x+1)y}$ (d) $\frac{100x^2}{(x+1)y}$

146. A lens of $\mu = 1.5$ produces a virtual image of a real object in air. If the whole set up is immersed in water, the image will

(a) shift closer to lens
(b) shift away from lens
(c) remain in same point
(d) depend on whether concave or convex lens

147. A lens behaves converging in air, diverging in water

(a) This is not possible.
(b) This is possible provided its radii of curvature are very high.

(c) This is possible provided its radii of curvature are very small.
(d) This is possible provided speed of light in it is more than that in water.

148. A lens of focal length f is cut into four identical parts. Focal length of each part is
(a) $\frac{f}{4}$ (b) $\frac{f}{2}$ (c) 2 f (d) 4 f

149. When light travels from one medium to another, there is no change in ratio of its
(a) velocity to wavelength (b) wavelength to time period
(b) time period to wave number (d) wave number to velocity

150. P and Q are two points on the screen of a typical YDSE. The path difference between the two rays reaching P is 3.2λ and that at Q is 4.8λ. The intensity ratio at P and Q is (larger: smaller)
(a) 1 (b) 1.5 (c) 2 (d) 3

151. In a YDSE, the number of fringes observed in a particular segment of screen changed by 4 when wavelength of the source is increased by 100 nm. If both wavelengths are in visible spectrum, (400 nm – 700 nm) the number of fringes now observed in the segment can possibly be
(a) 20 (b) 25 (c) 15 (d) 10

152. If white light is used in YDSE, there will be
(a) no perfectly white fringe (b) no perfectly dark fringe
(c) no coloured fringe (d) no fringe

153. Ina YDSE with light of wavelength 450 nm, bright fringe of order 10 is at point P of the screen. Now a thin film of $\mu = 1.6$ is placed in front of one slit. The central fringe shifts to P. The thickness of the film is
(a) 5 μm (b) 7.5 μm
(c) 10 μm (d) none of the above

154. A source of light emits waves of wavelength λ and frequency ν and period T. The number of waves counted by a stationary observer in time t second is
(a) T.t (b) $\frac{\nu}{t}$ (c) $\frac{t}{T}$ (d) $\frac{T}{t}$

155. A source emits light of wavelength 4000 Å, 5000 Å and 6500 Å. The light passes through red glass before being analysed by a spectrometer. Which wavelength is seen in the spectrum?
(a) All the three (b) Only 4000 Å (c) Only 5000 Å (d) Only 6500Å

156. The objective of a small telescope has focal length 120 cm. If its magnifying power is 40, the eyepiece has a focal length of
(a) 2 cm (b) 3 cm (c) 4 cm (d) 5 cm

157. A point source of light in space produces
(a) plane wave front (b) cylindrical wave front (c) spherical wave front (d) circular wave front

158. Coherent sources produce waves of
(a) same wavelength (b) same phase or constant phase difference
(c) same amplitude (d) (a) and (b)

159. Two coherent waves with intensities 9I and 16I are reaching a point with phase difference zero. At the point of interference, the intensity is
(a) 7I (b) 25I (c) 36I (d) 49I

160. If the distance of the nth dark band from the central fringe in a Young's double slet experiment is $\frac{31}{2}\frac{D\lambda}{d}$ then n is equal to
(a) 31 (b) 30 (c) 15 (d) 16

161. If wavelength of monochromatic light in Young's double slit arrangement is doubled without any other changes
(a) Only bandwidth of bright bands get doubled.
(b) Only bandwidths of dark bands gets doubled.
(c) The widths of both dark and bright bands get doubled.
(d) The widths of both dark and bright bands get halved.

162. If the monochromatic source is replaced by white light in Young's double slit experiment
(a) all fringes become white
(b) all fringes become coloured
(c) central fringe is white and all other fringes coloured
(d) alternate white and coloured fringes produced

163. The frequency of light of wavelength 3000 Å in vacuum is
(a) 10^{15} Hz (b) 9×10^{13} Hz (c) 3000 Hz (d) 90 Hz

164. The colour of light is characteristic of
(a) amplitude (b) wavelength (c) velocity (d) frequency

165. Time taken by sunlight to pass through a window of thickness 4 mm, of refractive index 3/2, is
(a) 2×10^{-11} s (b) 2×10^{8} s (c) 2×10^{11} s (d) 0.2×10^{-8} s

166. When light travels from an optically rarer medium to an optically denser medium, the velocity decreases because of change in
(a) wavelength (b) frequency (c) amplitude (d) phase

167. When light wave suffers reflection at the interface of air and glass, the change in phase of the reflected wave is equal to
(a) Zero (b) $\pi/2$ (c) π (d) $\pi/3$

168. The refractive index of a given piece of transparent quartz is greatest for
(a) Red light (b) Violet light (c) Green light (d) Yellow light

169. The minimum magnifying power of a telescope is M. If the focal length of its eye-lens is halved, the magnifying power will become
(a) M/2 (b) 2 M (c) 4 M (d) M/4

170. In normal adjustment of an astronomical telescope, final image is at
(a) 25 cm (b) 25 m (c) 25 mm (d) infinity

Assertion–Reason Type Questions

Directions: **Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

171. **Statement 1**
A rectangular glass slab does not disperse white light when light passes through the parallel surfaces only.
and

Statement 2
If refracting angle is zero, all colours travel with same speed.

172. Statement 1

If a lens gives virtual image for a real object, then it will always give a real image for a virtual object.

and

Statement 2

By principle of reversibility object and image locations are interchangeable.

173. Statement 1

Focal length of a mirror does not change with the surrounding medium.

and

Statement 2

The law of reflection, namely $i = r$ holds whatever be the surrounding medium.

174. Statement 1

For a prism, there are two angles of incidence for a given deviation, other than for minimum deviation.

and

Statement 2

By principle of reversibility of ray, incident and emergent rays can be interchanged.

175. Statement 1

A source of white light, when viewed through a non-parallel side of a rectangular glass slab can appear to be red but certainly not violet.

and

Statement 2

Critical angle for violet is less than that for red.

176. Statement 1

If there be no atmosphere, duration of night would increase.

and

Statement 2

Atmospheric refraction causes night to be shorter.

177. Statement 1

In YDSE, if slit separation $>>\lambda$, no fringe pattern can be observed.

and

Statement 2

Larger the fringe width less is the number of fringes observable per unit area of screen.

178. Statement 1

In YSDE, if slit separation $< \lambda$, no fringe pattern can be observed.

and

Statement 2

Larger the fringe width, less is the number of fringes observable per unit area of screen.

179. Statement 1

In a typical Young's double slit experiment, it is possible to introduce two films of different materials, one each in front of each slit, so that there is absolutely no change in the interference pattern.

and

Statement 2

Increase in path length is $t(\mu - 1)$ due to each film.

180. **Statement 1**

A convex mirror can produce only a virtual image.

and

Statement 2

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ and f is positive for convex mirror (symbols have usual meaning).

Linked Comprehension Type Questions

Directions: **This section contains 3 paragraphs. Based upon the paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

Passage I

A concave mirror is placed on a horizontal table with its axis vertical. An object is placed on the axis. The size of the image I is m times that of object. The inside of the mirror is filled with water $\left(n = \frac{4}{3}\right)$. The new image I' has the same size as I.

181. I is

(a) real, erect (b) real, inverted (c) virtual, erect (d) virtual, inverted

182. I' is

(a) real, erect (b) real, inverted (c) virtual, erect (d) virtual, inverted

183. Modulus value of m is:

(a) 3 (b) $\frac{1}{3}$ (c) 7 (d) $\frac{1}{7}$

Passage II

A typical Young's double slit experiment is performed with a slit separation of 0.5 mm and screen at 1 m. Among the points with intensity I_0, point P is closest to the centre of the screen and directly opposite to slit 1.The intensity at P remains unchanged when a film is introduced in front of slit 1, irrespective of whether the film is transparent with refractive index 1.5 or opaque

184. Phase difference between the waves reaching P from the two slits is

(a) 30° (b) 60° (c) 120° (d) 150°

185. The wavelength of light or electromagnetic wave used is (in nm)

(a) 300 (b) 375 (c) 500 (d) 600

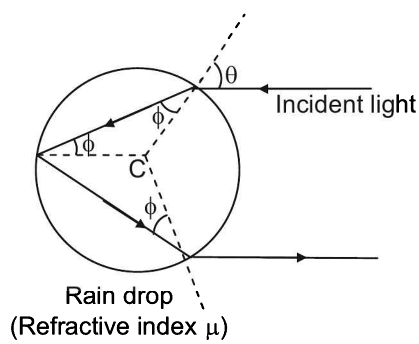
186. The minimum possible thickness of the transparent film is (in μm)

(a) 0.5 (b) 2.5

(c) 12.5 (d) 25

Passage III

According to the theory of formation of the rainbow, a ray of sunlight is refracted as it enters a spherical rain drop, undergoes a single internal reflection, and is refracted as it leaves the drop, as shown. The rainbow is formed from the rays whose deviation from the original direction is either maximum or minimum. This condition takes the effect of refractive index into account, which is different for different colours.



187. The total deviation D of the light ray as it emerges out of the drop is

(a) $2\theta - 2\phi + \pi$ (b) $2\theta - \phi + 2\pi$ (c) $2\theta - 4\phi + \pi$ (d) $4\theta - 2\phi + \pi$

188. For an extreme value of D, we set $\frac{dD}{d\theta} = 0$. This condition corresponds to

(a) $\frac{d\theta}{d\phi} = 1$ (b) $\frac{d\theta}{d\phi} = \frac{1}{2}$ (c) $\frac{d\phi}{d\theta} = \frac{1}{2}$ (d) $\frac{d\phi}{d\theta} = \frac{1}{4}$

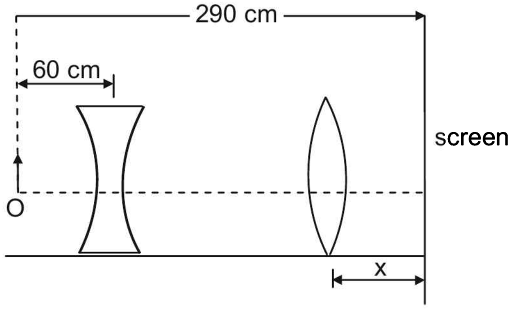
189. When the total deviation D has an extreme value, then

(a) $\cos\phi = \frac{2\cos\theta}{\mu}$ (b) $\sin\theta = \frac{2\sin\phi}{\mu}$ (c) $2\cos\phi = \frac{\cos\theta}{\mu}$ (d) $\cos\phi = \frac{\cos\theta}{\mu}$

Multiple Correct Objective Type Questions

Directions: **Each question in this section has four suggested answers out of which ONE OR MORE answers is/are correct.**

190.



A screen is placed at 290 cm in front of an object O and a concave lens of f = 15 cm, is placed at 60 cm in front of the object. The convex lens of focal length 20 cm is placed at a distance x from the screen

(a) An image is formed on the screen when x = 22 cm (b) An image is formed on the screen when x = 32 cm
(c) An image is formed on the screen when x = 220 cm (d) maximum magnification $|m| = 2$

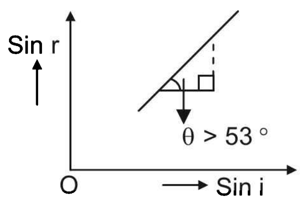
191. Which among the following quantities pertaining to a lens varies with wavelength of the incident light?

(a) Magnitude of power (b) Sign of power
(c) Magnification (for a given object at a given distance) (d) Focal length

192. By suitable combination of two prisms of different materials, it is possible to achieve

(a) both dispersion and deviation (for mean ray) (b) dispersion without deviation
(c) deviation without dispersion (d) no dispersion and no deviation

193. Graph of sin i vs sin r at an interface is as below.



(a) The line will pass through origin.
(b) The ray passes from denser to rarer medium.
(c) The refractive index of the second medium with respect to first medium is less than 0.75.
(d) The critical angle (for TIR) of first medium with respect to air is less than $\sin^{-1} 0.75$.

194. A convex lens of focal length f_1 and a mirror of focal length $f_2 \neq f_1$ are kept coaxial with separation d. A parallel beam of light incident on the lens is returned as a parallel beam. Then which among the following are possible?
(a) $d = |f_1|$ (b) $d = |f_1| + 2|f_2|$ (c) $d = |f_1| - 2|f_2|$ (d) f_2 can be infinite

195. A small cylindrical object is placed with its axis along that of a concave mirror at 30 cm from the mirror producing a real image. The lengths of the object and the image are such that one is four times the other. Which among the following are possible?
(a) The focal length is 10 cm. (b) The focal length is 6 cm.
(c) The distance between object and image is 30 cm. (d) The distance between object and image is 15 cm.

196. Which pair among the following four light waves will produce interference pattern?
(i) $E_1 \sin \omega_1 t$
(ii) $E_2 \sin (\omega_1 t + \frac{\pi}{4})$
(iii) $E_1 \sin \omega_2 t$
(iv) $E_2 \sin (\omega_2 t + \pi)$
(a) (i) and (ii) (b) (i) and (iii) (c) (iii) and (iv) (d) no pair

197. In a typical YDSE, if a thin film (thickness t, refractive index n) is placed in front of S_1, the central fringe will shift: (symbols having usual meanings)
(a) towards S_1 (b) away from S_1 (c) by $t(n-1)\frac{D}{d}$ (d) by $t\frac{nD}{d}$

Matrix-Match Type Questions

Directions: **Match the elements of Column I to elements of Column II. There can be single or multiple matches.**

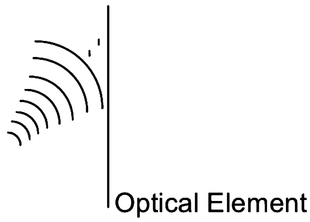
198. Two light waves have their electric vectors $E_1 = A\cos(\omega_1 t + \phi_1)$ and $E_2 = A\cos(\omega_2 t + \phi_2)$ and intensities of I_1 each respectively. At a point in space, they interfere and the resultant intensity is I. Match the following:

Column I	**Column II**
(a) $\omega_1 \neq \omega_2; \phi_1 \neq \phi_2$	(p) $I = I_{max}$
(b) $\omega_1 = \omega_2; \phi_1 \neq \phi_2$	(q) $I = 2I_1$
(c) $\omega_1 \neq \omega_2; \phi_1 = \phi_2$	(r) $I = 4I_1$
(d) $\omega_1 = \omega_2; \phi_1 = \phi_2$	(s) $I = 2I_1[1 + \cos(\phi_1 - \phi_2)]$

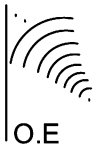
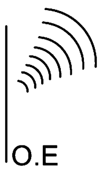
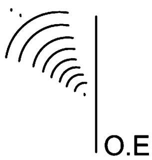
199. Given that object is virtual, match the columns:

Column I **If**	**Column II** **The image can be**
(a) Concave mirror, $|u| < |f|$	(p) real
(b) Concave mirror, $|u| > |f|$	(q) virtual
(c) Convex mirror, $|u| < |f|$	(r) enlarged
(d) Convex mirror, $|u| > |f|$	(s) diminished

200. Incident wave front on an optical element is as shown below:



Subsequently, the wave front is as per column 1. The possible optical elements are listed in column 2 Match the columns.

Column I	Column II
(a) 	(p) plane mirror
(b) 	(q) concave mirror
(c) 	(r) convex mirror
(d) 	(s) concave lens
	(t) convex lens

SOLUTIONS

ANSWERS KEYS

Topic Grip

1. $A_1 = 8°, A_2 = 5°$
2. $-\frac{13}{7}$ R, to the right of pole of surface (virtual image).
3. $\frac{2}{3}$ cm
4. (i) 12.5 cm infront of mirror, $\frac{9}{34}$ cm tall
   (ii) $\frac{295}{98}$ cm infront of lens (in air), inverted $\frac{9}{28}$ cm tall
5. I_1 at 60 cm to left of lens, inverted magnified, 2 cm tall
   I_2 at $\frac{150}{7}$ cm infront of mirror, inverted, $\frac{5}{7}$ cm tall.
   I_3 at 41.5 cm to left of lens, erect, $\frac{10}{13}$ cm tall.
6. (i) $P_1 = +4$ D; $P_2 = -5$ D
   (ii) $R_1 = 25$ cm; $R_2 = -20$ cm
   (iii) $d > 10$ cm
7. (i) 6.5 mm
   (ii) 5.42 mm
8. (i) S_2
   (ii) 1.67 μm
9. (i) 2.5 μm
   (ii) $\frac{5}{3}$ μm
10. $2KA^2$

11. (d)	**12.** (d)	**13.** (c)
14. (b)	**15.** (c)	**16.** (c)
17. (c)	**18.** (c)	**19.** (d)
20. (a)	**21.** (a)	**22.** (c)
23. (a)	**24.** (c)	**25.** (a)

26. (b)
27. (a), (c), (d)
28. (a), (c)
29. (a), (b), (c)
30. (a) → (q), (s)
    (b) → (q), (r)
    (c) → (p), (q)
    (d) → (q), (r)

IIT Assignment Exercise

31. (d)	**32.** (d)	**33.** (a)
34. (b)	**35.** (b)	**36.** (d)
37. (c)	**38.** (c)	**39.** (a)
40. (c)	**41.** (b)	**42.** (c)
43. (d)	**44.** (c)	**45.** (b)
46. (a)	**47.** (b)	**48.** (b)
49. (d)	**50.** (d)	**51.** (d)
52. (c)	**53.** (d)	**54.** (d)
55. (a)	**56.** (b)	**57.** (d)
58. (c)	**59.** (d)	**60.** (c)
61. (a)	**62.** (b)	**63.** (a)
64. (c)	**65.** (a)	**66.** (b)
67. (c)	**68.** (c)	**69.** (d)
70. (b)	**71.** (c)	**72.** (c)
73. (b)	**74.** (a)	**75.** (b)
76. (c)	**77.** (b)	**78.** (c)
79. (a)	**80.** (b)	**81.** (c)
82. (d)	**83.** (b)	**84.** (a)
85. (d)	**86.** (b)	**87.** (d)
88. (b)	**89.** (a)	**90.** (c)
91. (c)	**92.** (a)	**93.** (c)
94. (d)	**95.** (b)	**96.** (c)
97. (b)	**98.** (c)	**99.** (c)
100. (b)	**101.** (a)	**102.** (c)
103. (b)	**104.** (d)	**105.** (d)
106. (b)	**107.** (c)	**108.** (d)
109. (b)	**110.** (a)	**111.** (c)
112. (d)	**113.** (a)	**114.** (d)
115. (c)	**116.** (c)	

117. (a), (c)
118. (a), (b), (d)
119. (a), (d)
120. (a) → (r)
     (b) → (p), (r)
     (c) → (q), (r)
     (d) → (r), (s)

Additional Practice Exercise

121. (a) $0 < \theta < \sin^{-1}\left(\sqrt{\left(\frac{\mu_1}{\mu_2}\right)^2 - 1}\right)$
     (b) $\mu_1 > \sqrt{2}$
122. 10^5
123. $|v| = 25.2$ cm, erect, $m = 1.6$
124. 5 cm behind left edge, virtual
125. 4.5 cm, in front of mirror, inverted, half the object size
126. $f = +12$ m. No difference
127. (i) $t(\mu_2 - \mu_1) - \mu_1 \frac{d.a}{D}$
     (ii)
     (1) $t = \frac{2m - 1}{|\mu_2 - \mu_1|}.\frac{\lambda}{2}$
     (2) $t = \frac{m\lambda}{(\mu_2 - \mu_1)}$; $m = 1, 2, 3 \ldots$
128. (i) $I_C = I_0\left[5 + 4\cos\left(\varphi - \frac{\pi}{3}\right)\right]$,
     where
     $\phi = \frac{2\pi}{\lambda}\left[t_2(\mu_2 - 1) - t_1(\mu_1 - 1)\right]$
     (ii) $t_2(\mu_2 - 1) - t_1(\mu_1 - 1) = \frac{\lambda}{6}$
129. (i) $I_P = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$,

where $\phi = \dfrac{2\pi}{\lambda}\left[\dfrac{d^2}{2D} - (\mu - 1)t\right]$

$I_Q = 4I_0 \cos^2\left(\dfrac{\phi'}{2}\right)$, where

$\phi' = \dfrac{2\pi}{\lambda}\left[\dfrac{d^2}{2D} + (\mu - 1)t\right]$

(ii) $t = \dfrac{m\lambda}{2(\mu - 1)}$; $m = 1, 2, 3 \ldots$

(iii) $I_C = 4I_0 \cos^2\left(\dfrac{\pi}{\lambda}(\mu - 1)t\right)$

130. 4.5 mm

131. (c) **132.** (b) **133.** (a)
134. (c) **135.** (d) **136.** (c)
137. (d) **138.** (b) **139.** (b)
140. (d) **141.** (a) **142.** (d)
143. (c) **144.** (c) **145.** (b)
146. (d) **147.** (d) **148.** (c)
149. (a) **150.** (a) **151.** (a)
152. (b) **153.** (b) **154.** (c)
155. (d) **156.** (b) **157.** (c)
158. (d) **159.** (d) **160.** (d)
161. (c) **162.** (c) **163.** (a)
164. (d) **165.** (a) **166.** (a)
167. (c) **168.** (b) **169.** (b)
170. (d) **171.** (c) **172.** (d)
173. (a) **174.** (a) **175.** (a)
176. (a) **177.** (d) **178.** (b)
179. (a) **180.** (d) **181.** (c)
182. (b) **183.** (c) **184.** (c)
185. (b) **186.** (a) **187.** (c)
188. (c) **189.** (a)
190. (a), (c), (d)
191. (a), (c), (d)
192. (a), (b), (c)
193. (a), (b), (c), (d)
194. (a), (b), (c), (d)
195. (a), (c), (d)
196. (a), (c)
197. (a), (c)
198. (a) → (q)
(b) → (s)
(c) → (q)
(d) → (p), (r), (s)
199. (a) → (p), (s)
(b) → (p), (s)
(c) → (p), (r)
(d) → (q), (r), (s)
200. (a) → (t)
(b) → (s), (t)
(c) → (t)
(d) → (p), (q), (r)

HINTS AND EXPLANATIONS

Topic Grip

1. Dispersive power $\omega = \dfrac{A(n_v - n_R)}{A(n_y - 1)} \Rightarrow$ condition for achromatic combination

$\Rightarrow \quad A_1(n_{v1} - n_{R1}) = A_2(n_{v2} - n_{R2})$,

$\therefore \quad \omega_1 A_1(n_{y1} - 1) = \omega_2 A_2(n_{y2} - 1)$

$(1.5 - 1)\, A_1 \times 0.036 = (1.6 - 1) \times A_2 \times 0.048$

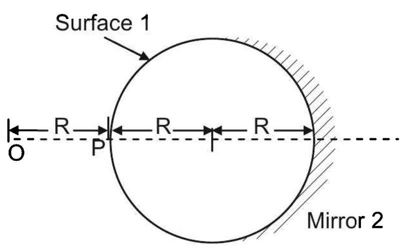
$\Rightarrow \quad A_1 = 1.6\, A_2$ ....(1)

$(1.5 - 1)\, A_1 - (1.6 - 1)\, A_2 = 1°$

$\Rightarrow \quad 0.5A_1 - 0.6A_2 = 1°$ .....(2)

(1) and (2) $A_2 = 5°$, $A_1 = 8°$

2. The process is

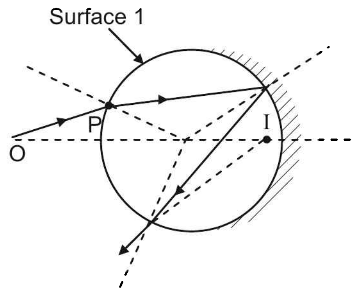


(i) Refraction on surface 1
(ii) Reflection on mirror
(iii) Refraction on surface 1.

(i) $\dfrac{1.5}{v'} - \dfrac{1}{-R} = \dfrac{(1.5 - 1)}{+R} \Rightarrow v' = -3R$

$\therefore$ u for silvered surface $= |3R + 2R|$

(ii)



$$\frac{1}{-5R} + \frac{1}{v''} = \left(\frac{1}{-R/2}\right) \Rightarrow v'' = -\frac{5}{9}R$$

$\therefore$ u for refraction at surface 1(second time)

$$= \left|\frac{5}{9}R - 2R\right| = \left|\frac{13}{9}R\right|$$

Note: The image is at right side of pole P at distance $\dfrac{13}{9}R$

(iii) $\dfrac{1.}{v} - \dfrac{1.5}{-\dfrac{13R}{9}} = \dfrac{(1 - 1.5)}{-R} \Rightarrow v = -\dfrac{13}{7}R$

i.e., $-\dfrac{13}{7}R$ to the right of pole of surface (virtual image)

3. $\text{Sin } i \approx i = \dfrac{0.5}{8}$ radian $= \dfrac{1}{16}$ radian $= 3.58°$

$\dfrac{i}{r} = \mu \Rightarrow r = \dfrac{i}{\mu} = \dfrac{1}{16 \times 1.5} = \dfrac{1}{24}$ radian $= 2.38°$

Use law of sines = (In Δ OPQ):

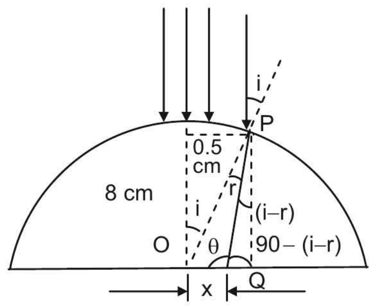
$\theta = 180° - [90° - (i - r)] = 90° + i - r$

$\dfrac{8}{\sin(90 + i - r)} = \dfrac{x}{\sin r}$ [use $\sin\theta = \theta$ in radian

Cos δ = I; for small angles]

$\Rightarrow \quad \dfrac{8}{\cos 1.2°} = 24x \Rightarrow x = \dfrac{1}{3}$cm

$\Rightarrow \quad$ diameter $2x = \dfrac{2}{3}$cm



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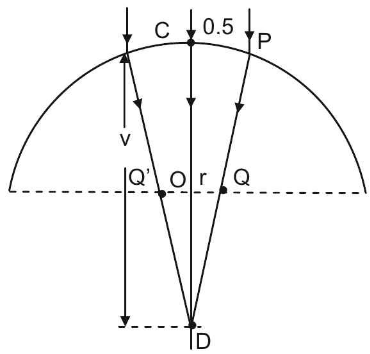


The parallel rays will be focused at a distance v, due to refraction at curved surface. v is given by

$$\frac{1.5}{v} - \frac{1}{\infty} = \frac{1.5-1}{R} = \frac{0.5}{8}$$

v = 24 cm. The beam converging to D meet the base at QQ'. Since it is a thin beam.

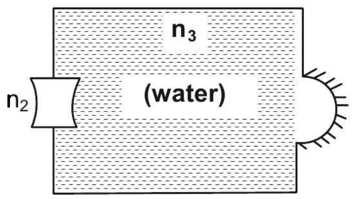
From similar triangles ΔDOQ and ΔDCP

$$\frac{CP}{OQ} = \frac{CD}{OD}; v = CD$$

$$\therefore \quad \frac{0.5}{r} = \frac{24}{(24-8)} \Rightarrow r = \frac{16}{24} \times 0.5$$

$$\text{Diameter } d = 2r = \frac{16}{24} = \frac{2}{3} \text{ cm}$$

4. (i) Refraction through concave lens



For the lens for refraction at first surface

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \Rightarrow \frac{n_2}{v} = \frac{n_1}{u} + \frac{n_2 - n_1}{R_1}$$

For second surface of the lens u' = v for refraction at 2nd surface.

$$\frac{n_3}{v} - \frac{n_2}{u'} = \frac{n_3 - n_2}{R_2}$$

Substitute for $\frac{n_2}{u'}$ from above.

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}; \; m = \frac{n_1 v}{n_3 u}$$

Here, $n_3 = \frac{4}{3}, n_1 = 1, n_2 = 1.5$

$v = v_1, u = -40, \; R_1 = -R, R_2 = +R$

Given f = –30 cm

$$\Rightarrow \frac{1}{-30} = (1.5-1)\left(\frac{1}{-R} - \frac{1}{+R}\right)$$

$$\Rightarrow R = 30 \text{ cm}, R_1 = -30, R_2 = +30$$

$$\therefore \frac{4}{3v_1} - \frac{1}{-40} = \frac{1.5-1}{-30} + \frac{\frac{4}{3} - 1.5}{+30}$$

$$\Rightarrow v_1 = \frac{-480}{17}, \; m_1$$

$$= \frac{1 \times \left(\frac{-480}{17}\right)}{\frac{4}{3} \times (-40)} = +\frac{9}{17}$$

(ii) Reflection by concave mirror

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}; \; m = \frac{-v}{u}$$

$$\text{Here } u = -\left(\frac{480}{17} + \frac{370}{17}\right) = -50 \text{ cm}; \; v = v_2$$

f = –10 cm (case i) = –20 cm (case ii)

∴ Case (i)

$$= \frac{1}{v_2} + \frac{1}{-50} = \frac{1}{-10}$$

$$m_2 = -\frac{(-12.5)}{-50} = -\frac{1}{4}$$

$$\therefore m_1 m_2 = \left(+\frac{9}{17}\right)\left(-\frac{1}{4}\right) = \frac{-9}{68}$$

$$\Rightarrow h' = 2\text{cm x} \left(-\frac{9}{68}\right) = -\frac{9}{34} \text{ cm}$$

Hence case (ii) final image

Real, 12.5 cm in front of mirror (in water)

Inverted, diminished, $\frac{9}{34}$ cm tall

Case (ii)

$$\frac{1}{v_2} + \frac{1}{-50} = \frac{1}{-20} \Rightarrow v_2 = -\frac{100}{3} = 33.3 \text{ cm, in}$$

front of the mirror

Not yet final answer

Since 33.3 cm > $\frac{370}{17}$ cm

There will be one more refraction through lens; before the image is formed.

$$m_2 = \frac{-\left(100/3\right)}{-50} = \frac{-2}{3}$$

(iii) Second refraction through lens

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}; \; m = \frac{n_1 v}{n_3 u}$$

Here $n_3 = 1, n_1 = \frac{4}{3}, n_2 = 1.5, v = v_3$

$$R_1 = -30, R_2 = +30, \; u = \frac{+100}{3} - \frac{370}{17} = \frac{+590}{51}$$

$$\therefore \frac{1}{v_3} - \frac{4/3}{590/51} = \frac{1.5 - \frac{4}{3}}{-30} + \frac{1-1.5}{30}$$

$$\Rightarrow \quad v_3 = \frac{+295}{28}$$

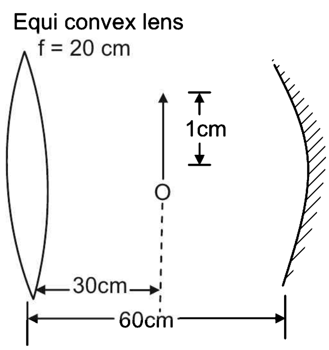
$$m_3 = \frac{\frac{4}{3} \times \frac{295}{98}}{1 \times \frac{590}{51}} = \frac{102}{294}$$

$$\Rightarrow \quad m_1 m_2 m_3 = \left(\frac{9}{17}\right)\left(-\frac{1}{4}\right)\left(\frac{102}{294}\right) = \frac{9}{196}$$

$$\Rightarrow \quad h' = 2\text{cm} \times \left(-\frac{9}{196}\right) = -\frac{9}{98}\text{cm}$$

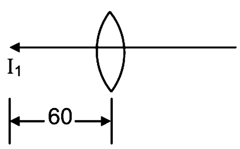
Hence case (ii) final image is real, $\frac{295}{98}$ cm in front of the lens (in air) inverted diminished, $\frac{9}{98}$ cm tall.

5. Let I_1 be real image of object formed by lens, I_1 will lie to the left of lens beyond 2f



$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{+20} \Rightarrow v = +60 \text{ cm (Real)}$$

$\Rightarrow$



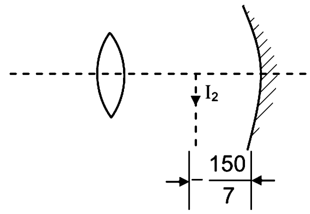
$$m = \frac{60}{= -30} = -2 \Rightarrow h' = 2\text{cm};$$

$\therefore$ I_1 = at 60 cm to left of lens, inverted magnified, height 2 cm

Let I_2 be the real image of object formed by the mirror. I_2 will be between f and 2f of mirror.

$$\frac{1}{v} + \frac{1}{-30} = \frac{1}{-12.5} \Rightarrow v = -\frac{150}{7}\text{cm}$$

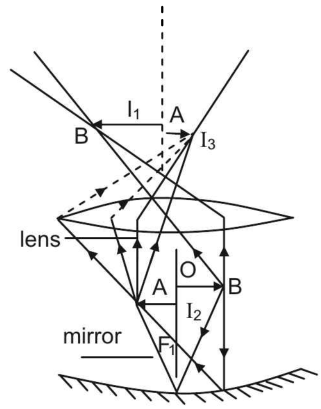
$\Rightarrow$



$$m = -\frac{\left(150/7\right)}{-30} = \frac{-5}{7} \Rightarrow h' = \frac{-5}{7}\text{cm} \text{ to left of}$$

mirror, inverted; diminished, height $= \frac{5}{7}$ cm

Now, I_2 acts as object for lens. It gives I_3 to the left of lens, beyond 2f.



$$u = -\left(60 - \frac{150}{7}\right) = \frac{-270}{7}\text{ cm}$$

$$\frac{1}{v} + \frac{7}{270} = \frac{1}{+20}$$

$$\Rightarrow \quad v = +\frac{540}{13}\text{ cm} = 41.5 \text{ cm}$$

$$m = \frac{540 \times 7}{13 \times (-270)} = \frac{-14}{13}$$

$$\therefore h' = \left(\frac{-5}{7}\right) \times \left(\frac{-14}{13}\right) = \frac{10}{13}\text{cm (Erect)}$$

$\therefore$ I_3 = 41.5 cm to left lens, erect, diminished, height $\frac{10 \text{ cm}}{13}$

6.

(i) Ratio of focal lengths

$$\frac{f_1}{f_2}=\frac{(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}{(\mu-1)\left(\frac{1}{R_1'}-\frac{1}{R_2'}\right)}=\frac{\frac{1}{R_1}-\frac{1}{R_2}}{\frac{1}{R_1'}-\frac{1}{R_2'}}$$

Ratio of focal lengths in liquid

$$\frac{f_{1\ell}}{f_{2\ell}}=\frac{\left(\frac{\mu}{\mu_\ell}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}{\left(\frac{\mu}{\mu_\ell}-1\right)\left(\frac{1}{R_1'}-\frac{1}{R_2'}\right)}=\frac{f_1}{f_2}=-\frac{5}{4} \quad \text{— (1)}$$

$$\frac{f_{1\ell}}{f_{2\ell}}=-\frac{5}{4} \quad \text{— (1)}$$

When kept in contact, considering f_{11} and f_{21} are in cm:

$$P=\frac{1}{F}=\frac{1.2}{\left(\frac{f_{1\ell}}{100}\right)}+\frac{1.2}{\left(\frac{f_{2\ell}}{100}\right)}$$

$$\Rightarrow \frac{-0.6}{100}=\frac{1.2}{f_{1\ell}}+\frac{1.2}{f_{2\ell}} \quad \text{—(2)}$$

Solving (1) and (2)

$$\Rightarrow -\frac{0.6}{100}=\frac{1.2}{f_{1\ell}}-\frac{1.2\times 5}{f_{1\ell}\times 4}$$

$\Rightarrow f_{1\ell} = +50$ cm

$f_{2\ell} = -40$ cm

$$\frac{f_2}{f_{2\ell}}=\frac{f_1}{f_{1\ell}}=\frac{\left(\frac{\mu}{\mu_\ell}-1\right)}{(\mu-1)}=\frac{\left(\frac{1.5}{1.2}-1\right)}{1.5-1}=\frac{1}{2}$$

$\Rightarrow$ $f_1 = +25$ cm; $f_2 = -20$ cm

$\Rightarrow$ $P_1 = +4D$; $P_2 = -5D$

(ii) We know

$$\frac{1.2}{f_{1\ell}}=(1.5-1.2)\left(\frac{1}{R_1}-\frac{1}{R_1'}\right)$$

$$\Rightarrow \frac{1.2}{50}=0.3\left(\frac{2}{R_1}\right)$$

$\Rightarrow R_1 = 25$ cm

Similarly

$$\frac{1.2}{f_{2\ell}}=(1.5-1.2)\left(\frac{1}{R_2}-\frac{1}{R_2'}\right)$$

$$\Rightarrow \frac{1.2}{-40}=0.3\left(\frac{-2}{R_2}\right)$$

$\Rightarrow R_2 = -20$ cm

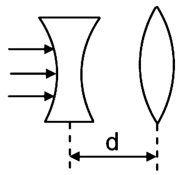
(iii) Case I , when converging lens meet the parallel beam.

$$\frac{1}{v}-\frac{1}{\infty}=\frac{1}{50} \Rightarrow v = 50 \text{ cm: for the second lens}$$

$$\frac{1}{v}-\frac{1}{(50-d)}=-\frac{1}{40} \Rightarrow \frac{1}{v}=\frac{-1}{40}+\frac{1}{(50-d)}:$$

For v to be + ve, (50-d) < 40 $\Rightarrow$ d >10 cm

Case II



When diverging lens meets in incident rays.

$$\frac{1}{v}-\frac{1}{\alpha}=-\frac{1}{40} \Rightarrow v = -40 \text{ cm}$$

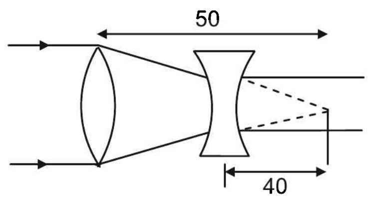
For second lens u = – (40 + d)

$$\frac{1}{v}-\frac{1}{-(40+d)}=\frac{1}{50}$$

$$\Rightarrow \frac{1}{v}=\frac{1}{50}-\frac{1}{(40+d)}$$

for ‘v’ to be +ve (40 + d) > 50 $\Rightarrow$ d > 10 cm.

Aliter:



7.

(i) $\beta = \lambda\frac{D}{d} = 500 \times \frac{10^{-9}\times 80\times 10^{-2}}{0.4\times 10^{-3}}$

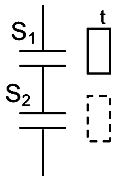
$= 10^{-3}$m = 1 mm

$$\text{distance} = \left(3\frac{1}{2}+3\right)\times 1 \text{ mm} = 6.5 \text{ mm}$$

(ii) $\lambda_{liquid} = \frac{\lambda_{air}}{n} \quad \therefore \beta' = \frac{\beta}{n}$

$$\frac{6.5\text{mm}}{1.2}=5.42 \text{ mm}$$

8.

(i) 

If C is centre of screen optical path length S_1C has now become less $\Rightarrow$ central fringe shifts towards S_2

(ii) Let t be thickness of film

Path difference at centre:

let t be thickness of the film

and imagine an equal thickness

in liquid. Time taken by the ray

to pass through film

$$T_1 = \frac{t}{c}.1.1;\ \text{through liquid } T_2 = \frac{t}{c}.1.2\,;\ T_2 > T_1$$

$$\therefore\quad \delta T = T_2 - T_1 = \frac{t}{c}(1.2 - 1.1)$$

Path difference = δ T . v_ℓ

$$= \frac{t}{c}(1.2-1.1).\frac{c}{1.2}$$

$$= t\left[1 - \frac{1.1}{1.2}\right]$$

$$\Rightarrow \text{path difference} = t\left(1 - \frac{1.1}{1.2}\right) = \frac{t}{12}$$

Aliter:

Path difference at the centre = $t(\mu_r - 1)$

$$= t\left(\frac{1.1}{1.2} - 1\right) = -\frac{t}{12}\ \text{(neglect sign)}$$

$$\Rightarrow\quad \text{phase difference } \phi = \frac{t}{12\lambda_{liq}} \cdot 2\pi \qquad \ldots(1)$$

If I_0 is earlier intensity at centre, I now, then we know

$$I = I_0 \cos^2\frac{\phi}{2} \Rightarrow \frac{I_0}{4} = I_0 \cos^2\frac{\phi}{2}$$

$$(\because \text{Here } I = \frac{I_0}{4}) \Rightarrow \cos\frac{\phi}{2} = \pm\frac{1}{2} \Rightarrow \min \phi = \frac{2\pi}{3}$$

Substitute in equation — (1) $\Rightarrow \dfrac{2\pi}{3} = \dfrac{t}{12}.\dfrac{2\pi}{\lambda_\ell}$

$$\therefore\quad (1)\ t_{min} = 4\lambda_{liq} = \frac{4 \times 500 \times 10^{-9}}{1.2} = 1.67\ \mu m$$

9. (i) The path difference between adjacent bright fringes is λ. Hence a shift of central maximum by 4 fringe widths corresponds to a path difference 4λ introduces by the film.

$$2t_p(\mu_p - 1) = 4\lambda$$

$$t_p = \frac{4\times 0.5}{2(1.4 - 1)} = 2.5\ \mu m$$

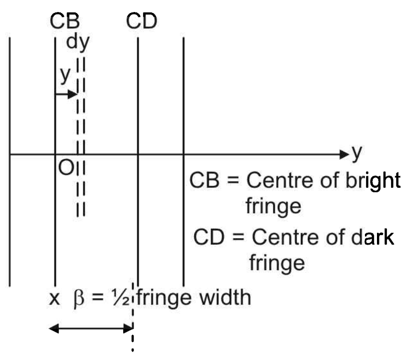
(ii) When the whole set up is immersed in liquid.

$$2t_p\left(\frac{\mu_p}{\mu_\ell} - 1\right) + t_a\left(\frac{1}{\mu_\ell} - 1\right) = 0$$

$$t_a\frac{(1.3 - 1)}{1.3} = 2 \times 2.5\left(\frac{1.4 - 1.3}{1.3}\right)$$

$$\Rightarrow\quad t_a = \frac{5}{3}\ \mu m$$

10. Let us consider two adjacent dark and bright fringes. We have to prove that averaged over the fringe width β, the value of intensity is equal to the sum of intensities of individual beams i.e., $2KA^2$ Take y-axis as shown and consider the intensity at a strip as shown. The value of intensity at $y = 4KA^2\cos^2\dfrac{\phi}{2}$



Value of intensity over the thin strip dy at y:

$$= 4KA^2\cos^2\frac{\phi}{2}\ dy$$

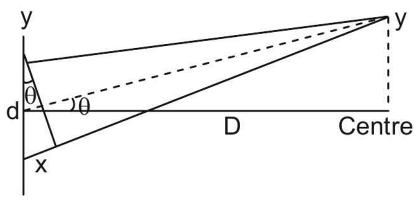
$\therefore$ average value over the half fringe width

$$= \frac{\int 4KA^2\cos^2\frac{\phi}{2}.dy}{\beta} \qquad \text{—(1)}$$

($\beta \rightarrow$ half fringe width)

Considering basic Young's Double slit arrangement. Path difference x to the position y is given by:

$$\frac{x}{d} = \frac{y}{D} \Rightarrow y = \frac{xD}{d} \Rightarrow dy = \frac{dx.D}{d}$$



Also phase difference

$$\phi = x\frac{2\pi}{\lambda} \Rightarrow d\phi = \frac{dx.2\pi}{\lambda} \Rightarrow dx = \frac{d\phi\lambda}{2\pi}$$

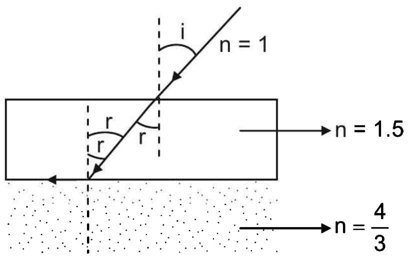
$\therefore \quad dy = \dfrac{d\phi\lambda D}{2\pi d}$ substitute in (1)

$$I_{av} = \frac{\int_0^{\pi} 4KA^2\left(\cos^2\frac{\phi}{2}\right)\frac{d\phi}{2\pi}\frac{\lambda D}{d}}{\frac{\lambda D}{2d}}$$

[$\because \phi$ varies from 0 to π from centre of bright fringe to centre of dark fringe and $\beta = \dfrac{\lambda D}{2d}$]

$$= \frac{4KA^2}{\pi}\int_0^{\pi}\frac{(1+\cos\phi)d\phi}{2} = \frac{4KA^2}{\pi}.\frac{\pi}{2} = 2KA^2$$

11.



Just before total internal reflection between glass and water, ray come out at grazing angle, i.e., $\theta = 90°$ (see figure)

$\dfrac{\sin r}{\sin 90°} = \dfrac{\mu_w}{\mu_g}$; for the incident ray at the top $\Rightarrow$

$$\frac{\sin i}{\sin r} = \mu_g \Rightarrow \sin i = \sin r \mu_g = \frac{\mu_w}{\mu_g} \times \mu_g = \mu_w > 1$$

$$\frac{\sin r}{\sin 90°} = \frac{\mu_g}{\mu_w} \Rightarrow \sin r => 1$$

$\Rightarrow$ Hence TIR not possible

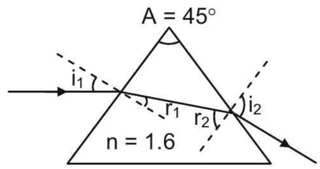
Alternative method:

For the ray passing through layers of different media: $n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$ just before total internal reflection at interface of glass–water $\theta_3 = 90$; and n_1 in air = 1

$\therefore \quad \sin\theta = n_3 \sin 90° = n_3 \Rightarrow \sin\theta > 1$ [$\therefore n_3 > 1$].

Not possible.

12.



$\sin i_1 = \dfrac{5}{3}\sin r_1$; $r_1 + r_2 = A$;

$r_2 = (A - r_1) = (45° - r_1)$

$\dfrac{5}{3}\sin(45° - r_1) = \sin i_2$

$\sin i_2 < 1 \Rightarrow \sin(45° - r_1) < \dfrac{1}{\frac{5}{3}}$

$\Rightarrow 45 - r_1 < 37° \Rightarrow r_1 > 8°$

$\Rightarrow \sin r_1 > \sin 8°$

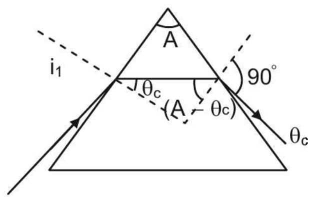
$\Rightarrow \dfrac{3}{5}\sin i_1 > \sin 8° \quad \Rightarrow \sin i_1 > \dfrac{5}{3}\sin 8°$

The angle is small enough so that $\sin\theta \approx \theta$ (radian) holds:

i_1 (in degree). $\dfrac{\pi}{180} > \dfrac{5}{3}\left(8 \times \dfrac{\pi}{180}\right)$

$\therefore \quad i_1$ (in degree) $> \left(\dfrac{5}{3} \times 8\right)^{\circ} > 13°$

13. Clearly $\delta = \delta_{min}$ ($\because$ parallel to base)



$A = 2\theta$ when $\theta = \theta_c$

Grazing incidence and grazing emergence

For T.I. R. to occur:

$\Rightarrow A > 2\theta_c = 2\sin^{-1}\dfrac{1}{\frac{5}{3}}$

$= 2 \times 37° = 74°$

14. $\beta = \lambda \frac{D}{d} \Rightarrow \frac{\beta_2}{\beta_1} \Rightarrow \frac{\lambda_2}{\lambda_1} = \frac{650}{500} = 1.3$

15. $t(n-1) = N\lambda$, $N = 5$

$$\Rightarrow t = \frac{5\lambda}{(n-1)} = \frac{5 \times 600 \times 10^{-9}}{0.5} = 6\ \mu m$$

16. Statement 1 is true but not 2.

If two prisms of same materials are used in the arrangement mentioned there will be neither dispersion nor deviation since the combination works as a slab.

17. Statement 2 is wrong

For a bright fringe, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}$

For a fringe of minimum intensity $= I = I_1 + I_2 - 2\sqrt{I_1 I_2}$, which will not vanish if $I_1 \neq I_2$

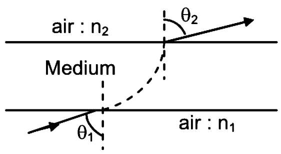
18. Statement 1 is true. Path difference for n^{th} bright fringe is $d \sin\theta = n\lambda$

$$d = \frac{n\lambda}{\sin\theta}$$

$$d_{min} = \frac{n_{min}\lambda}{(\sin\theta)_{max}} = \frac{1 \times \lambda}{1} = \lambda$$

Statement 2 is false. There can be phase difference but it should be a constant over time.

19.



Will be parallel

Consider statement 2 (which of course is true) with each $\Delta x \to 0$ (Consider successive parallel layers f infinitesimal thickness. Let n_i, n_j, n_k, ...n_z be the refractive indices of these layers. Then

$$n_1 \sin\theta_1 = n_i \sin\theta_i$$
$$n_i \sin\theta_i = n_j \sin\theta_j$$
$$n_z \sin\theta_z = n_2 \sin\theta_2$$

Hence for initial and final layer, which are both air:

$n_1 \sin\theta_1 = n_2 \sin\theta_2 \Rightarrow n_1 = n_2 = 1$

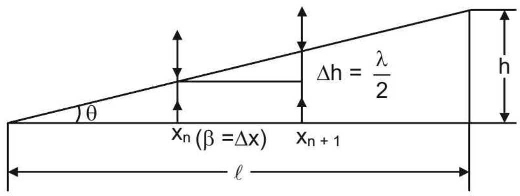
$\Rightarrow \theta_1 = \theta_2$

20. Self explanatory (choose a medium to make $n < 1$)

21. For rays at almost normal incidence if x_n and x_{n+1} are positions of nth and $(n+1)$th bright fringes, $2\Delta h = \lambda$ (since they are adjacent fringes). If β is fringe width, from geometry of the figure

$$\frac{\Delta h}{\beta} = \tan\theta = \frac{h}{\ell} \Rightarrow \frac{\lambda}{2\beta} = \frac{h}{\ell} \Rightarrow \beta = \frac{\ell\lambda}{2h}$$

Fringe width constant for given λ, but varies with λ.



22. Solution same as above.

23. At the line of contact $\Delta h = 0$, path difference is zero. But phase reversal at 2nd surface reflection, gives a phase difference π. Hence destructive interference.

24. $\alpha = \frac{h}{u}$, unaided viewing $\alpha' = \frac{h}{D}$

$$M = \frac{\alpha}{\alpha'} = \frac{D}{u}$$

25. As per Cartesian convention, with the light incident from left to right

$$\frac{1}{(-x)} - \frac{1}{(-u)} = \frac{1}{f}$$

$$\frac{1}{u} = \frac{1}{f} + \frac{1}{x}$$

26. $M = \frac{D}{u} = \frac{D}{f} + \frac{D}{D} = 1 + \frac{D}{f}$

27. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow$ (take differentials):

$$\frac{-\Delta v}{v^2} - \frac{\Delta u}{u^2} = 0 \Rightarrow \frac{\Delta v}{\Delta u} = -\frac{v^2}{u^2}$$

$\Rightarrow$ since both object and image are real, $\frac{v}{u}$ is positive

$\frac{v^2}{u^2}$ is positive

$\therefore\ -\frac{v^2}{u^2}$ is negative $\Rightarrow \frac{\Delta v}{\Delta u}$ is negative

Hence image moves in the opposite direction of the displacement of object,

$\therefore \quad \Delta u = -0.1, \Delta v = +0.4$

$$\frac{\Delta v}{\Delta u} = -4 = -\frac{v^2}{u^2} \Rightarrow \frac{v}{u} = 2$$

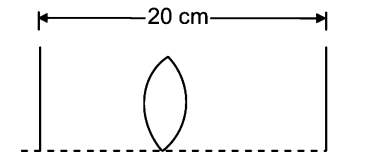
(since both are –ve value)

$$m = -\frac{v}{u} = -2 \Rightarrow |v| = |2u| = 30 \text{ cm}$$

Applying signs appropriately $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$-\frac{1}{30} - \frac{1}{15} = \frac{1}{f} \Rightarrow f = -10 \text{ cm}$$

28.



Use the formula (Solved example 15.27)

$$|u| = \frac{\ell}{2}\left[1 \pm \sqrt{1 - \frac{4f}{\ell}}\right]$$

$$= \frac{20}{2}\left[1 \pm \sqrt{1 - \frac{4 \times 4.2}{20}}\right] = 10\left[1 \pm \sqrt{1 - 0.84}\right]$$

= 10 [1 ± 0.4] = 14 cm or 6 cm, both are interchangeable values of u and v.

u = 6 cm, v = 14 cm gives real image with magnification. Images are inverted in both cases

29. First case: shift of central band

$$= (\mu - 1)\, t_1 \left(\frac{D}{d}\right) = 3\beta = 3\frac{\lambda D}{d}$$

$$t_1 = \frac{3\lambda}{(\mu - 1)} = \frac{3 \times 6000 \times 10^{-10}}{0.6} = 3\ \mu\text{m}$$

2nd case since centre is a dark band, the shift is $\left(2 + \frac{1}{2}\right)\beta$

$$\therefore \quad (\mu - 1)\, t_2 \frac{D}{d} = 2.5\,\beta = 2.5\frac{\lambda D}{d}$$

$$t_2 = \frac{2.5 \times 6000 \times 10^{-10}}{0.6} = 2.5\ \mu\text{m}$$

When both films are used, path difference at the centre is

$$\Delta y = (\mu - 1)t_1 - (\mu - 1)t_2 = (t_1 - t_2)(\mu - 1)$$

$$= (3 - 2.5) \times 10^{-6} \times 0.6$$

Corresponding phase difference = $\Delta y \frac{2\pi}{\lambda}$

$$= \frac{0.5 \times 10^{-6} \times 0.6 \times 2\pi}{6000 \times 10^{-10}} = \pi$$

Hence a dark band at, the centre

30. The lens equation gives

$$\frac{1}{D - u} - \frac{1}{-u} = \frac{1}{f}$$

On simplifying the above quadratic equation,

$$u = \frac{D \pm \sqrt{D^2 - 4Df}}{2}$$

D < 4f:

no position of lens forms real image since u is imaginary. For a concave lens, the image of a real object is always virtual

(a) → q, s

D > 4f:

For a convex lens this gives two positions of the lens resulting in a real image. Also

$$\frac{uv}{u + v} = f \Rightarrow uv = (u + v)f = Df > 4f^2$$

∴ (b) → r

D = 4f

In this case, $u = \frac{D}{2} = 2f$ and $v = \frac{D}{2} = 2f$ for a convex lens. For a concave lens, the image is virtual

∴ (c) → p, q

D = ∞

This means the object is at ∞. The concave lens produces a virtual image. $uv > 4f^2$

∴ (d) → q, r

IIT Assignment Exercise

31. $\frac{1}{v} + \frac{1}{-5} = \frac{1}{f} \quad \frac{1}{v} + \frac{1}{-5} = \frac{1}{-7.5} \Rightarrow v = +15$

Magnification = $\frac{-v}{u} = \frac{-15}{-5} = 3$

32. The image will be virtual, erect and behind the mirror. The image will be enlarged.

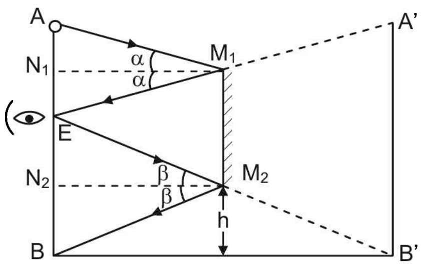
33. (a) is wrong

All others are normal behaviour of plane mirror.

$P = 0; \therefore f = \text{infinity}$

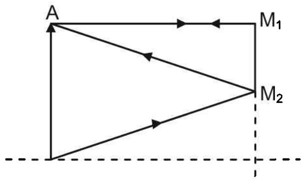
$\delta = 180 - 2i$

34.



$$M_1M_2 = N_1N_2$$
$$= N_1E + EN_2$$
$$= ½ (AE) + ½ (BE)$$
$$= ½ (AE + BE)$$
$$= ½ \times AB = ½ \times \text{height of man} = 0.9 \text{ m}$$

Aliter:



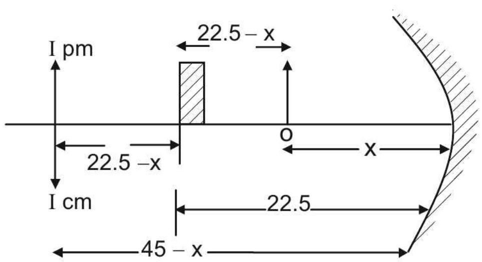
It can be verified that wherever the mirror M_1M_2 is kept with respect to the object OA $M_1M_2 = \dfrac{OA}{2}$

35. $h = BN_2 = EB - EN_2 = ½ \, EB$
$= ½ \times 1.6 \text{ m} = 0.8 \text{ m}$

36. The image formed by a mirror will always be an object for the other, so number of images is infinite.

37. Principle of optical reversibility: if any ray is reversed it will retrace its path through an optical system.

38.



$u = -x$

$v = -(45 - x)$

$f = -10$ cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-x} + \frac{1}{-(45-x)} = \frac{1}{-10}$$

$\Rightarrow \quad x^2 - 45x + 450 = 0$

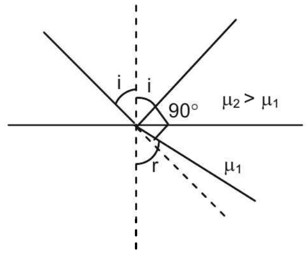
$\Rightarrow \quad x = 30 \text{ or } 15, x < 22.5 \Rightarrow x = 15 \text{ cm}$

The above answer can be obtained without any calculation. The image of the real object, formed by a plane mirror is behind the mirror i.e., at distance greater than 22.5 cm from concave mirror. i.e., at distance greater than 2f (= 20 cm) from plane mirror. This means object has to be placed in between f and 2f of concave mirror. i.e., 10 cm < x < 20 cm. The only value which meets this requirement is 15 cm.

39. If mirror has speed v, the image has speed 2v.

$v = at = \dfrac{x_0}{a}.a = x_0 \Rightarrow \text{Image speed} = 2x_0$

40.



$$\theta_c = \sin^{-1}\left(\frac{\mu_1}{\mu_2}\right)$$

$\mu_2 \sin i = \mu_1 \sin r$

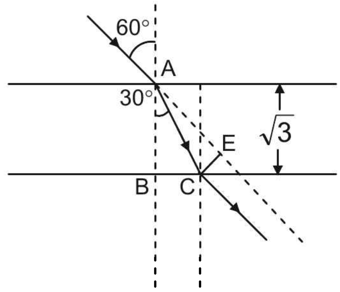
Also $r = 90 - i$

$$\therefore \frac{\mu_1}{\mu_2} = \frac{\sin i}{\sin(90 - i)} = \tan i$$

$$\sin \theta_c = \frac{1}{{}_1\mu_2} = \frac{\mu_1}{\mu_2} = \tan i$$

$\theta_c = \sin^{-1} (\tan i)$

41.



$$AC = \frac{AB}{\cos 30°} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}} = 2$$

$$\frac{CE}{AC} = \sin 30° = \frac{1}{2}$$

$$\text{Shift } CE = \frac{AC}{2} = 1 \text{ m}$$

Aliter:

$$\text{Lateral shift} = \frac{t\sin(i-r)}{\cos r} = \frac{\sqrt{3}\sin 30°}{\cos 30°}$$

$$= \sqrt{3}.\tan 30° = 1 \text{ m}$$

42. Light rays bend towards normal when entering to medium of larger μ and 'vice versa.'

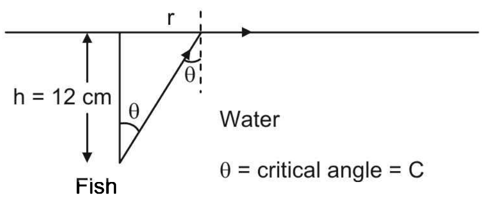
$\mu_g \approx 1.5$; $\mu_{water} \approx 1.33$; $\mu_{diamond} \approx 2.42$

43. At critical angle $\rightarrow \sin C = \frac{1}{{}_{m2}\mu_{m1}} = \frac{1}{\mu_{m1}/\mu_{m2}}$

$$= \frac{1.33}{1.57} = 0.875$$

44. Concave

45.



$$\frac{r}{h} = \tan C$$

$$\sin\theta = \frac{1}{\mu} = \frac{3}{4}$$

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{\frac{(16-9)}{16}} = \frac{\sqrt{7}}{4}$$

$\tan\theta = 3/\sqrt{7} = r/12 \quad \therefore\ r = (12 \times 3)/\sqrt{7} = 36/\sqrt{7}.$

Aliter:

$$R = h\tan C = h\frac{\sin C}{\cos C} = h\frac{\sin C}{\sqrt{1-\sin^2 C}}$$

$$= \frac{h}{\mu}.\frac{1}{\sqrt{1-\frac{1}{\mu^2}}} \qquad \left(\because \sin C = \frac{1}{\mu}\right)$$

$$= \frac{h}{\sqrt{\mu^2-1}} = \frac{12 \text{ cm}}{\sqrt{\left(\frac{4}{3}\right)^2 - 1}} = \frac{12 \text{ cm} \times 3}{\sqrt{4^2-3^2}} = \frac{36}{\sqrt{7}} \text{ cm}$$

46. Apparent depth in water

= 10.0/1.33 = 7.52 cm

Apparent depth in liquid = 10/1.6 = 6.25 cm

$\therefore$ Distance to be shifted = 7.52 – 6.25

= 1.27 cm upward.

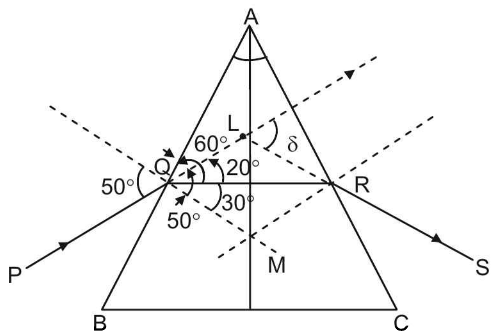
47. $\text{Shift} = t\left(1-\frac{1}{\mu}\right) = 10\left[1-\frac{1}{1.6}\right] = 3.75$

48. $A_1(\mu_1 - 1) = A_2(\mu_2 - 1)$ for no deviation

$4(1.54 - 1) = 3(\mu_2 - 1)$

$\Rightarrow \mu_2 = 1.72$

49.



QR parallel to BC

$\therefore$ $\angle AQR = 60°$

$\angle MQR = (90° - 60°) = 30°$

$\angle LQR = (50° - 30°) = 20°$

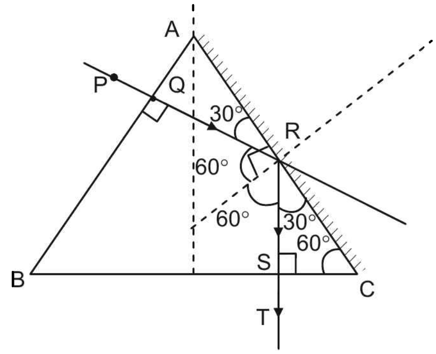
Similarly $\angle LRQ = 20°$

$\therefore$ $\delta = 20° + 20° = 40°$ or

Aliter:

$D = 2i - A = 100° - 60° = 40°$

50. From the geometry of the figure, it is easily seen that emergent ray is perpendicular to BC



51. $r_1 + r_2 = A = 75° \Rightarrow$ for T.I. R;

$\sin r_2 = \frac{1}{\mu} = \frac{1}{\sqrt{2}} \Rightarrow r_2 = 45°$

$\therefore \quad r_1 = 75° - 45° = 30° \Rightarrow \frac{\sin i_1}{\sin r_1} = \sqrt{2}$

$\Rightarrow \quad \sin i_1 = \frac{1}{2}.\sqrt{2} = \frac{1}{\sqrt{2}}$

$i_1 = 45°$

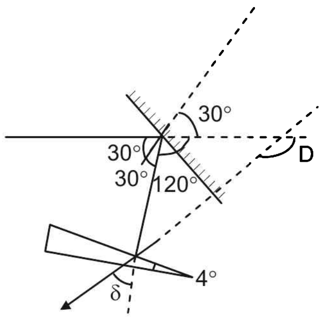
52. For zero deviation $(\mu_1 - 1) A_1 - (\mu_2 - 1) A_2 = 0$

$\Rightarrow \quad (1.54 - 1) 4 = (1.72 - 1) \theta$

$\theta = 3°$

53. $\delta = (\mu - 1) A = (1.5 - 1) 5 = 2.5°$

54.



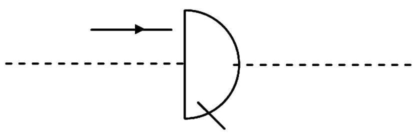
Thin prism

Hence $\delta = A(\mu - 1)$

$\delta = 4(1.5 - 1) = 2°$

D = Total deviation = $120° + \delta = 122°$

55.



Rays falling normally on the plane surface pass without deviation, so that the same parallel beam falls on the curved surface.

For refraction at the curved surface

$u = -\infty, \quad v = f, \quad R = -20$ cm

$\frac{1}{f} - \frac{1.5}{-\infty} = \frac{1 - 1.5}{-20}$

$\frac{1}{f} = \frac{1}{40}$; $f = 40$ cm

Distance from plane surface = 40 + 20 = 60 cm

56. Since lens is concave $f = -12$ cm

$-1/u + 1/v = 1/f \Rightarrow 1/v = 1/f + 1/u = \frac{1}{-12} + \frac{1}{-4}$

$v = -3$ cm

57. Since magnification is taking place, the lens is convex.

The image is real when object is at 16 cm and virtual when object is at 8 cm

$M = \frac{v}{u} = -3 \Rightarrow v = -3 \times -16 = 48$

$\frac{1}{48} - \frac{1}{-16} = \frac{1}{f} \Rightarrow f = 12$

$f = 12$ cm.

Aliter:

Magnification is taking place $\Rightarrow$ lens is convex, eliminates (a) and (b). When object is placed between f and 2f, magnified real image is formed. So one of the u values is greater than f and the other u value is less than f. Hence the value of f lies in between the two values of u. i.e., f is in between 16 cm and 8 cm, eliminates (c). Hence correct answer is (d).

58. $P = 1/f = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f} = 5 = 0.5\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ and

$-1 = \left(\frac{1.5}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\Rightarrow$ Solving: $\Rightarrow -5 = \frac{1}{2}\frac{1}{\left(\frac{1.5}{n_1} - 1\right)}$

$n_1 = \frac{5}{3}$

Aliter:

$F = \frac{1}{P} = \frac{1}{5} = 0.2$ m = 20 cm

$f_m = -100$ cm ($\because$ diverging)

$f_m = \frac{f\, n_m (n_\ell - 1)}{(n_\ell - n_m)}$

$\Rightarrow \quad -100 = \frac{20 \times n_m (1.5 - 1)}{(1.5 - n_m)}$

$\Rightarrow \quad -10 = \frac{n_m}{(1.5 - n_m)} \Rightarrow -15 + 10\, n_m = n_m$

$\Rightarrow \quad n_m = \frac{5}{3}$

59. $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}-\frac{1}{(-20)}=\frac{1}{15}$

$\Rightarrow \frac{1}{v}=\frac{1}{15}-\frac{1}{20} \Rightarrow v = 60$ cm

The image formed by lens will be the object for the mirror. It is at a distance of 60 – 5 = 55 cm behind the mirror. : To create the image at the object, the rays should retrace after reflection at the mirror; i.e., the rays are incident normally at the mirror, i.e., u = R

$\Rightarrow u = R \Rightarrow R = 55$ cm

60. $f = \frac{d^2 - x^2}{4d}$

$16 = \frac{d^2-(60)^2}{4d}$

d = 100 cm

61. Shortest possible distance between the object and real image is 4f.

For that the minimum distance of object from lens is 2f.

$3f - \left[1-\frac{1}{\mu}\right]t = 2f$

$\mu = \frac{t}{t-f}$

62. Initially

$\frac{1}{36}+\frac{1}{45}=\frac{1}{F} \Rightarrow F = 20$ cm

Finally,

$\frac{1}{48}+\frac{1}{\left(5+\frac{40}{\mu}\right)}=\frac{1}{F}=\frac{1}{36}+\frac{1}{45}$

$\Rightarrow 5+\frac{40}{\mu}=\frac{240}{7} \Rightarrow \mu = 1.37$

63. $\frac{1}{f}=\left(\frac{3}{2}-1\right)\left(\frac{1}{R}-\frac{1}{-R}\right)=\frac{1}{R}$

f = R

$\frac{1}{v}-\frac{1}{+\frac{R}{3}}=\frac{1}{+R}$ $\qquad v=\frac{R}{4}$

$m=\frac{v}{u}=\frac{+R/4}{+R/3}=\frac{3}{4}$

64. $\frac{1}{f_1}=(\mu-1)\left(\frac{1}{\infty}-\frac{1}{R}\right)=-\frac{\mu-1}{R}$

$\frac{1}{f_2}=\left(\frac{4\mu}{3}-1\right)\left(\frac{1}{R}-\frac{1}{\infty}\right)=\left(\frac{4\mu}{3}-1\right)\frac{1}{R}$

$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{\mu}{3R}$ $\qquad f=\frac{3R}{\mu}$

65. $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\frac{1}{3R}=(\mu-1)\left(\frac{1}{R}-\frac{1}{\infty}\right)$

$\mu = 1+\frac{1}{3}=\frac{4}{3}=1.33$

66. $P=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

When immersed in medium of μ’.

$\frac{-P}{2}=\frac{\mu'}{f_1}=\mu'\left(\frac{\mu}{\mu'}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ $\therefore \mu' = 1.75$

Aliter:

$P_m = P_{air}\frac{(n_\ell - n_m)}{(n_\ell - 1)}$

$\Rightarrow -\frac{P}{2}=P\frac{(1.5-n_m)}{(1.5-1)}$

$\Rightarrow \frac{1}{4}=1.5-n_m$

$\Rightarrow n_m = 1.5 + 0.25 = 1.75$

67. $D=\frac{1}{f} \Rightarrow D'=\frac{1}{f/2}=2D$

68. $\frac{[ma]^2}{a^2}=16, m=4, \frac{v}{u}=4$

$\Rightarrow v = 4\times u = 120$ cm

69. frequency

70. $\frac{c}{v}=\mu \Rightarrow \frac{f\lambda}{f\lambda_1}=1.5$

$\Rightarrow \lambda_1=\frac{\lambda}{1.5}=\frac{6000}{1.5}$

$= 400Å = 400$ nm

71. $\frac{c}{v} = n$;

$\therefore \quad v \propto \frac{1}{n}; v = f\lambda^1 \Rightarrow v \propto \lambda^1$

$\lambda' = \frac{v}{f} = \frac{c}{nf} = \frac{f\lambda}{nf} = \frac{\lambda}{n}$

72. $\frac{c}{v} = n$

73. $v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8 \text{ m s}^{-1}$

74. $v = \frac{c}{n} \Rightarrow \frac{v_1}{v_2} = \frac{n_2}{n_1}; v_1 > v_2 \Rightarrow n_2 > n_1$

75. $c = \lambda f \Rightarrow f = \frac{c}{\lambda} = \frac{3 \times 10^8}{6000 \times 10^{-10}}$

$= 0.5 \times 10^{15}$ Hz

76. Shift by one fringe width corresponds to a path difference λ. Hence path difference $t(\mu - 1) = n\lambda$

$t = \frac{n\lambda}{(\mu - 1)}$

$= \frac{5(4000)}{\frac{3}{2} - 1} = 10(4000)$Å

$= 4\ \mu$m

77. Phase difference $= \frac{\pi}{2}$

$\Rightarrow \quad E_0\left[\sin\left(\omega t + \frac{\pi}{3}\right) + \sin\left(\frac{\pi}{2} + \omega t + \frac{\pi}{3}\right)\right]$

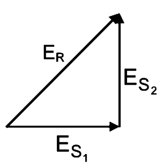
$\Rightarrow \quad 2E_0 \sin\left(\omega t + \frac{\pi}{3} + \frac{\pi}{4}\right)\cos\frac{\pi}{4}$

$\Rightarrow \quad 2E_0 \frac{1}{\sqrt{2}}\sin\left(\omega t + \frac{7\pi}{12}\right)$

$\therefore \quad E_r = \sqrt{2}E_0$

Aliter:

Consider phasor addition which is identical to vector addition:



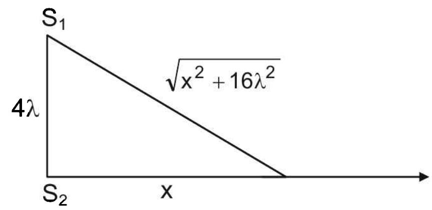
$\Rightarrow \quad |E_{S_1}| = |E_{S2}| = E_0$

$E_R = \sqrt{2}\,E_0$

78. $\left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = n$

$\frac{I_1}{I_2} = \left(\frac{\sqrt{n} + 1}{\sqrt{n} - 1}\right)^2$

79.



$\sqrt{x^2 + 16\lambda^2} - x = \lambda$

$\sqrt{x^2 + 16\lambda^2} = x + \lambda$

$x^2 + 16\lambda^2 = (x + \lambda)^2$

$x^2 + 16\lambda^2 = x^2 + \lambda^2 + 2x\lambda$

$15\lambda = 2x$

$x = 7.5\ \lambda$

80. $n\lambda = (\mu - 1)\,t$

$10 \text{ x } \lambda = 0.6 \times 10^{-5}$

$\lambda = 0.6 \times 10^{-6}$ m

$= 6000 \times 10^{-10}$ m $= 6000$ Å

81. $m = -\frac{1}{n} = -\frac{v}{u}$

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{n}{u} + \frac{1}{u} = \frac{1}{f}$

$\therefore \quad u = (n + 1)f \Rightarrow$ Both u and f should be –ve, i.e., in same direction

Aliter:

$m = \frac{f}{f - u}$ for mirror.

Here $m = \frac{1}{n}$ ($\because$ real image, m is –ve)

$f = -f$ ($\because$ concave mirror)

$u = -u$ ($\because$ real object)

$\therefore \quad -\frac{1}{n} = \frac{-f}{-f+u} \Rightarrow f - u = -nf$

$\Rightarrow (n+1)f = u$

82. Real image $m = -\frac{v}{u} = -3$

[$\therefore$ the image is inverted]

$v = 3u$; $f = -30$ cm

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{3u} + \frac{1}{u} = \frac{1}{-30}$

$\therefore \quad u = -40$ cm

Virtual image $m = -\frac{v}{u} = 3$

[$\because$ the image is virtual]

$\Rightarrow \quad v = -3u \Rightarrow \frac{-1}{3u} + \frac{1}{u} = \frac{1}{-30}$

Then $u = -20$ cm

Aliter:

$m = -3$ for real image

$f = -30$ cm ($\because$ concave mirror)

u is –ve for real image

We have $m = \frac{f}{f-u} \Rightarrow -3 = \frac{-30}{-30+u}$

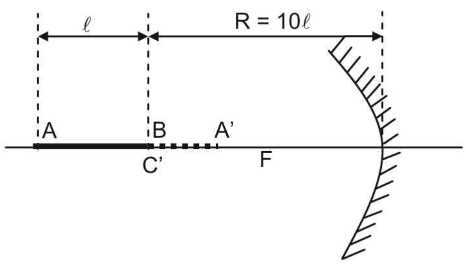
$\Rightarrow \quad 90 - 3u = -30 \Rightarrow u = \frac{120}{3} = 40$ cm

$m = +3$ for virtual image

$\Rightarrow \quad +3 = \frac{-30}{-30+u} \Rightarrow -90 + 3u = -30$

$\Rightarrow u = \frac{60}{3} = 20$ cm

83.



Since the image is diminished the object should be beyond 2F.

One end of the rod must be at the centre of curvature C, the other end beyond C

$\therefore \quad R = 2f = 10\ell$

For A :

$u = -11\ell$

$f = -5\ell$

$\therefore \frac{1}{v} - \frac{1}{11\ell} = \frac{1}{-5\ell}$

$v = -\frac{55\ell}{6} \rightarrow$ position of A'

$\therefore$ A'B $= 10\ell - \frac{55\ell}{6} = \frac{5}{6}\ell$

$\therefore$ magnification $= \frac{5}{6}$

84. Focal length of a mirror does not depend on the medium (no refraction)

85. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$u = +30$ cm

$f = +20$ cm

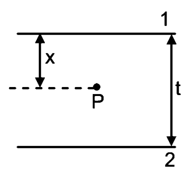
$v = 60$ cm

$m = -\frac{v}{u} = -2$

$|m| > 1$

$\therefore$ Image is virtual, inverted and enlarged.

86.

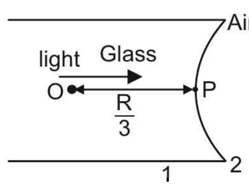


The apparent depth from side 1 $= \frac{x}{\mu} = d$

From side 2 , $\frac{t-x}{\mu} = 3d$

$\therefore \quad \frac{t}{\mu} = 4d, \quad t = 4\mu d$

87.



$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\frac{1}{v} - \frac{1.5}{\left(-\frac{R}{3}\right)} = \frac{1-1.5}{+R}$$

$$\frac{1}{v} = -\frac{0.5}{R} - \frac{4.5}{R} = \frac{-5}{R}$$

$v = -\frac{R}{5}$ [–ve sign shows, left of P]

88. $\mu = \dfrac{\sin\frac{A+D_m}{2}}{\sin\frac{A}{2}} \Rightarrow \sqrt{2} = \dfrac{\sin\frac{A+D_m}{2}}{\sin 30°}$

$\because A = 60°,\ \mu = \sqrt{2}$

$$\sin\frac{A+D_m}{2} = \sqrt{2}\sin 30° = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\frac{A+D_m}{2} = 45° \Rightarrow D_m = 90° - A$$

$\Rightarrow \quad D_m = 30°$

89. $i = \dfrac{D_{min} + A}{2} = \dfrac{30° + 60°}{2} = 45°$

90. $\dfrac{1}{f_0} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$ ----(1)

$$\frac{1}{f'} = \left(\frac{\mu}{\left(\frac{5\mu}{6}\right)} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$= \dfrac{1}{5}\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]$ ----(2)

Divide (1) by (2) $\Rightarrow$

$$\frac{f'}{f_0} = \frac{(\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}{\frac{1}{5}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} = 5(\mu - 1)$$

$f' = 5(\mu - 1) f_0$

Aliter:

$$f_m = \frac{f_{air} \times n_m (n_\ell - 1)}{(n_\ell - n_m)}$$

$$\Rightarrow \quad f_m = f_0 \times \frac{5\mu}{6} \times \frac{(\mu - 1)}{\left(\mu - \frac{5\mu}{6}\right)} = f_0\ 5(\mu - 1)$$

$\therefore \quad f_m = 5 f_0 (\mu - 1)$

91. $P = \dfrac{100}{f_0} + P'$

$$P' = P - \frac{100}{f_0}$$

$$\therefore f' = \frac{1}{P - \frac{100}{f_0}} m = \frac{100\,cm}{P - \frac{100}{f_0}} = \frac{100 f_0}{Pf_0 - 100} cm$$

Aliter:

$P = P_1 + P_2 \Rightarrow P = \dfrac{100}{f_1} + \dfrac{100}{f_0}$,

where, f_1 and f_0 are in cm.

$$\Rightarrow \frac{P}{100} - \frac{1}{f_0} = \frac{1}{f_1}$$

$$\frac{Pf_0 - 100}{100 f_0} = \frac{1}{f_1} \Rightarrow \frac{100 f_0}{(Pf_0 - 100)} = f_1$$

92. $\dfrac{1}{f_1} = \left(\dfrac{\mu_2}{\mu_1} - 1\right)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

$$= \left(\frac{\frac{3}{2}}{2} - 1\right)\left(-\frac{1}{R} - \frac{1}{R}\right) = -\frac{1}{4} \times \frac{-2}{R} = \frac{1}{2R}$$

$\therefore$ f = 2R ($\therefore$ converging lens)

93. $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{-R} - \dfrac{1}{\infty}\right) = \dfrac{\mu - 1}{-R} = -\dfrac{5}{3} m^{-1}$

Power $= -\dfrac{5}{3}$ D

94. $n_{water} < n_{glass} \Rightarrow$ speed increases $\Rightarrow \lambda$ increases

95. $\mu_{violet} > \mu_{green} > \mu_{red}$

96. $\dfrac{n\lambda_b D}{d} = (n - 1)\dfrac{\lambda_r D}{d}$

$n\lambda_b = (n - 1)\lambda_r$

$n(5200) = (n - 1)7800$

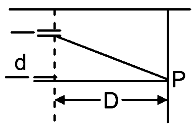
$n = 3$

97. $\beta = \frac{\lambda D}{d}$; $\Delta\beta = \frac{\lambda}{d}\Delta D$

$$30 \times 10^{-6} = \frac{\lambda}{10^{-3}}\left(5 \times 10^{-2}\right)$$

$\lambda = 6000 Å$

98.



Path difference at P $= (D^2 + d^2)^{\frac{1}{2}} - D$

$$= D\left[1 + \frac{d^2}{D^2}\right]^{\frac{1}{2}} - D$$

$$= D + D.\frac{1}{2}\frac{d^2}{D^2} - D\left[\because \frac{d}{D} << 1\right] = (2n-1)\frac{\lambda}{2}$$

(for dark band)

$$\lambda = \frac{d^2}{(2n-1)D}$$

For destructive interference at P

n = 1, 2, 3.....

$$\therefore \lambda = \frac{d^2}{D}, \frac{d^2}{3D}, \frac{d^2}{5D}, \ldots.$$

99. $\beta = \frac{\lambda D}{d}$;

$$\beta' = \frac{(2\lambda)D}{2d} = \beta$$

100. $I = \left(\sqrt{I_0} + \sqrt{I_0}\right)^2 = 4I_0$ for central maxima

101. Two sources are coherent only when λ is equal

102. Remains the same

103. Energy

104. $\beta = \frac{D\lambda}{d}$, $d \to \frac{d}{2}$ and $D \to 3D$

$\beta = 6$ times

105. $I \propto$ (amplitude)^2
Resultant amplitude $(A_1 + A_2)$ at the maximum and $A_1 - A_2$ at the minimum

$$\frac{I_1}{I_2} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{(4+2)^2}{(4-2)^2} = \frac{36}{4} = 9$$

Ratio $I_{min} : I_{max} = 1 : 9$

106. The emerging rays are parallel to incident rays.

107. In figure (c), both the curved surfaces have same R towards the same side. Hence no dispersion is exhibited.

For a lens, $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

(Lens maker's formula)

For no dispersion, $d\left(\frac{1}{f}\right) = 0$

$\therefore \quad 0 = (d\mu)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ or $R_1 = R_2$ is the condition for no dispersion.

108. Scattering is inversely proportional to wavelength

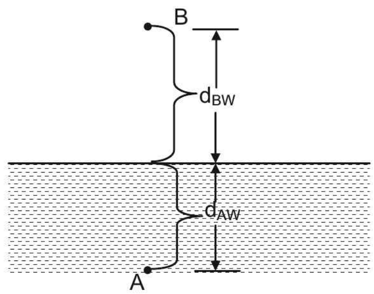
109. Resolving power $\propto \frac{1}{\lambda}$

110. Because of reflection at boundary, additional phase difference π equivalent to path difference of $\lambda/2$ happens : destructive interface $= n\lambda$

111. A coloured glass absorbs all other colours

112. Not possible to choose a medium rarer than air. i.e., n cannot be increased.

113.



$d = d_{BW} + d_{AW}$ (given)

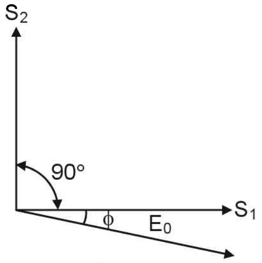
$d_{BA} = d_{AW} + \mu d_{BW} \quad (\because \mu > 1)$

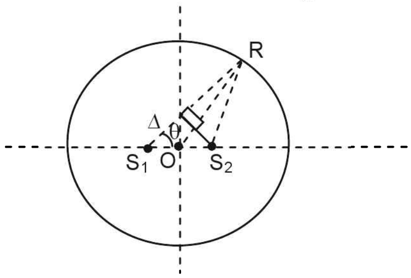
$\therefore \quad d_{BA} > d$

$d_{AB} = d_{BW} + \frac{d_{AW}}{\mu} (\because \mu > 1)$

$\therefore \quad d_{AB} < d$

114.





$\Delta = 2d\cos\theta$

$\phi = \dfrac{2\pi}{\lambda}(2d\cos\theta)$

For maxima $\phi + \dfrac{\pi}{2} = 2m\,\pi$, m = 1, 2, 3, …

$\dfrac{4\pi d\cos\theta}{\lambda} = 2m\pi - \dfrac{\pi}{2}$

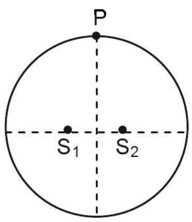
$\cos\theta = \dfrac{\lambda}{4d}\left(2m - \dfrac{1}{2}\right), m = 1,2,3,...$

115. $\dfrac{2m - \dfrac{1}{2}}{20} \le 1,\ 2\,m \le 20.5$

m = 1, 2, 3, … 10

10 points.

116.

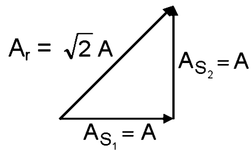


The path difference at P from S_1 and S_2 is zero. But the sources are having a phase difference of $\dfrac{\pi}{2}$. Hence if A is the individual amplitude in phasor addition

$A_r = \sqrt{2}$ A. $\therefore$ $I \propto \left(\sqrt{2}\,A\right)^2 \propto 2A^2$;

$I_0 \propto 4\,A^2 \Rightarrow I = \dfrac{I_0}{2}$

Aliter:



$I = 4\,A^2\cos^2\dfrac{\phi}{2} = 4\,A^2\cos^2 45° = \dfrac{4A^2}{2}$

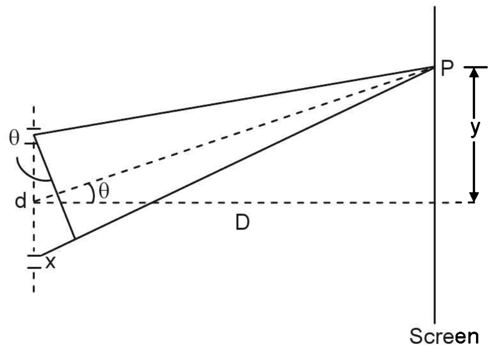
$= \dfrac{I_0}{2}$

117. ($I \propto A^2$ is written as $I = A^2$ for convenience)

If A is the amplitude of interfering beams, then $I_0 = (A + A)^2 = 4A^2$ (when they are in phase). If δ is the phase difference: $(A')^2 = A^2 + A^2 + 2\,AA\cos\delta = 2A^2(1+\cos\delta)$

$= 2\,A^2\left(1 + 2\cos^2\left(\dfrac{\delta}{2}\right) - 1\right) = 4A^2\cos^2\left(\dfrac{\delta}{2}\right)$

$= I_0\cos^2\dfrac{\delta}{2}$



$I_0' = (A')^2 = I_0\cos^2\dfrac{\delta}{2}$

From the figure:

$\sin\theta = \tan\theta = \dfrac{x}{d} = \dfrac{y}{D}$

$\therefore$ Path difference at P:

$x = \dfrac{yd}{D} \Rightarrow$ corresponding

phase difference $\delta = \dfrac{2\pi}{\lambda}\dfrac{yd}{D} = \dfrac{2\pi y}{\left(\lambda D/d\right)} = \dfrac{2\pi y}{\beta}$

$I' = I_0\cos^2\left(\dfrac{\pi y}{\beta}\right)$

When y = 0.25 mm and β = 1 mm ⇒

$I' = I_0\cos^2\dfrac{\pi}{4} = \dfrac{I_0}{2}$

118. Here u = x (+ve since in the direction of beam)

$$\frac{1}{v}-\frac{1}{|x|}=\frac{-1}{10}\Rightarrow\frac{1}{v}=\frac{1}{|x|}-\frac{1}{10}=\frac{10-|x|}{10|x|}$$

$$v=\frac{10|x|}{10-|x|}$$

$\Rightarrow$ v +ve, i.e., on RHS, if $|x| < 10$

Hence (a), (b) are correct

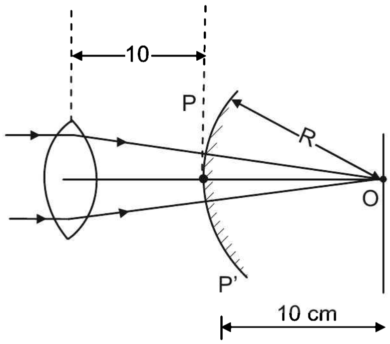
When x = 10; v = ∞ i.e., parallel beam is produced.

(c) $\rightarrow$ wrong

When $|x| = 20 \Rightarrow v = \dfrac{10(20)}{10-20} = -20$ cm

$\Rightarrow$ (d) is correct

119.



Distance from lens to O is: $f=\dfrac{1}{5}=0.2\,\text{m}$

$\therefore$ OP = 10 cm. When convex mirror is used

R = 2f = 10 cm, Hence rays fall normal on surface and retraces the path

Obviously concave mirror does not create a parallel beam

When a concave lens is at PP', u = + 10 cm

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow\frac{1}{v}-\frac{1}{10}=-\frac{1}{10}\Rightarrow\frac{1}{v}=0$$

$\Rightarrow$ v = ∞

A parallel beam proceeds towards right. However, a convex lens at this position does not produce parallel beam.

120. For any δ, $\delta = (i_1 + i_2) - A$

When $\angle A$ and $\angle i_1$ are small, $\sin i_1 = \mu \sin r_1$

$\Rightarrow$ $i_1 = \mu r_1$ and $i_2 = \mu r_2$

$\therefore$ $\delta = (i_1 + i_2) - (r_1 + r_2) = (\mu - 1)(r_1 + r_2)$
$= (\mu - 1)A$

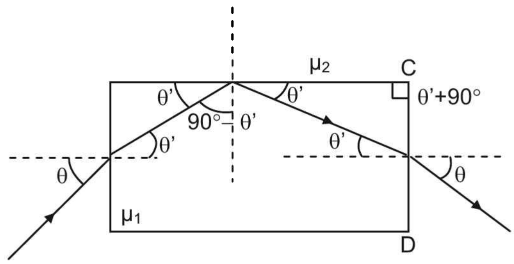
δ_{min} occurs when $i_1 = i_2 = i$

$\delta_{min} = 2i - A$

δ_{max} occurs when $i_1 = \dfrac{\pi}{2} \Rightarrow \delta = \dfrac{\pi}{2} + i_2 - A$

Additional Practice Exercise

121.



(a) $\mu_2 \sin\theta = \mu_1 \sin\theta'$ —(1)

$\sin(90° - \theta^I) \geq \dfrac{\mu_2}{\mu_1}$ for internal reflection at BC

$\cos\theta^I \geq \dfrac{\mu_2}{\mu_1}$ —(2) Let the critical value be θ'_{c}. $\cos\theta'_c =$ μ_2/μ_1 and since it is cosine function the acceptable of $\theta' < \theta_c$ (for $\cos\theta' > \cos\theta_c'$ to be met)

From Equation (1) and (2)

Let incident angle be θ_c for θ'_c. Then from (1)

$$\text{Sin}\,\theta_c=\frac{\mu_1}{\mu_2}\sin\theta'_c\Rightarrow\sin^2\theta_c=\frac{\mu_1^2}{\mu_2^2}\sin^2\theta'_c$$

$$\text{From (2): }\sin^2\theta_c'=1-\cos^2\theta_c'=\left(1-\frac{\mu_2^2}{\mu_1^2}\right)$$

$$\therefore\quad\sin^2\theta_c=\frac{\mu_1^2}{\mu_2^2}\left(1-\frac{\mu_2^2}{\mu_1^2}\right)=\left(\frac{\mu_1^2}{\mu_2^2}-1\right)$$

But $\theta < \theta_c \Rightarrow$

$\therefore$ $\sin\theta < \sqrt{\left(\dfrac{\mu_1}{\mu_2}\right)^2-1}$ $\rightarrow$ for the ray to be internally reflected at BC

Range of values of θ is

$$0<\theta<\sin^{-1}\left(\sqrt{\left(\frac{\mu_1}{\mu_2}\right)^2-1}\right)\qquad\text{— (3)}$$

At the surface CD

Angle of incidence = θ'

Hence will emerge at θ

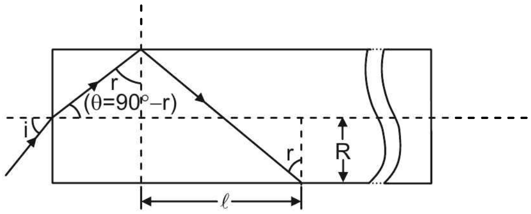
(b). The ray after T.I.R at BC will emerge from CD by refraction, provided, when θ = 90 ;
Sin θ = 1 ⇒ from equation (3)

$$\left(\frac{\mu_1}{\mu_2}\right)^2 - 1 > 1$$

$$\frac{\mu_1}{\mu_2} > \sqrt{2}$$

If $\mu_2 = 1$ (Air) then $\mu_1 > \sqrt{2}$

122.



$\sin 40° = 1.3 \sin\theta \Rightarrow 0.65 = 1.3 \sin\theta$

$$\theta = \sin^{-1} 0.5 = \frac{\pi}{6}$$

$$r = \frac{\pi}{2} - \theta = \frac{\pi}{3}\,\text{rad}$$

for TIR to happen $\sin r \geq \dfrac{1}{\mu}$

$\mu \,.\, \sin r = 1.3 \,.\, \sin\dfrac{\pi}{3} = 1.3 \times 0.86 > 1$

$\Rightarrow$ TIR $\Rightarrow$ from the fig $\tan r = \dfrac{\ell}{2}.\dfrac{1}{R}$

$\Rightarrow \ell = 2R\tan r$

$\therefore \ell = 2 \times 10^{-5} \times \sqrt{3} = 3.5 \times 10^{-5}\,\text{m}$ $\therefore$ Total no: of reflections $\simeq \dfrac{3.5}{3.5 \times 10^{-5}} = 10^5$

123. $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$\mu_2 = 1$, $\mu_1 = \dfrac{4}{3}$, u = – 21 cm, R = – 14 cm,

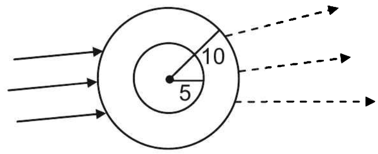
v = – 25.2 cm

Virtual image, lateral magnification

$$m = \frac{\mu_1 v}{\mu_2 u} = \frac{\frac{4}{3}(-25.2)}{1(-21)} \simeq 1.6 \text{ erect}$$

and magnified.

124.



The image formed by the first refraction is given by

$$\frac{\mu}{v} - \frac{1}{\infty} = \frac{\mu - 1}{R_1} \Rightarrow \frac{1.5}{v} = \frac{1.5 - 1}{10}$$

1^{st} Surface $\Rightarrow \dfrac{1.5}{v_1} - \dfrac{1}{\infty} = \dfrac{1.5-1}{10} = \dfrac{0.5}{10}$

$$v_1 = \frac{1.5 \times 10}{0.5} = 30 \text{ cm}$$

2nd Surface ⇒ virtual object on RHS of the surface

$\Rightarrow u_2 = 30 - 5 = 25$ cm

$$\frac{1}{v_2} - \frac{1.5}{25} = \frac{1 - 1.5}{5} = -\frac{1}{10} \Rightarrow v_2 = -25 \text{ cm}$$

3rd Surface $\Rightarrow u_3 = -25 - 10 = -35$ cm

$$\frac{1.5}{v_3} - \frac{1}{-35} = \frac{1.5 - 1}{-5} \Rightarrow \frac{1.5}{v_3} = -\frac{1}{10} - \frac{1}{35}$$

$$\Rightarrow v_3 = \frac{-35}{3} \text{ cm}$$

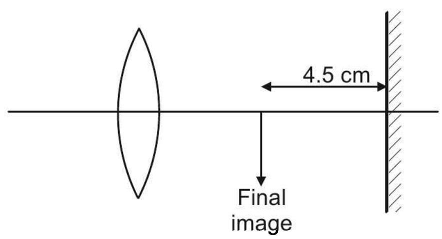
4th Surface $u_4 = \dfrac{-35}{3} - 5 = \dfrac{-50}{3}$ cm

$$\frac{1}{v_4} - \frac{1.5}{\frac{-50}{3}} = \frac{1 - 1.5}{-10}$$

$$\Rightarrow \frac{1}{v_4} = \frac{1}{20} - \frac{4.5}{50} \Rightarrow v_4 = -25 \text{ cm}$$

i.e., Virtual image; 5 cm behind left edge.

125.



For the first image of O formed by the lens:

$$\frac{1}{v} - \frac{1}{-45} = \frac{1}{15}$$

$$v = \frac{45}{2} = 22.5 \text{ cm}$$

$\therefore$ Real, inverted image I,

$$m_1 = \frac{v}{u} = \frac{22.5}{-45} = -\frac{1}{2}$$

I_1 serves as object to the mirror

$$\frac{1}{v^I} + \frac{1}{+4.5} = \frac{1}{\infty};$$

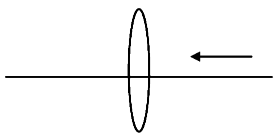
$v^I = -4.5$ cm

Real image

$$m_2 = -\frac{(-4.5)}{+4.5} = +1$$

The effect of the mirror is to shift the image 9 cm closer to lens. The final image is a real image, inverted and half the size of the object. Since their image is within the focal length of the lens, it does not produce another real image

126.



Lens maker's formula for a thin lens

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$R_1 = -120$ cm, $R_2 = -80$ cm, $\mu_1 = \frac{5}{4}$, $\mu_2 = \frac{3}{2}$

$$\frac{1}{f} = \left(\frac{6}{5} - 1\right)\left(\frac{1}{-120} - \frac{1}{-80}\right)$$

$f = +1200$ cm $= +12$ m.

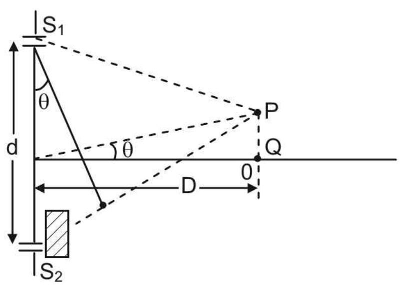
$\Rightarrow$ a converging lens.

Light incident from right side.

$$\frac{1}{f^I} = \left(\frac{3/2}{5/4} - 1\right)\left(\frac{1}{+80} - \frac{1}{+120}\right)$$

No difference.

127.



(i) $\ell_1 = (S_1 P)\,\mu_1$

$\ell_2 = (S_2 P - t)\,\mu_1 + t\mu_2 = (S_2 P)\,\mu_1 + t\,(\mu_2 - \mu_1)$

$\ell_2 - \ell_1 = \mu_1\,[S_2 P - S_1 P] + t\,(\mu_2 - \mu_1)$

$S_2P - S_1P = \Delta$

$$\tan\theta = \frac{\Delta}{d} = \frac{a}{D}$$

$$\therefore \quad \ell_2 - \ell_1 = \mu_1\left(\frac{da}{D}\right) + t(\mu_2 - \mu_1)$$

The optical path difference at P.

$$= \mu_1\left(\frac{da}{D}\right) + t\,(\mu_2 - \mu_1)$$

$\ell_1' = (S_1Q)\,\mu_1$, $\ell_2' = (S_2Q)\,\mu_1 + t\,\mu_2$

$= (S_2Q)\,\mu_1 + t\,(\mu_2 - \mu_1)$

$\ell_2' - \ell_1' = \mu_1\,(S_2Q - S_1Q) + t\,(\mu_2 - \mu_1)$

$$= \mu_1\left(\frac{-da}{D}\right) + t\,(\mu_2 - \mu_1)$$

The optical path difference at Q

$$= t\,(\mu_2 - \mu_1) - \mu_1\,\frac{da}{D}$$

(ii) Central point O.

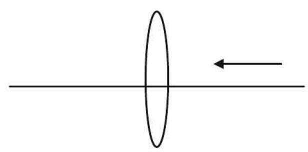
Optical path difference

$= [(S_2O - t)\mu_1 + t\,\mu_2] - (S_1O)\,\mu_1$

$= (S_2O - S_1O)\,\mu_1 + t\,(\mu_2 - \mu_1) = 0 + t(\mu_2 - \mu_1)$

Condition for dark fringe at O.

$$t\,|\mu_2 - \mu_1| = (2m - 1)\frac{\lambda}{2}, \; m = 1, 2, 3, \ldots$$



$$t = \frac{(2m-1)}{|\mu_2 - \mu_1|}\frac{\lambda}{2}$$

for bright fringe $t\,|\mu_2 - \mu_1| = m\lambda$

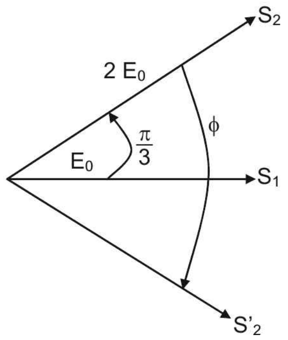
$$t = \frac{m\lambda}{|\mu_2 - \mu_1|}, m = 1,2,3\ldots$$

128. (i) $\ell_1 = (S_1O - t_1)\,1 + t_1\mu_1$

$\ell_2 = (S_2O - t_2)\,1 + t_2\mu_2$

$\ell_2 - \ell_1 = (t_1 - t_2) + t_2\mu_2 - t_1\mu_1$

$= t_2\,(\mu_2 - 1) - t_1\,(\mu_1 - 1)$



(a) **Optical path difference = $\ell_2 - \ell_1 = \ell$ say > 0**

Then $\ell_2 > \ell_1$ so that S_2 lags S_1 by $\frac{2\pi}{\lambda}(\ell) = \phi$

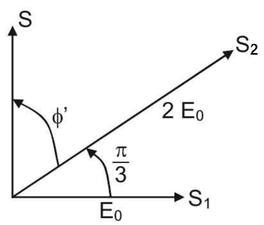
The resultant amplitude

$$= \left[E_0^2 + (2E_0)^2 + 2E_0(2E_0)\cos\left(\phi - \frac{\pi}{3}\right)\right]^{\frac{1}{2}}$$

$$= \left[E_0^2\left[5 + 4\cos\left(\phi - \frac{\pi}{3}\right)\right]\right]^{\frac{1}{2}}$$

$$I_C = I_0\left[5 + 4\cos\left(\phi - \frac{\pi}{3}\right)\right]$$

(b) **If $\ell_2 - \ell_1 < 0$. then say $\ell_1 - \ell_2 = \ell'$**



S_2 leads S_1 by $\frac{2\pi}{\lambda}\ell' = \phi'$

The resultant amplitude

$$= \left[E_0^2\left[5 + 4\cos\left(\phi' + \frac{\pi}{3}\right)\right]\right]^{\frac{1}{2}}$$

$I_c = I_0\,[5 + 4\cos(\phi' + \frac{\pi}{3})\,]$

(ii) For the central point O to be the bright fringe.

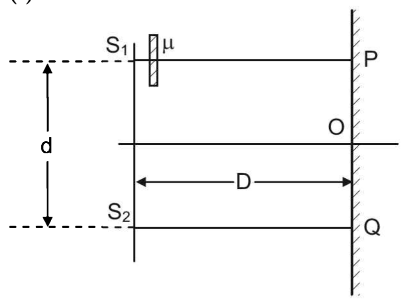
$\Rightarrow$ only possible if $\ell_2 > \ell_1$, such that $\phi - \frac{\pi}{3} = 0$ or $m\lambda$

OPD = $m\lambda$, m = 0 —(1) a $\Rightarrow \phi - \frac{\pi}{3} = 0$

$$\frac{2\pi}{\lambda}\left|t_2(\mu_2 - 1) - t_1(\mu_1 - 1)\right| = \frac{\pi}{3}$$

$$t_2(\mu_2 - 1) - t_1(\mu_1 - 1) = \frac{\lambda}{6}$$

129. (i)



$\ell_1 = (S_1P - t) + \mu t = S_1P + (\mu - 1)t$

$\ell_2 = S_2P$

$\ell_2 - \ell_1 = S_2P - S_1P - (\mu - 1)t$

$S_2P - S_1P = (D^2 + d^2)^{1/2} - D$

$= D\left[1 + \left(\frac{d}{D}\right)^2\right]^{\frac{1}{2}} - D$. Since $\frac{d}{D} << 1$;

$$S_2P - S_1P = D\left(1 + \frac{1}{2}\left(\frac{d}{D}\right)^2\right) - D$$

$$= D + \frac{d^2}{2D} - D = \frac{d^2}{2D}$$

$\therefore\ \ell_2 - \ell_1 = \frac{d^2}{2D} - (\mu - 1)t$, say positive.

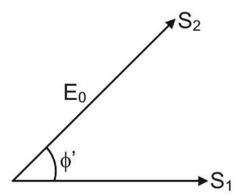
Then $\ell_2 > \ell_1$, S_2 lags S_1 by

$$\phi = \frac{2\pi}{\lambda}\left(\frac{d^2}{2D} - (\mu - 1)t\right)$$

$I_P = I_0 + I_0 + 2I_0\cos\phi$

$= 2I_0(1 + \cos\phi)$

$= 4I_0\mathrm{Cos}^2\left(\frac{\phi}{2}\right)$



Point Q

$\ell_1' = (S_1Q - t) + \mu t$; $\qquad \ell_2' = S_2Q$

$$\ell_1' - \ell_2' = (S_1Q - S_2Q) + \mu t$$

$$= \frac{d^2}{2D} + (\mu - 1)t$$

$$S_2 \text{ leads } S_1 \text{ by } \phi' = \frac{2\pi}{\lambda}\left[\frac{d^2}{2D} + (\mu - 1)t\right]$$

$$I_Q = 4I_0\cos^2\left(\frac{\phi'}{2}\right)$$

(ii) Condition for $I_P = I_Q$

$\cos\phi = \cos\phi'$

$\phi = \phi^I \to t = 0$, not acceptable.

$\phi^I = 2\,m\,\pi + \phi$

$$\frac{2\pi}{\lambda}\left[\frac{d^2}{2D} + (\mu - 1)t\right] = 2m\pi + \frac{2\pi}{\lambda}\left[\frac{d^2}{2D} - (\mu - 1)t\right]$$

$$t = \frac{m\lambda}{2(\mu - 1)}, m = 1,2,3,...$$

(iii) Intensity at O.

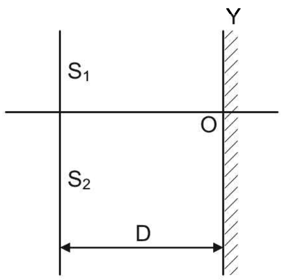
Optical path difference $= (\mu - 1)t$

Phase difference $\phi = \frac{2\pi}{\lambda}(\mu - 1)t$.

S_2 leading S_1.

$$I_C = 4\,I_0\,\cos^2\left(\frac{\pi}{\lambda}(\mu - 1)t\right)$$

130.



For λ_1 the m^{th} fringe is located at $y_m = \frac{mD\lambda_1}{d}$

For λ_2 the n^{th} fringe is located at $y_n' = \frac{nD\lambda_2}{d}$

As the fringes have to match $y_m = y_n'$

$$\lambda_1\frac{mD}{d} = \frac{nD}{d}\lambda_2$$

$$\frac{\lambda_1}{\lambda_2} = \frac{750}{900} = \frac{n}{m} = \frac{5}{6}$$

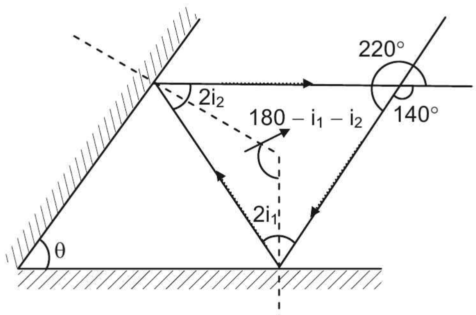
Taking n = 5 or m = 6

$$Y_6 = y_5' = \frac{5(2)(900 \times 10^{-9})}{2 \times 10^{-3}} = 4.5 \text{ mm}$$

This is the first position at which the bright fringes overlap.

131. $\frac{360°}{37°} = 9.7, ..,$ hence 9

132.



$2i_1 + 2i_2 = 140°$

$\Rightarrow\ i_1 + i_2 = 70°$

$\theta = i_1 + i_2 = 70°$ (cyclic quadrilateral)

$\therefore$ No. of images $= \frac{360}{70°} = 5 \ldots = 5$

133. concave mirror, with source at F

134. $\frac{dv}{du} = -\frac{v^2}{u^2}$

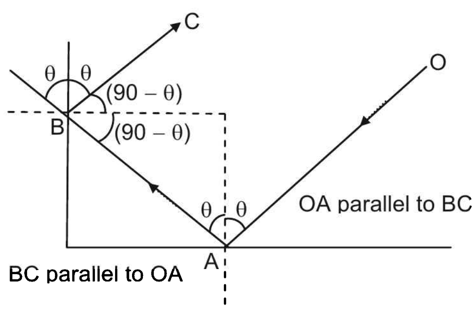
slope $= \infty$ at $v = \infty$, or $u = 0$, $|u| = f$ at

$v = \infty, |v| = f$ at $u = \infty$

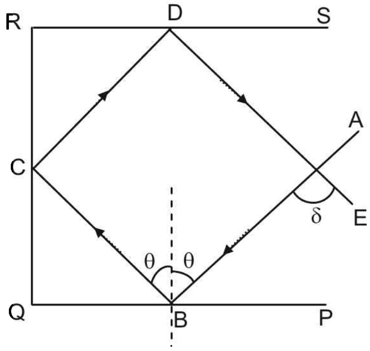
slope = 1 at $v = u = 2f$

135. Knowledge based

136.



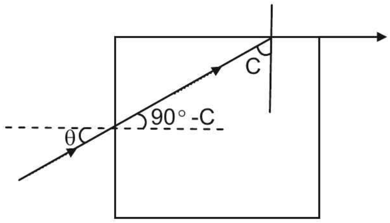
137.



DE parallel to BC (see previous solutions)

$\therefore \quad \delta = 2\theta$, where θ is angle of incidence.

138.



$$n = \frac{\sin\theta}{\sin 90 - C} = \frac{\sin\theta}{\cos C};\ \cos C = \frac{\sin\theta}{n}$$

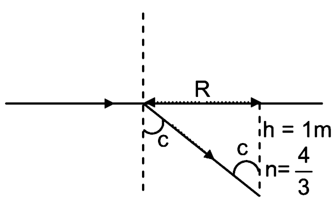
$$n = \frac{1}{\sin C} \Rightarrow \sin C = \frac{1}{n}$$

$$\therefore \quad 1 = \cos^2 C + \sin^2 C = \frac{\sin^2\theta}{n^2} + \frac{1}{n^2}$$

$$1 = \frac{1}{n^2}(1 + \sin^2\theta)$$

$$\Rightarrow \quad n = \sqrt{1 + \sin^2\theta}$$

139.



$$\sin C = \frac{1}{n} = \frac{3}{4}$$

$$\tan C = \frac{3}{\sqrt{7}} = \frac{R}{h} = \frac{R}{1} \Rightarrow R = \frac{3}{\sqrt{7}}\ \text{m}$$

140. $$2\cos\frac{A}{2} = \frac{\sin\frac{(A+D)}{2}}{\sin\frac{A}{2}} \Rightarrow \sin A = \sin\frac{A+D}{2}$$

$$\Rightarrow \quad A = \frac{A+D}{2} \text{ OR } A = 180 - \frac{A+D}{2}$$

$$\Rightarrow \quad D = A \text{ or } D = 360 - 3A = 3(120 - A)$$

[Note that A should be < 120°. Since

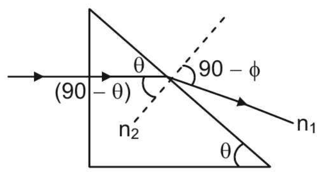
$$n = 2\cos\frac{A}{2} > 1 \Rightarrow \cos\frac{A}{2} > \frac{1}{2} \Rightarrow \frac{A}{2} < 60°$$

$$\Rightarrow \quad A < 120°]$$

141. $$i_1 + i_2 = 2i = A + D \Rightarrow i = \frac{A+D}{2} = 180 - A$$

$A < 120° \Rightarrow i > 60°$.

142.



$$n_2 \sin(90 - \theta) = n_1 \sin(90 - \phi)$$

$$\Rightarrow \quad n_2 \cos\theta = n_1 \cos\phi$$

$$\Rightarrow \quad \cos\theta = \frac{n_1}{n_2}\cos\phi$$

Both in water and air incident angle $(90 - \theta)$ is same

In water: $\cos\theta = \dfrac{n_1}{n_2}\cos\phi$

$$= \frac{n_1}{n_2}\cos\frac{\theta}{2}\ \left[\because \phi = \frac{\theta}{2} \text{ given date}\right]$$

In air: $\cos\theta = \dfrac{1}{n_2}\cos 0 = \dfrac{1}{n_2} \Rightarrow$

$$\therefore \quad \frac{n_1}{n_2}\cos\frac{\theta}{2} = \frac{1}{n_2} \Rightarrow \cos\frac{\theta}{2} = \frac{1}{n_1}$$

$$\Rightarrow \quad \cos\frac{\theta}{2} = \frac{3}{4} \Rightarrow \theta = 2\cos^{-1}\frac{3}{4}$$

143. For thin prism,

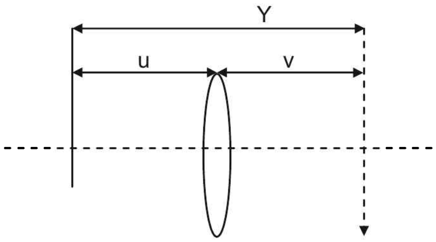
$$n = \frac{\frac{A+\delta}{2}}{\frac{A}{2}} \Rightarrow \delta = A(n-1) \text{ (in radian)}$$

converting both sides to degrees $\delta° = \theta\,(n - 1)$

144. $\delta_V - \delta_R = (\mu_V - \mu_R)\,A$

$\Rightarrow \quad \omega = (1.59 - 1.55)\,8 = 0.32°$

145. $P = \dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u}$



For the given condition

$$\frac{v}{|u|} = |m| = x \Rightarrow v = |u|x$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow v + |u| = y \Rightarrow |u|x + |u|$$

$= y \Rightarrow \quad |u|\,(x + 1) = y$

$$|u| = \frac{y}{(x+1)}; v = |u|x = \frac{yx}{(x+1)}$$

$$\frac{1}{v} - \frac{1}{-|u|} = \frac{x+1}{xy} + \frac{x+1}{y} = \frac{x+1}{y}\left(\frac{1}{x} + 1\right)$$

$$= \frac{x+1}{y} \cdot \frac{x+1}{x}$$

$$\frac{1}{f} = \frac{(x+1)^2}{xy} \text{ all in cm}$$

$$\therefore \quad P = \frac{1}{f'(\text{metre})} = \frac{1}{f/100} = \frac{100(x+1)^2}{xy}$$

146. When immersed in water, a converging lens becomes less converging (focal length increases $\Rightarrow |v|$ becomes lesser $\Rightarrow$ image is closer to lens).

A diverging lens becomes less diverging (focal length becomes more negative $\Rightarrow$ v becomes more negative $\Rightarrow$ image moves away)

For convex lens forming virtual image:

$\dfrac{1}{v} - \dfrac{1}{-|u|} = \dfrac{1}{f}$ (f always +ve; $|u|$ constant in air and water)

$\dfrac{1}{v} = \dfrac{1}{f} - \dfrac{1}{|u|} \Rightarrow$ L. H. S –ve, if $|u| < f$ (virtual image)

In water: $f\uparrow; \dfrac{1}{f}\downarrow; \left|\dfrac{1}{f} - \dfrac{1}{|u|}\right|\uparrow; \left|\dfrac{1}{v}\right|\uparrow; v\downarrow$

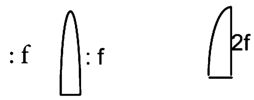
For concave lens; always virtual image, f: –ve;

$$\frac{1}{v} = -\frac{1}{|f|} - \frac{1}{|u|}$$

$$= -\left|\frac{1}{|f|} + \frac{1}{|u|}\right|;\ |f|\uparrow;\ \left|\frac{1}{|f|} + \frac{1}{|u|}\right|\downarrow;\ \left|\frac{1}{v}\right|\downarrow; v\uparrow$$

147. So that its n is less than that of water

148.



(similar case for concave lens also)

149. frequency: f, wavelength: λ, time period: $\dfrac{1}{f}$ wave number: $\dfrac{2\pi}{\lambda}$, velocity $v = f\lambda$.

f remains same in both media

150. $I = I_0 \cos^2 \dfrac{\phi}{2}$, ϕ: phase difference $= \dfrac{\delta}{\lambda}.2\pi$

$$\Rightarrow \quad \frac{\phi}{2} = \frac{\delta\pi}{\lambda}$$

$$\text{Required Ratio} = \frac{\cos^2 3.2\pi}{\cos^2 4.8\pi}$$

$$= \frac{\cos^2(4\pi - 0.8\pi)}{\cos^2(4\pi + 0.8\pi)} = 1$$

151. $\beta = \dfrac{\lambda D}{d}$

If n be number of fringes over a length l. Then $n = \dfrac{\ell}{\beta}$

$$= \frac{\ell}{\cancel{\frac{\lambda D}{d}}} \Rightarrow n\lambda = \frac{\ell d}{D} = \text{constant}$$

$\Rightarrow \quad n\lambda = (n - 4)\,(\lambda + 100)$ (λ in nm)

$\qquad = n\lambda - 4\lambda - 400 + 100\,n$

$\Rightarrow \quad \lambda = 25\,n - 100$

since $400 < \lambda < 600$

(visible spectrum 400 – 700 nm; $\lambda + 100) < 700$)

$25n - 100 > 400 \Rightarrow n \geq 20$;

$25n - 100 < 600 \Rightarrow n < 28$

$\Rightarrow \quad n - 4 > 16\ \&\ n - 4 < 24$

$\therefore \quad 20 < n < 28 \Rightarrow 16 < (n - 4) < 24$

152. Central white fringe, all others coloured no perfectly dark.

153. $\therefore$ $t(\mu - 1) = 10\,\lambda_1$
$\Rightarrow$ $t\,(1.6 - 1) = 10 \times 450$ nm
$\Rightarrow$ $t = 7.5$ μm

154. If ν in the frequency, in 1 second ν cycles are seen
$\therefore$ cycles counted in t second $= t\nu = \frac{t}{T}\left[\because \frac{1}{T} = \nu\right]$

155. Red wavelength range = 6200Å – 7800 Å
All others will be filtered by the red glass.

156. $m = \frac{f_o}{f_e} \Rightarrow f_e = \frac{f_o}{m} = \frac{120 \text{ cm}}{40} = 3$ cm

157. Wavefronts are spherical which are at the same distance from the source.

158. By definition of coherent source.

159. When coherent waves interfere, amplitude vectors add up. Amplitude $\propto \sqrt{I}$.
$\therefore$ Resultant Amplitude $= \sqrt{9I} + \sqrt{16I} = 7\sqrt{I}$; Intensity $= (7)^2\, I = 49\, I$

160. $y = \left[n - \frac{1}{2}\right]\frac{D\lambda}{d} = \frac{31}{2}\frac{D\lambda}{d}$
$\Rightarrow n = 16$

161. $\beta = \frac{\lambda D}{d}$, β doubles for both dark and bright fringes

162. Only at the central maximum will all colours meet with zero phase difference

163. $c = \nu\lambda$

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{3 \times 10^{-7}} = 10^{15} \text{ Hz}$$

164. frequency

165. time $= \frac{4 \times 10^{-3}}{3 \times 10^8} 1.5 = 2 \times 10^{-11}$s

166. Wave length

167. When light moves from a medium of higher velocity to a medium of lower velocity reflection at interface causes a phase reversal.

168. $\mu = A + \frac{B}{\lambda^2}$

169. Magnifying power $M = f_o/f_e$

170. Infinity.

171. Dispersion is zero only because deviation is zero for every colour. (emergent ray is parallel to incident ray for every colour)

172. Not always, if the lens is concave. A concave lens will give virtual image for a virtual object. If object distance is more than focal distance.

$$[\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{-|f|} + \frac{1}{|u|}$$

$\Rightarrow$ if $|u| > |f|$, v is negative ($\Rightarrow$ virtual image)

173. Yes. Derivation of mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ is done using i = r and definition of f is by using $u = \infty$ in the above formula

174. $\delta = i + e - A$

175. $\mu_v > \mu_r \Rightarrow$ violet undergoes TIR, while red light emerges through non-parallel side. The two non-parallel sides act like a glass prism of prism angle 90°.

176. Self explanatory, Sun at dawn or setting Sun is seen even when it is below horizon, due to presence of atmosphere.

177. $\beta = \frac{\lambda D}{d} \Rightarrow \frac{d}{\lambda} = \frac{D}{\beta}$

Large d makes β too small to be observed

178. Path difference $\delta = d \sin\theta$

order of fringe, $n = \frac{\delta}{\lambda} = \frac{d}{\lambda}\sin\theta$.
$\Rightarrow$ $n < 1$ if $d < \lambda$
$\Rightarrow$ no fringe pattern.

179. If $t_1(\mu_1 - 1) = t_2(\mu_2 - 1)$, there will be no change

180. $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$; f is positive

Statement II is right. For real image, v is negative L. H. S of above equation is negative]

$\Rightarrow$ $\therefore$ $\frac{1}{f} - \frac{1}{u} < 0$

Possible is u is positive, ie., virtual object and also $|u| < |f|$

181. When filled with water, magnitude of f will decrease. If I was real, I' will also be real and of more size.

If v and v' are initial and final values of image distances then $\left|\frac{v}{u}\right| = \left|\frac{v'}{u}\right|$ (given data) $\therefore$ $|v| = |v'|$ but f reduces to F. Hence v is virtual image and v' is real image

$\therefore$ I is virtual. $\therefore$ erect.

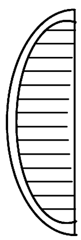
182. size of I' is same as I $\Rightarrow$ clearly magnification changes sign from + to – $\Rightarrow$ I' is real, $\therefore$ inverted.

183. Focal length of the concave mirror

$|f| = \frac{|R|}{2}$. The water inside the mirror forms a plano – convex lens of focal length f' given by:

$$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{\infty}\right)$$

$$= \left(\frac{4}{3} - 1\right)\frac{1}{R} = \frac{1}{3R} = \frac{1}{3(2|f|)} = \frac{1}{6|f|}$$



where, f is the focal length of the mirror. Focal length of the combination is $\left[\frac{1}{F} = \frac{1}{f} - \frac{2}{f(\text{lens})}\right]$

$$\frac{1}{F} = \frac{1}{f} + \frac{2}{6f} \Rightarrow F = \frac{3}{4}f \text{ (Both F \& f are –ve)}$$

$\Rightarrow$ In both cases we have $|v| = |mu|$

First case, virtual image, v +ve; u, f are –ve; take m, modulus value) + $\frac{1}{m|u|} - \frac{1}{|u|} = -\frac{1}{|f|}$

$$\Rightarrow \frac{1}{|u|}(x - 1) = -\frac{1}{|f|} \quad (x = \frac{1}{m}) \text{ — (1)}$$

second case, real image, u, v, F are –ve

$$\frac{1}{m|u|} + \frac{1}{|u|} = \frac{4}{3|f|} \Rightarrow \frac{1}{|u|}(x + 1) = \frac{4}{3|f|} \text{ — (2)}$$

Divide 1 by 2 $\Rightarrow \frac{x - 1}{x + 1} = -\frac{3}{4}$

$\Rightarrow$ $4x - 4 = -3x - 3$

$x = \frac{1}{7} \Rightarrow m = 7$

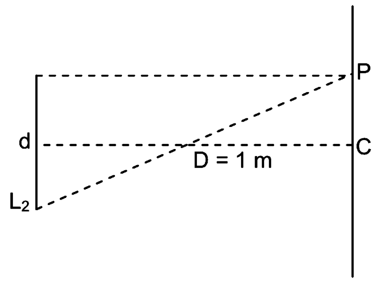
184. With opaque film in front of a slit, intensity (everywhere) is I_0 $\Rightarrow$ without film, maximum intensity (at centre) = $4I_0$.

$$I = 4I_0 \cos^2\frac{\theta}{2} = I_0 \text{ (given)}$$

$$\cos^2\frac{\theta}{2} = \frac{1}{4}$$

$$\Rightarrow \cos\frac{\theta}{2} = \frac{1}{2} \Rightarrow \theta = 120°$$

185.



Path difference at P

$$\delta = L_2P - L_1P = \sqrt{D^2 + d^2} - D$$

$$\simeq [(1 + .25 \times 10^{-6})^{1/2} - 1]$$

$$\simeq 1 + \frac{1}{2}(0.25) \times 10^{-6} - 1$$

$$= 0.125 \times 10^{-6}$$

From previous solution phase difference θ

$= 120°$ corresponds to $\lambda/3$

$$\Rightarrow \delta = \frac{\lambda}{3}$$

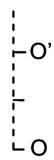
$$\Rightarrow 0.125 \times 10^{-6} = \frac{\lambda}{3}$$

$$\Rightarrow \lambda = 0.375 \times 10^{-6}$$

$= 375$ nm (UV radiation)

186. $\delta = t(\mu - 1) = t(1.5 - 1) = t(1.5 - 1) = 0.5\,t$

Let the central bright fringe shift (towards slit 1 side) from O to O'



Clearly OP = PO' (for minimum thickness of film)

$\Rightarrow$ $OO' = 0.5\text{ mm} = \delta\dfrac{D}{d}$

$\Rightarrow$ $0.5 \times 10^{-3} = 0.5\text{ t}.\dfrac{1}{0.5\times10^{-3}}\,1$

$\Rightarrow$ $t = 0.5 \times 10^{-6}\text{ m} = 0.5\ \mu\text{m}$

187. Deviation from first refraction = $\theta - \phi$, from internal reflection $\pi - 2\phi$. From second refraction $\theta - \phi$, all Total deviation $D = 2\theta - 4\phi + \pi$

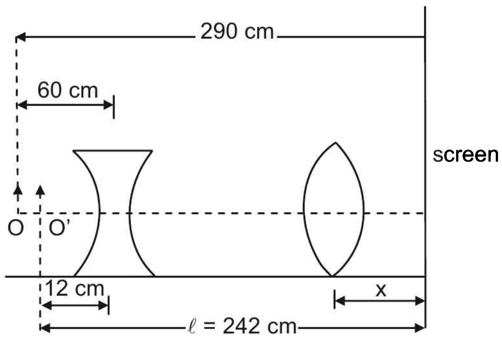
188. $\dfrac{dD}{d\theta} = 2 - 4\dfrac{d\phi}{d\theta} = 0$

$\dfrac{d\phi}{d\theta} = \dfrac{1}{2}$

189. $\sin\theta = \mu\sin\phi \Rightarrow \cos\theta = \mu(\cos\phi)\,\dfrac{d\phi}{d\theta} = \dfrac{\mu\cos\phi}{2}$

$\cos\phi = \dfrac{2\cos\theta}{\mu}$

190.



For the first image by concave lens

$\dfrac{1}{v} - \dfrac{1}{-60} = \dfrac{1}{-15} \Rightarrow v = -12\text{ cm}$

Now use formula : $|u| = \dfrac{\ell}{2}\left[1 \pm \sqrt{1 - \dfrac{4\times f}{\ell}}\right]$

$\ell = (290 - 60 + 12) = 242\text{ cm}$

$|v| = \dfrac{242}{2}\left[1 \pm \sqrt{1 - \dfrac{4\times 20}{242}}\right]$

$= \dfrac{242}{2}\left[1 \pm \sqrt{\dfrac{81}{121}} = 121\left[1 \pm \dfrac{9}{11}\right]\right]$

$|v| = 220$ cm or 22 cm. These are the two positions of x when image is formed

Maximum magnification when v = 220, u = 22

$\Rightarrow$ $m_2 = -10$

For the concave lens

$\Rightarrow$ $m_1 = \dfrac{-12}{-60} = \dfrac{1}{5}$

$\therefore$ $m_1 \times m_2 = -10 \times \dfrac{1}{5} = -2$

191. $P = \dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

n varies with wavelength.

$\therefore$ f varies with n

$\therefore$ v varies for a given u

$\therefore$ $m = \dfrac{v}{u}$ varies

192. Knowledge based

193. $n_1 \sin i = n_2 \sin r$

Clearly, if sin i = 0

$\Rightarrow$ sin r = 0 (passes through origin)

$n_{21} = \dfrac{n_2}{n_1} = \dfrac{\sin i}{\sin r} = \dfrac{1}{\sin r/\sin i}$

$= \dfrac{1}{\text{slope}} = \dfrac{1}{\tan\theta} < \dfrac{1}{\tan 53^\circ}$

$\Rightarrow$ $n_{21} < \dfrac{1}{4/3} \Rightarrow n_{21} < 3/4$

$\Rightarrow$ $\dfrac{n_2}{n_1} < \dfrac{3}{4}$

$\Rightarrow$ 1: denser, 2: rarer.

$n_2 < \dfrac{3}{4}n_1$

But, any medium, $n \geq 1$

$\therefore$ $1 \leq n_2 \leq \dfrac{3}{4}n_1$

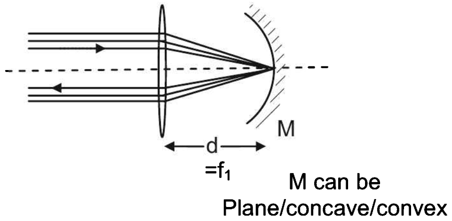
$\Rightarrow$ $n_1 \geq \dfrac{4}{3}$

$\Rightarrow$ $\dfrac{1}{n_1} \leq \dfrac{3}{4}$

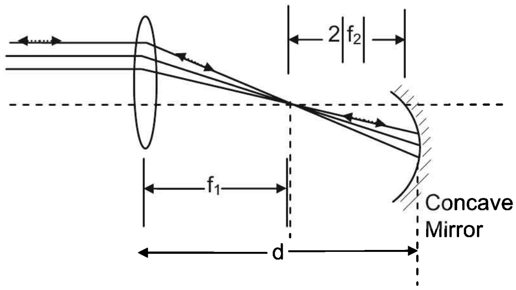
$\Rightarrow$ $\sin C \leq \dfrac{3}{4}$

194. Posibilities:

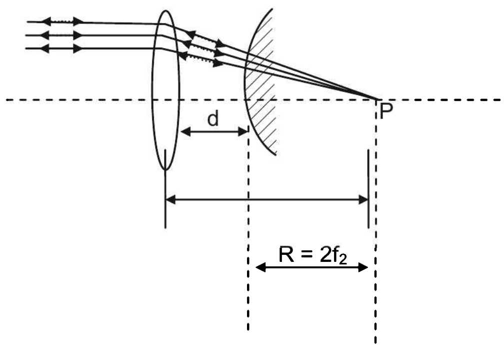
(i)



(ii)



(iii)



195. longitudinal magnification,

$$m' = \frac{dv}{du} = \frac{-v^2}{u^2} = -m^2$$

$\therefore \quad m' = -4$ or $-\frac{1}{4}$ (longitudinally inverted).

Also $m = -2$ ($\because$ inverted real image) or $-\frac{1}{2}$

$\Rightarrow$ using $|m| = \dfrac{1}{\dfrac{|u|}{|f|} - 1}, |u| = \dfrac{3}{2}f$ or $3f$

$\Rightarrow$ $30 \text{ cm} = \frac{3}{2}f$ or $3f \Rightarrow f = 20$ cm or 10 cm

$|v| = |m|\,|u| = 60$ cm or 15 cm

Distance between object and image is 30 cm or 15 cm

196. Condition: ω should be same, constant phase difference can exist. Amplitude can be different.

197. Knowledge based

198. $I_1 = k\langle E_1\rangle^2 = \dfrac{kA^2}{2}$; similarly $I_2 = \dfrac{kA^2}{2} = I_1$

where k is a factor relating the intensity and the square of the electric field

Now if the two waves interfere,

$$I = \left\langle k\left(E_1 + E_2\right)^2\right\rangle = I_1 + I_2 + \langle 2kE_1E_2\rangle$$

$$= 2I_1 + \langle 2kE_1E_2\rangle$$

where

$2kE_1E_2 = 2kA^2\cos(\omega_1 t + \phi_1)\cos(\omega_2 t + \phi_2)$

$= kA^2[\cos\{(\omega_1 + \omega_2)t + (\phi_1 + \phi_2)\} + \cos\{(\omega_1 - \omega_2)$
$t + (\phi_1 - \phi_2\}]$

when $\omega_1 \neq \omega_2$, the long time average (average over many cycles) is zero

$\therefore \quad I = 2I_1$

$\therefore \quad$ (a) $\rightarrow$ q; (c) $\rightarrow$ q

when $\omega_1 = \omega_2$, the long time average of the first term is zero but the average of the second term is k $A^2\cos(\phi_1 - \phi_2) = 2I_1\cos(\phi_1 - \phi_2)$

$\therefore \quad$ (b) $\rightarrow$ s; (d) $\rightarrow$ s

when $\omega_1 = \omega_2$ and $\phi_1 = \phi_2$, $I = 2I_1 + 2I_1\cos(0)$

$= 4I_1 = I_{max}$

$\therefore$ (d) $\rightarrow$ p, r

199. Mirror eqn: $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow \dfrac{u}{v} + \dfrac{u}{u} = \dfrac{u}{f}$

$\Rightarrow \quad \dfrac{u}{v} = \dfrac{u - f}{f} \Rightarrow \dfrac{v}{u} = \dfrac{f}{u - f}$

$\Rightarrow \quad -m = \dfrac{f}{u - f}$

$\Rightarrow \quad m = \dfrac{f}{f - u} = \dfrac{1}{1 - \dfrac{u}{f}}$

$u = +|u|$ ($\because$ virtual object)

$\because$ For concave mirror $\Rightarrow f = -|f| \Rightarrow m = \dfrac{1}{1 + \dfrac{|u|}{|f|}}$

$\Rightarrow$ $0 < m < 1$

m is positive means v and u have opp signs. Since object is virtual, image is real, $\therefore$ both (a) and (b) ; p, s.

For convex mirror $f = +|f|$ $m = \dfrac{1}{1 - \dfrac{|u|}{|f|}}$

If $\dfrac{|u|}{|f|} < 1$, m is positive and > 1

$\therefore$ (c): p, r

If $\dfrac{|u|}{|f|} < 1$, m is negative and can be diminished or enlarged (example: $\dfrac{|u|}{|f|} = 1.5$, $\Rightarrow m = -2$; $\dfrac{|u|}{|f|} = 3, m = -0.5$)

(d) q, r, s

200. Waves on right side means lens; left means mirror

(a) converging $\Rightarrow$ convex lens

(b) diverging $\Rightarrow$ concave lens

(c) convex lens with object at F.

(d) diverging $\Rightarrow$ virtual image; can be plane mirror (or) convex mirror (or) concave mirror with object within f.

(a) t (b) s, t, (c) t (d) p, q, r

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CHAPTER

2

MODERN PHYSICS

CHAPTER OUTLINE

Preview

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Electromagnetic Waves

Photons

- Concept Strands (1-3)

Electromagnetic Spectrum

The Photoelectric Effect

- Concept Strands (4-5)

Matter Waves

- Concept Strand (6)

Bohr Theory

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CONCEPT CONNECTORS

- 25 Connectors

TOPIC GRIP

- Subjective Questions (10)
- Straight Objective Type Questions (5)
- Assertion–Reason Type Questions (5)
- Linked Comprehension Type Questions (6)
- Multiple Correct Objective Type Questions (3)
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IIT ASSIGNMENT EXERCISE

- Straight Objective Type Questions (80)
- Assertion–Reason Type Questions (3)
- Linked Comprehension Type Questions (3)
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ADDITIONAL PRACTICE EXERCISE

- Subjective Questions (10)
- Straight Objective Type Questions (40)
- Assertion–Reason Type Questions (10)
- Linked Comprehension Type Questions (9)
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ELECTROMAGNETIC WAVES

Electromagnetic waves are characterized by mutually perpendicular oscillatory magnetic and electric fields. Gamma rays, X-rays, visible light and radio waves are examples of electromagnetic waves. The two fields are mutually sustaining and supporting because the oscillating magnetic field induces the perpendicular electric field and the oscillatory electric field induces back the oscillating magnetic field. They have the same frequency and phase. An electromagnetic wave propagating in the x-direction is represented by the associated electric and magnetic fields, respectively, by the equations

$$\bar{E} = \bar{E}_0 \sin(kx - \omega t)$$

$$\bar{B} = \bar{B}_0 \sin(kx - \omega t)$$

where $k = \frac{2\pi}{\lambda}$ is the wave vector and $\omega = 2\pi\nu$ is the angular frequency. In vacuum, all electromagnetic waves travel with the speed of light c, where

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

in a direction defined by the vector product $\bar{E} \times \bar{B}$. Their amplitude ratio is

$$\frac{E_0}{B_0} = c$$

Electromagnetic waves carry energy. The energy density (energy contained in unit volume) of electromagnetic waves in empty space is the sum of the energy densities in the associated electric and magnetic fields.

$$u = u_E + u_M = \frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\frac{B_{rms}^2}{\mu_0} = \varepsilon_0 E_{rms}^2 \left(\because \frac{B_{rms}}{E_{rms}} = \frac{1}{c} \right)$$

where the rms value is defined in a period of the wave as

$$E_{rms} = \left[\frac{1}{T}\int_0^T E_0^2 \sin^2(kx - \omega t)dt \right]^{\frac{1}{2}} = \frac{E_0}{\sqrt{2}}$$

The energy of electromagnetic wave is carried equally by its electric field and magnetic field.

PHOTONS

The energy density of an electromagnetic wave is

$$u = \varepsilon_0 E_{rms}^2 = \frac{\varepsilon_0}{2} E_0^2$$

As u is proportional to the square of the amplitude of the electric field vector, we find that the more energetic a wave, the larger is its amplitude. Fig. 2.1 shows electromagnetic waves of a given wavelength λ, of three different energies (amplitudes).

Modern physics tells us that the energy of an electromagnetic wave is an integral multiple of a quantum of energy known as the ***photon***, originally proposed by Max Planck in 1900.

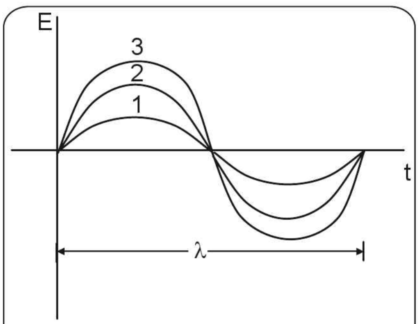


Fig. 2.1

Photon energy is given by

$$E = h\nu = \frac{hc}{\lambda}$$

where h is Planck's constant whose value is 6.63×10^{-34} J s (4.14×10^{-15} eV s). Thus the three waves in Fig. 2.1 have energies

$$E_1 = n_1 h\nu;\ E_2 = n_2 h\nu;\ E_3 = n_3 h\nu$$

where n_1, n_2, n_3 are integers and hν is the energy quantum of the photon corresponding to this wave of frequency ν (wavelength λ). n_1, n_2 and n_3 are the numbers of photons in the three waves.

The following properties of photons are to be noted:

(i) Photon is a discrete quantity of electromagnetic energy and it has a particle nature. Total energy of an electromagnetic wave is the sum of energies of all photons in it.

(ii) Different photons (like γ-ray, X-ray, UV ray etc.) have different wavelengths (or different frequencies) and hence photons of different frequencies have different energies.

(iii) As per Einstein's theory of relativity, when a mass m gets converted completely into energy, the equivalent energy E is given by E = mc^2. Equating this to the photon energy, we get

$$E = mc^2 = h\nu = \frac{hc}{\lambda}$$

$$\Rightarrow \quad mc = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} = p$$

is the *linear momentum* of a photon. Direction of the momentum is the direction in which the electromagnetic wave is moving.

(iv) $m = \frac{h\nu}{c^2} = \frac{h}{\lambda c}$

is called *the dynamic mass* of a photon. The rest mass of a photon is zero.

(v) When a photon is absorbed by a surface, the impulse it exerts on that surface is equal to the change in momentum;

$$\left|\Delta\bar{p}\right| = \frac{h\nu}{c} = \frac{h}{\lambda} = \left|\bar{j}\right|$$

Thus photon incident on a surface can exert pressure on the surface.

(vi) When a photon is reflected normally by a surface, the impulse it exerts on that surface is equal to

$$\left|\bar{j}\right| = \left|\Delta\bar{p}\right| = \frac{2h\nu}{c} = \frac{2h}{\lambda}$$

(vii) In vacuum, all photons travel with the same speed, equal to the speed of light (visible photons) $c \approx 3.0 \times 10^8$ m s^{-1}.

(viii) In other media, different photons travel with different speeds. For example, inside glass, the speed of violet photon is less than the speed of red photon.

(ix) The speed of all photons in any media (other than vacuum) is always less than c.

(x) Joule (J) is too large a unit for measuring the energy of a photon. The more appropriate unit is the electron volt (eV). Hence the energy of a photon E = $\frac{h\nu}{e}$ eV.

$$E = \frac{h\nu}{e} = \frac{hc}{e\lambda}eV = \frac{1240}{\lambda(nm)}eV$$

(xi) The power P (in watt) of a source of electromagnetic radiation is given by

$$P = nh\nu = \frac{nhc}{\lambda}$$

where, n = number of photons emitted per second by the source.

(xii) The intensity I (in W m^{-2}) of an electromagnetic radiation is given by

$$I = Nh\nu = \frac{Nhc}{\lambda}$$

where N = number of photons incident per unit area per unit time (m^{-2} s^{-1}).

(xiii) The intensity I (W m^{-2}) of an electromagnetic radiation in vacuum is given by I = Energy density × speed

$$\Rightarrow I = \frac{1}{2}\varepsilon_0 E_0^{\,2} c = \frac{1}{2}\frac{B_0^{\,2}}{\mu_0}c = \frac{E_0 B_0}{2\mu_0} \text{ where,}$$

E_0 = amplitude of electric field $\left(V m^{-1}\right)$,

B_0 = amplitude of magnetic field (T),

c = speed of light in vacuum (m s^{-1}) and

$$\frac{E_0}{B_0} = \frac{E_{rms}}{B_{rms}} = \frac{E}{B} = c$$

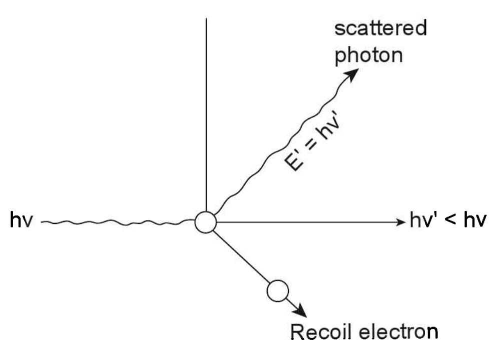
The intensity of an electromagnetic wave is represented by Poynting vector $\left(\bar{S}\right)$, which is represented as $\bar{S} = \frac{1}{2\mu_0}\bar{E}_0 \times \bar{B}_0$

Since energy is carried by a wave in the direction of its propagation, $\bar{E}_0 \times \bar{B}_0 \left(\text{or } \bar{E} \times \bar{B}\right)$ represents the direction of propagation of an electromagnetic wave i.e., $\bar{E} \times \bar{B}$ represents the direction of $\bar{c}$).

(xiv) Photons are electrically neutral. Hence they are not deflected by electric fields or by magnetic fields.

(xv) A photon may collide with a material particle (like an electron) and in such collisions the total linear momentum and total energy remain conserved. The photon may get absorbed and/or a new photon or particle may be emitted in such a process.

(xvi) During collision of a photon with material particle, unlike the partial transfer of energy in classical theory, the photon gives either its entire energy or none of its energy to the particle.
(Note: There are exceptions.)



An example is Compton.

CONCEPT STRANDS

Concept Strand 1

A sodium lamp of power 16 W emits light of wavelength 5893 Å,

(i) What is the energy of a photon?
(ii) What will be the number of photons striking a screen of area 1 m^2 normally per second, when the screen is at a distance of 2 m from the source?

Solution

(i) Photon energy $E_{(eV)} = h\nu$

$$= \frac{hc}{\lambda}\text{joule} = \frac{hc}{\lambda e}\text{eV} = \frac{6.6\times10^{-34}\times3\times10^{8}}{5.893\times10^{-7}\times1.6\times10^{-19}}\text{eV}$$

= 2.1 eV [Note: use hc = 1242eV–nm fore casy soution meV unit.]

(ii) Number of photons/m^2 s at 2 m

$$= \frac{P}{eE_{(eV)}4\pi d^2} = \frac{16}{1.6\times10^{-19}\times2.1\times4\pi\times2^2}$$

$= 9.47 \times 10^{17} \approx 101^8$

Concept Strand 2

Sun's radiating power is 3.9×10^{26} W. The radiation is emitted isotropically. Therefore, at the surface of the Earth, which is at a distance of 15×10^{10} m from Sun, what

(i) is the energy incident normally per unit area per unit time?
(ii) are the rms values of the electric and magnetic fields?

Solution

(i) $$I = \frac{P}{4\pi d^2} = \frac{3.9\times10^{26}}{4\pi\left(15\times10^{10}\right)^2} = 1380\,\text{W/m}^2$$

(ii) The intensity of sunlight I = 1380 W/m^2

$$= \frac{1}{2}\varepsilon_0 cE_0^{\,2} = \varepsilon_0 cE_{rms}^2 \left(\because E_{rms} = \frac{E_0}{\sqrt{2}}\right)$$

Half of the intensity is due to the energy of the electric field and the other half due to the energy of the magnetic field.

$$\therefore \quad E_{rms} = \sqrt{\frac{I}{\varepsilon_0 c}}$$

$$\therefore \quad E_{rms} = \sqrt{\frac{1380}{8.85\times10^{-12}\times3\times10^{8}}}\,\frac{V}{m} = 7.2\times10^2\ \text{V/m}$$

$$\therefore \quad B_{rms} = \frac{E_{rms}}{c} = \frac{7.2\times10^{2}}{3\times10^{8}} = 2.4\times10^{-6}\,\text{T}$$

Note the relative magnitudes of E and B.

Concept Strand 3

Intensity of a parallel monochromatic beam of light of wavelength 500 nm is 50 W m^{-2}.

(i) Determine the energy and momentum of a photon.
(ii) Determine number of photons crossing 1 mm^2 area held perpendicular to the beam in 1 second.

Solution

(i) $$E = \frac{hc}{\lambda} = \frac{1240\,\text{eV}}{\lambda(\text{nm})} = 2.48\ \text{eV};$$

$$p = \frac{E}{c} = 0.83\times10^{-8}\,\text{eV s m}^{-1}$$

(ii) No. of photons/mm^2/s

$$= \frac{\text{Power}(W)\times\text{Area}(m^2)}{\text{Energy}(\text{joule})} = \frac{50\times\left(10^{-3}\right)^2}{2.48\times1.6\times10^{-19}}$$

$= 1.3 \times 10^{14}\ s^{-1}$

ELECTROMAGNETIC SPECTRUM

Classification of different electromagnetic waves in the order of their frequencies (or wavelength in vacuum) is called electromagnetic spectrum. It is given in the table. The ranges given are only approximate. There is no abrupt demarcation from one species to the neighbouring.

Name of Electro-magnetic wave	Range of Frequency (Hz)	Range of Wavelength in vacuum (m)
Radio waves	$3 \times 10^4 - 3 \times 10^9$	$10^4 - 10^{-1}$
Microwaves	$3 \times 10^9 - 3 \times 10^{11}$	$10^{-1} - 10^{-3}$
Infrared waves	$3 \times 10^{11} - 4 \times 10^{14}$	$10^{-3} - 8 \times 10^{-7}$
Visible light	$4 \times 10^{14} - 8 \times 10^{14}$	$8 \times 10^{-7} - 4 \times 10^{-7}$
Ultraviolet rays	$8 \times 10^{14} - 3 \times 10^{16}$	$4 \times 10^{-7} - 10^{-8}$
X-rays	$3 \times 10^{16} - 3 \times 10^{19}$	$10^{-8} - 10^{-11}$
Gamma rays	$3 \times 10^{19} - 3 \times 10^{21}$	$10^{-11} - 10^{-13}$

From radio waves to gamma rays frequency increases whereas wavelength decreases.

Radio waves

Radio waves were discovered by Marconi.

Radio waves are generated by electronic circuits. They are used in radio communication, radar and TV transmission. Radio waves can be further classified as:

(i) Low frequency (LF) 3×10^4Hz – 3×10^5Hz
(ii) High frequency (HF) 3×10^5Hz – 3×10^6Hz
(iii) Medium frequency (MF) 3×10^6Hz – 3×10^7Hz
(iv) Very high frequency (VHF) 3×10^7Hz – 3×10^8Hz
(v) Ultra high frequency (UHF) 3×10^8Hz – 3×10^9Hz

Microwaves

Microwaves were discovered by Hertz. They are generated by devices like klystrons, magnetrons and gunn diodes.

They find applications in satellite communication, microwave ovens and radar systems. They can also be used to study atomic and molecular properties.

Infrared radiation

Infrared radiations were discovered by Herschel in 1800. Hot bodies are sources of infrared radiations. The hotter the body, the greater the frequency of infrared radiation emitted. Sun is a natural source of infrared radiation. Glubar, Nernst filament and pointolite lamp are some of the laboratory sources used to produce infra red rays.

Heat sensitive instruments like bolometers and thermopiles can detect infrared rays. Photoelectric cells and photographic plates are sensitive to infrared rays.

Infrared radiations are highly absorbed by mirrors, lenses and prisms made of glass. Hence to reflect infrared rays mirrors made of stainless steel are used. Prisms to deviate infrared rays are made of quartz or rock salt.

The extent of scattering is inversely proportional to the fourth power of wavelength. Hence infrared rays are less scattered compared to visible light. Thus infrared rays have high penetrating power and so are used in long distance photography, especially when there is fog and mist.

In medical field infrared rays are used for the diagnosis of superficial tumours and treatment of muscle sprains, paralysis, fracture of bones etc.

The phenomenon of green house effect is because of infrared radiation.

The Earth's atmosphere admits visible light and infrared rays, ultraviolet rays being filtered out by the ozone layer in the atmosphere.

When these admitted radiations fall on Earth's surface, Earth gets slightly heated. The resulting emission from its surface is only infrared rays. These emitted radiations are not allowed to leave the Earth's atmosphere because of the downward reflection on low lying clouds and carbon dioxide gas. This leads to the increase in temperature of the atmosphere. This phenomenon is called green house effect.

If the amount of carbon dioxide in atmosphere increases abnormally due to industrial pollution, the resulting rise in temperature will be fatal to life on earth.

Visible light

The part of the electromagnetic spectrum sensitive to human eye is called visible light. The wavelength range of visible light is from 4000 Å to 7000 Å and the corresponding frequency range is 7.5×10^{14} Hz to 4.2×10^{14} Hz. There are 7 dominant colours in it . They are violet, indigo, blue, green, yellow, orange and red (VIBGYOR). Violet is of the highest frequency while red is of the lowest frequency. The sensitivity of human eye is maximum for the wavelength 5600 Å (yellow green).

Ultraviolet rays

Ultraviolet rays were discovered by Ritter. Sun is the natural source of UV radiation. They are harmful. The ozone layer in Earth's atmosphere absorbs ultraviolet rays and thus living beings are protected from its harmful effects. The continuous damage done to ozone layer by industrial pollution will lead to catastrophic results.

Some of the laboratory sources of ultraviolet radiations are:

(i) mercury vapour lamp
(ii) incandescent tungsten filament lamp
(iii) discharge tube.

Ultraviolet rays can affect photographic plates, produce photoelectric effect, phosphorescence or fluorescence. Any of these properties can be used to detect it.

Ultraviolet rays are absorbed by ordinary glass. Hence prisms and lenses for the detection of ultraviolet rays are made of quartz.

In ultramicroscopes, ultraviolet rays are used instead of ordinary visible light. Because of the smaller wavelength of ultraviolet rays, the instrument has a greater resolving power. (Resolving power of a microscope is inversely proportional to the wavelength of light used).

Ultraviolet rays are used for sterilisation, to examine fingerprints, to detect erasures in documents, in burglar alarms etc.

X-rays

X-rays were discovered by W. Roentgen. They are produced when fast moving electrons hit a metal target of high atomic number. The greater the energy of the electrons, the greater the frequency of the X-rays produced. X-rays of higher frequency are known as hard X-rays and those of lower frequency are known as soft X-rays.

X-rays produce phosphorescence, fluorescence and photoelectric effect. They can ionise gases through which they pass. They are invisible but affect photographic plates. They have very high penetrating power. They destroy living cells.

X-rays can be used to take photographs of internal parts of a body. They are used for the treatment of skin diseases and tumours.

They are used to detect flaws inside metal pieces. X-ray diffraction studies in crystals reveal crystal structure.

Gamma rays (γ rays)

γ-rays were discovered by Willard in 1900. They are of nuclear origin. γ-rays are produced during disintegration of nuclei. Cosmic rays are also sources of γ-rays.

γ rays produce photoelectric effect, ionisation, fluorescence and phosphorescence. They can be detected by scintillation detectors. They are more harmful than X-rays.

Monochromatic radiation

Radiation consisting of a single wavelength is called monochromatic radiation. For example, light from a Laser is a monochromatic radiation.

Composite radiation

Radiation consisting of more than one wavelength is called composite radiation. White light which contains seven dominant colours is an example.

THE PHOTOELECTRIC EFFECT

The phenomenon of emission of electrons by metals when visible or ultraviolet radiation illuminates them is known as ***photoelectric effect****. The electrons ejected from the surface of the metal are known as* ***photoelectrons***. A typical photoelectric experiment is shown in Fig. 2.2. Ultraviolet radiation from source S passes through a quartz window Q and falls on a photosensitive plate P inside an evacuated glass tube T. P is earthed to zero potential. Collecting anode C can be maintained at a desired positive (or) negative potential with respect to P. Voltmeter V measures the potential difference between P and C and the micro ammeter A measures the current flowing in the circuit.

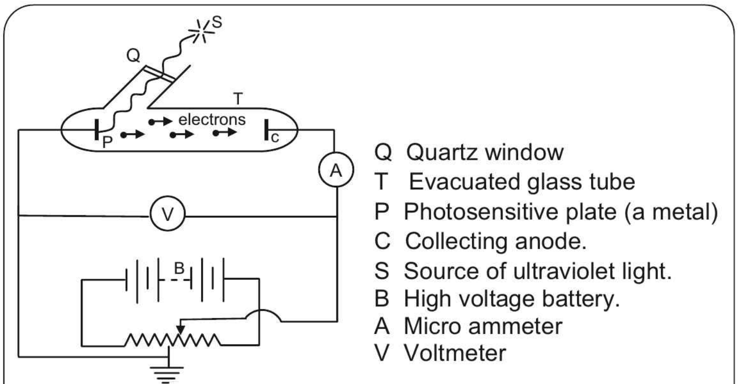


Fig. 2.2

(i) Effect of intensity of light on photoelectric current

Ultraviolet light of a particular frequency is allowed to fall upon plate P. The anode C is made positive with respect to P (by about 10 V), so that all the electrons ejected from P are immediately swept on to C.

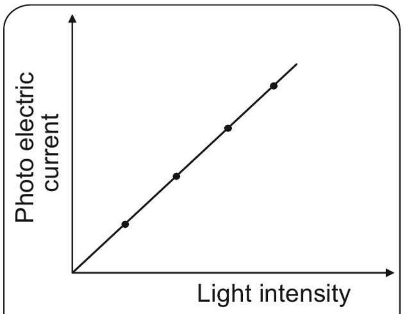


Fig. 2.3

Keeping frequency of light and the accelerating potential fixed, intensity of incident light is varied and the corresponding photocurrent is measured. It is found that the photocurrent is directly proportional to the intensity of incident light (Fig. 2.3).

The number of photoelectrons ejected from the photosensitive plate per second is proportional to the intensity of incident light.

(ii) Effect of potential difference between P and C on photocurrent

Now, the frequency and the intensity of light are held constant and the potential difference is varied, keeping it positive (i.e., V_C is positive; $V_P = 0$ (Earthed)).

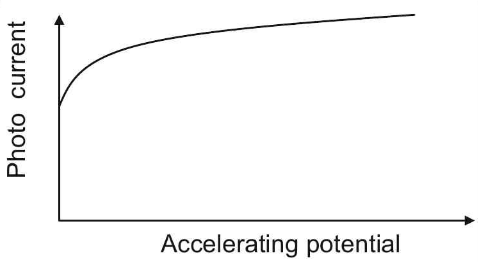


Fig. 2.4

The current is measured for various potentials. It is found that photocurrent increases with increase in accelerating potential, till, for a certain positive potential of C, the current becomes maximum. Beyond this, the current does not increase any further for any increase in the accelerating potential. This maximum value of current is called ***saturation current***.

When the potential of the collecting anode C is reduced to zero, the current does not become zero. If the potential of C is made negative, current decreases. If C is made gradually more negative, current decreases rapidly till at a sharply defined negative potential V_0 of C the current becomes zero.

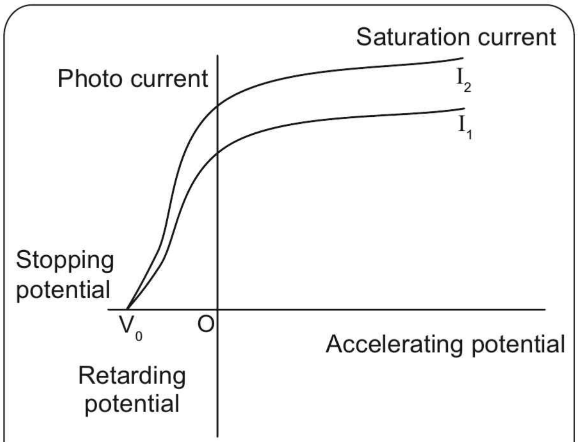


Fig. 2.5

The retarding potential for which the photoelectric current just becomes zero is called ***cut-off*** (or) ***stopping potential*** for this particular frequency of incident light, and for the element used in the photosensitive plate.

If the experiment is repeated with light of the same frequency at a higher intensity, the saturation current is found to be greater (proportional to the light intensity, as per effect no.1 detailed earlier). But the stopping potential, V_0 remains the same. Thus the stopping potential is indepe ndent of the intensity of incident light provided frequency remains the same.

The occurrence of non-zero current for small negative potentials of C (> V_0) can be explained as follows: For light of a given frequency, photoelectrons are emitted with velocities ranging from zero to a maximum value. Electrons with greater velocities (hence greater kinetic energies) are able to overcome the small retarding potential to reach C. When the retarding potential is V_0, current drops to zero which means work done by this V_0 on the fastest electrons emitted must be equal to their kinetic energy, making them just unable to reach C.

$$\frac{1}{2}mv^2_{max} = eV_0$$

where m is the mass of electron and e the charge of electron. Hence, until the retarding potential attains the value V_0, a small photocurrent will be present.

(iii) Effect of frequency (on stopping potential)

If the experiment is repeated with different frequencies, keeping saturation current unchanged (by adjusting the intensity at each frequency suitably), and if the variation of frequency ν with stopping potential V_0 is plotted, we get the curves as shown in Fig. 2.6 and Fig. 2.7. ($\nu_1 > \nu_2 > \nu_3$).

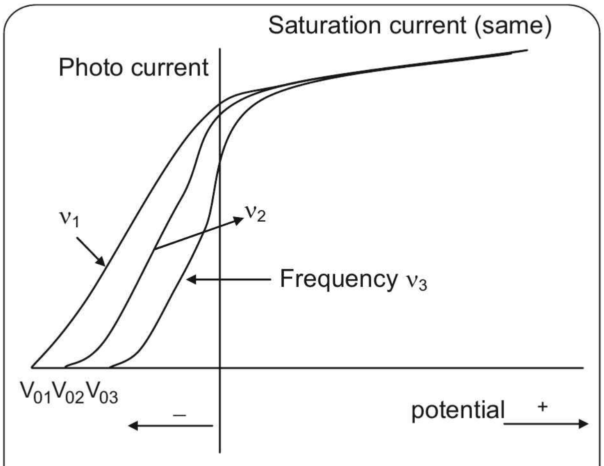


Fig. 2.6

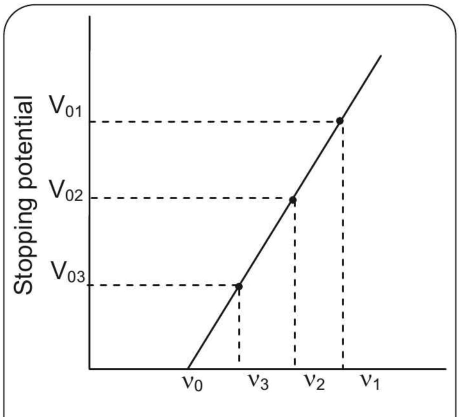


Fig. 2.7

We find that as the frequency of incident light is increased, the magnitude of the stopping potential increases (proportionally) indicating that the maximum kinetic energy increases linearly.

Also, on reducing the frequency, there is a minimum frequency (where the line meets the ν axis), ν_0, for which the stopping potential is zero. At a frequency just lower than ν_0, the metal emits no photoelectrons, whatever be the intensity. ν_0 is called ***threshold frequency***. The corresponding wavelength of the electromagnetic wave is called threshold wavelength and it is the maximum wavelength (λ_0) that can produce photoelectric emission from this surface.

If the frequency of incident light exceeds the threshold frequency, the photoelectron emission starts instantaneously, ($\sim 10^{-9}$ s) even if the incident light is dim (low intensity).

Laws of photoelectric emission

The experimental results are summarized as follows:

(i) For a given photosensitive material, there is a certain minimum frequency (called threshold frequency), below which no photoelectric emission takes place, no matter how great the intensity of light is.

(ii) Photoelectric emission is an instantaneous process. As soon as the frequency of light exceeds the threshold frequency, emission starts without any apparent time lag (in $\sim 10^{-9}$ s).

(iii) For light of any frequency (provided the frequency is above the threshold frequency), the photocurrent is proportional to the intensity of light.

(iv) The maximum kinetic energy of the photoelectrons increases with increase in frequency and is independent of the intensity of the incident light.

Einstein's photoelectric equation (Quantum theory)

Classical wave theory of light cannot explain the experimental results of photoelectric emission. According to classical theory, the law of conservation of energy requires that even if a dim light falls on a metal, if it falls for a long time, photoelectrons should be emitted. But it does not happen.

Albert Einstein provided the correct explanation of the photoelectric effect in 1905 for which he was later awarded the Nobel prize. His explanation is based on the revolutionary hypothesis of photons made five years earlier by Max Planck. Light of frequency ν is made up of photons, each of energy hν.

When a photon collides with an electron just within the surface of a metal, it may transfer its energy to the electron. This transfer is an 'all or nil' process, the electron getting all of hν or none at all. The photon then simply drops out of existence. The energy acquired by the electron may enable it to escape from the surface of the metal if it is moving in the right direction. One photon can interact with only one electron.

As the electron tries to leave the surface, it has to overcome the potential barrier of the metal surface. The existence of a potential barrier is explained as follows: The fact that the electron is confined to the metal implies that it has lower potential energy than when it is outside the metal. So, in order to come out of the metal, it has to acquire potential energy equal to this difference. This requirement acts as a potential barrier preventing the electron from escaping the metal. If the electron is given energy equal in magnitude to the potential barrier, it will surmount the potential barrier and escape the metal. This potential energy barrier is called **work function** ϕ of the metal. If the energy $h\nu$ acquired by the electron is less than ϕ, it cannot come out of the surface. $h\nu$ must be $\geq \phi$. This defines the threshold frequency ν_0 as that frequency for which

$$h\nu_0 = \phi$$

Now, suppose $h\nu > h\nu_0$. Each photon may not cause an electron emission; some electrons may not travel in the right direction, others may suffer more collisions and lose their initially possessed energy $h\nu$ and therefore may not be emitted. Even if travelling in the right direction (towards surface) they may still undergo collisions and lose energy. If all energy is lost, no emission takes place. If 100 photons strike, 100 electrons will not be emitted. Thus we have to take into account the efficiency of photoelectric emission. (If problems do not mention this efficiency, assume 100% efficiency).

Those electrons, which lose part of their energy prior to leaving the surface, will have kinetic energy less than $h\nu - \phi$. Only those electrons which have not lost any energy, will come out of the surface with kinetic energy $= h\nu - \phi$, and these will have maximum possible kinetic energy $= \frac{1}{2}mv^2_{max} = h\nu - \phi = h\nu - h\nu_0.$

This leads to the ***photoelectric equation***

$$h\nu = h\nu_0 + \frac{1}{2}mv^2_{max}$$

So, the electrons, coming out of the surface of the metal have kinetic energies ranging from zero to $\frac{1}{2}mv^2_{max}$.

Explanation of the experimental results

It has already been stated that the emitted photoelectrons have a range of kinetic energies. Those which have small energy form a cloud of space charge around the plate. Sufficiently energetic electrons penetrate this cloud to come out and thereby reach the anode.

Therefore, even with no potential applied to the anode a photoelectric current flows in the circuit. If accelerating potential is increased, more electrons reach the anode. This explains why current increases with increase in accelerating potential (Fig. 2.4). At sufficiently high potential all emitted electrons reach anode leading to maximum current.

Any further increase in potential has no effect on the photocurrent. The current attains its saturation value.

If intensity increases, then the number of photons is more and the number of emitted electrons reaching anode increases for the same accelerating potential (given a particular photoelectric efficiency or a 100 % efficiency) resulting in the increase of current. Saturation current is also larger. (Fig. 2.5)

If on the other hand, instead of increasing the potential, if the potential is reduced, the current decreases. Even when there is no potential difference between cathode and anode, i.e., potential is made zero, a current still flows. If the potential is sufficiently negative to prevent even the fastest electron reaching the anode, (Fig. 2.6) we have

$$eV_0 = \frac{1}{2}mv^2_{max}$$

The photoelectric equation can now be written as

$$eV_0 = h\nu - h\nu_0 \Rightarrow V_0 = \frac{h}{e}\nu - \frac{h}{e}\nu_0$$

This equation represents a straight line, (compare with $y = mx + c$)

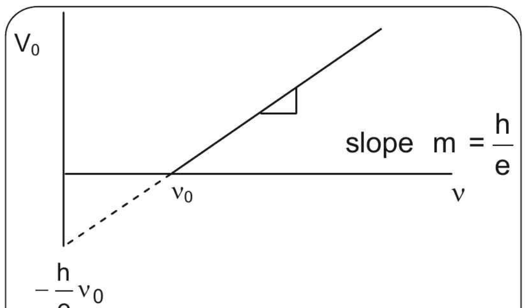


Fig. 2.8

where the slope of the line $= \frac{h}{e} = 4.14 \times 10^{-15}$ V s and the Y intercept is $-\frac{h}{e}\nu_0$

Fig. 2.7 and Fig. 2.8 can be interpreted as follows:

As the frequency of radiation is increased keeping the intensity constant, according to the photoelectric equation, the maximum kinetic energy of the electron increases. Thus

$$eV_0 = \frac{1}{2}mv^2_{max} = h\nu - h\nu_0.$$

Therefore, V_0 is a linear function of the frequency ν.

CONCEPT STRANDS

Concept Strand 4

For a metal used as cathode in a photoelectric effect experiment, a stopping potential of 3.0 V was required for light of wavelength 300 nm, 2.0 V for 400 nm and 1.0 V for 600 nm. Determine the work function for this metal and the value of Planck's constant.

Solution

300 nm $\Rightarrow$ $\nu = \dfrac{c}{\lambda} = 1\times10^{15}\,s^{-1}$

400 nm $\Rightarrow$ $\nu = 0.75 \times 10^{15}\,s^{-1}$

600 nm $\Rightarrow$ $\nu = 0.5 \times 10^{15}\,s^{-1}$

$$V_0 = \frac{h}{e}\nu - \frac{\phi}{e} \quad \text{— (1)}$$

Slope of the line

$$\frac{h}{e} = \frac{2-1}{(0.75-0.5)\times10^{15}} = 4\times10^{-15}$$

$$h = 4 \times 10^{-15} \times 1.6 \times 10^{-19}$$

$$= 6.4 \times 10^{-34}\text{ J s}$$

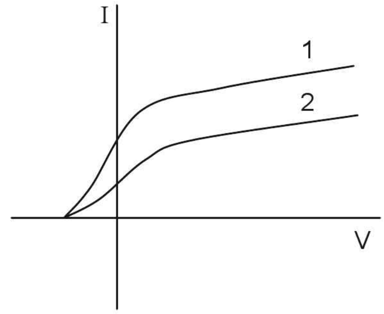
Substituting in equation (1),

$$3.0 = 4 \times 10^{-15} \times 1 \times 10^{15} - \frac{\phi}{e}$$

$$\Rightarrow \frac{\phi}{e} = 1\text{ V} \Rightarrow \phi = 1\text{ eV}$$

Concept Strand 5

A photoelectric substance is separately illuminated by two different monochromatic sources. Both sources are kept at the same distance from the substance. The dependence of photoelectric current on voltage applied between cathode and anode is as shown in the figure. Compare the (i) frequencies and (ii) power, of the two sources?



Solution

The stopping potential is the same for both sources

$\Rightarrow$ KE_{max} is same.

Since $KE_{max} = h\nu - \phi$ and

ϕ = work function of the metal is the same for both ($\because$ same metallic surface)

$\Rightarrow$ ν same.

Therefore, the two sources have the same frequency

$\therefore$ More current means more intensity of source.

$\Rightarrow$ higher power

$\Rightarrow$ source 1 has more power (being at the same distance from substance).

MATTER WAVES

Radiation has dual nature. Radiant energy behaves as waves in some experiments and particles (or photons) in others. Therefore, it is reasonable to expect that particles of matter like electrons, protons, neutrons and the like also exhibit the dual nature as particles or waves under suitable circumstances.

de Broglie's equation

Louis de Broglie proposed that the wavelength λ associated with a particle of momentum p is given by

$$\lambda = \frac{h}{p}$$

where h is Planck's constant. This equation which relates the wave aspect λ on the left to the particle aspect p on the right of the equation, is known as ***de Broglie's equation***. The argument leading to this equation is as follows: A photon's energy E = hν. If photon is considered to have mass m, its energy, according to theory of relativity, is mc^2. Thus

$E = h\nu = mc^2$. If p is momentum of photon $p = mc = \frac{h\nu}{c}$

$\Rightarrow p = \frac{h}{\lambda}$ de Broglie assumed that this equation $p = \frac{h}{\lambda}$ is equally applicable to both photons and particles of matter, like electrons or neutrons.

So, the de Broglie wavelength associated with a particle of mass m moving with a velocity v has wavelength $\lambda = \frac{h}{mv} = \frac{h}{p}$.

The de-Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2m(KE)}}$$

where m = mass of particle

v = speed of particle

KE = kinetic energy of particle = $\frac{1}{2}mv^2$

p = linear momentum of particle

h = Planck's constant

(i) It is interesting to see what the de Broglie wavelength of a body of mass 1 g moving with a velocity of 1 m/s will be

$$\lambda = 6.625 \times 10^{-34}/10^{-3} = 6.625 \times 10^{-31}\text{m}$$

The wavelength is too small to be significant in any practical situation.

(ii) If a charged particle of mass m, carrying a charge q is accelerated through a potential difference V, its de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

The de Broglie wavelength associated with an electron can be calculated as follows: Electron's charge is e and mass is m. If accelerated through a potential V, the electron attains velocity v.

Then, the kinetic energy $\frac{1}{2}mv^2 = eV$

$$v = \sqrt{\frac{2eV}{m}};$$

$$\lambda = \frac{h}{m\sqrt{\frac{2eV}{m}}} = \frac{h}{\sqrt{2eVm}}$$

Substituting the values of h, e and m

$$\therefore \lambda = \frac{6.625 \times 10^{-34}}{\left(2 \times V \times 1.6 \times 10^{-19} \times 9.1 \times 10^{-31}\right)^{\frac{1}{2}}}$$

$$\Rightarrow \lambda \frac{12.27}{\sqrt{V}}\text{Å} = \frac{1.227}{\sqrt{V}}\text{nm} = \sqrt{\frac{150}{V}}\text{Å}$$

For example, if V = 100, λ = 0.123 nm

(iii) For a gas molecule of mass m at an absolute temperature T, its speed = $v_{rms} = \sqrt{\frac{3kT}{m}}$, where k = Boltzmann's constant.

$$\text{Kinetic energy, KE} = \frac{1}{2}mv_{rms}^2 = \frac{1}{2}m \times \frac{3kT}{m} = \frac{3}{2}kT$$

The de-Broglie wavelength of the gas molecule is given by $\lambda = \frac{h}{\sqrt{2m(KE)}} = \frac{h}{\sqrt{2m\frac{3}{2}kT}}$

$$\therefore \lambda = \frac{h}{\sqrt{3mkT}}$$

Note:

The small wavelengths of fast moving electrons (of the order of 1 Å) is made use of in electron microscopes. The resolving power of electron microscope is very high as resolving power is inversely proportional to wavelength (Resolving power $\propto \frac{1}{\lambda}$).

CONCEPT STRAND

Concept Strand 6

An electron and a photon each has a wavelength of 2.0 Å. What are their
(i) momenta?
(ii) energies?

Solution

(i) $p_e = \frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{2 \times 10^{-10}} = 3.3 \times 10^{-24} \text{kg ms}^{-1}$

(for electron)

p_p = same as above (for photon)

(ii) $K = \frac{p^2}{2m} \Rightarrow K_e = \frac{p^2}{2m_e} = 5.98 \times 10^{-18}$ J

$= \frac{5.98 \times 10^{-18}}{e} \text{eV} = \frac{5.98 \times 10^{-18}}{1.6 \times 10^{-19}} \text{eV} = 37.4 \text{ eV}$

(for electron)

$K_p = \frac{hc}{e\lambda}(\text{eV}) = \frac{12422}{2} = 6211 \text{ eV} = 6.2 \text{ keV}$

(for photon)

BOHR THEORY

Atomic structure

Ernest Rutherford and his students, Hans Geiger and Ernest Marsden, in 1910 conducted experiments in which charged particles were projected onto thin foils of elements and their deflections observed. Rutherford's projectiles were alpha particles (helium nuclei) from naturally radioactive elements. The target atom, being neutral, had no charge. Therefore, when the alpha particles arrive at the neutral atom, just outside the atom there is no force on the particle.

But inside, the repulsive force due to the positive charge of the atom deflects the particle, as if the positive charge were concentrated at the center of the atom. Rutherford called this concentration of positive charge as the ***nucleus***. His experiments established that the atom consists of a central nucleus of approximate dimension no greater than 10^{-14} m in diameter.

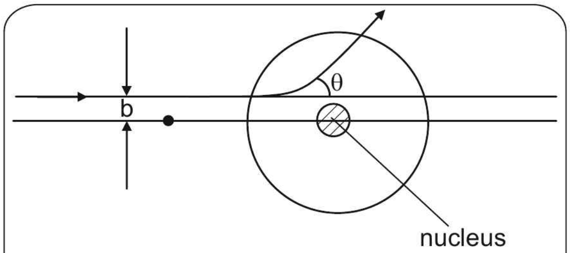


Fig. 2.9

The angle of deflection θ of the path of the alpha particle inside the atom (Fig. 2.9) depends on the charge of the nucleus Ze, the energy of the alpha particle E and the distance of line of approach of α-particles from the nucleus called the ***impact parameter*** denoted by b. Rutherford derived the formula

$$b = \frac{Ze^2 \cot\left(\theta/2\right)}{4\pi\varepsilon_0 E}$$

The beam of alpha particles has a distribution of impact parameters. Therefore, the alpha particles will be scattered in all directions. If the probability for the occurrence of different b is known, θ can be calculated. Calculations and experimental results agree remarkably well. α-particles having impact parameter b = 0 are reflected back. The closest distance 'd' upto which such α-particles can come near the nucleus is called the distance of closest approach, which is of the order of the size of the nucleus. We can show that

$$d = \frac{2Ze^2}{4\pi\varepsilon_0 E}$$

where $E = \frac{1}{2}mv^2$ = kinetic energy of α-particles.

Bohr's postulates

(i) Bohr accepted Rutherford's concept of nuclear atom model, which assumes that the atom consists of a central nucleus. According to Bohr the electrons revolve around the nucleus, in circular orbits, the centripetal force required for rotation being provided by the electrostatic attraction between the electron and the nucleus.

(ii) According to laws of classical mechanics, an electron can revolve in an orbit of any radius, provided its velocity is of suitable magnitude. Further, an accelerating electron will continuously radiate energy, which leads to continuously decreasing orbital radius, eventually making the electron collapse into the nucleus. To avoid this catastrophe Bohr deviated from classical physics and assumed that an electron could revolve only in a few widely separated, permitted orbits. While revolving in these orbits around the nucleus, electrons do not radiate energy. These non-radiating orbits are called **stationary orbits**.

(iii) The permissible orbits of an electron revolving around a positive nucleus are those for which the angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$ where h is Planck's constant. Thus for any permitted orbit,

$$L = mvr = \frac{nh}{2\pi}$$

where m is the mass of electron, v its velocity in the orbit of radius r and n, a positive integer called ***quantum number***.

(iv) The emission or absorption of energy (i.e., photons) takes place when an electron jumps or falls i.e., undergoes a transition from one permitted orbit to another. If E_1 and E_2 are the energies of the electron in the orbits 1 and 2, respectively, then

$$h\nu = E_2 - E_1$$

where ν = frequency of photon (absorbed/emitted).
If $E_2 > E_1$, the difference in energy is emitted and if $E_2 < E_1$, the difference in energy is absorbed.

Velocity, radius and energy of the orbital electrons of hydrogen or hydrogen like ions

A hydrogen like ion is that which has only one electron in the orbit around its nucleus. (e.g., He^+, Li^{++} etc.)
From Postulate 1 we get

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{r^2} \quad \text{—(1)}$$

where Ze is the charge of the nucleus.

(Ze = e for hydrogen, Ze = 2e for singly ionized helium etc.)

From Postulate 3,

$$mvr = \frac{nh}{2\pi} \quad \text{—, (2)}$$

Combining (1) and (2) and eliminating r, velocity

$$v = \frac{Ze^2}{2\varepsilon_0 h}\left(\frac{1}{n}\right)$$

$\Rightarrow$ $v \propto \frac{1}{n}$ and $v \propto Z$. Multiplying and dividing with c, we get

$$v = \frac{Z}{n}.c\left(\frac{e^2}{2\varepsilon_0 hc}\right) = \frac{Z}{n}.c\alpha, \text{ where } \alpha = \frac{e^2}{2\varepsilon_0 hc} = \frac{1}{137}$$

= ***fine structure constant***

$$\therefore \quad v = \left(\frac{Z}{n}\right)\frac{c}{137}$$

Again, from (1) and (2), eliminating v, radius of orbit

$$r = \frac{\varepsilon_0 h^2}{\pi m Z e^2}\left(n^2\right)$$

$\Rightarrow$ $r \propto n^2$ and $r \propto \frac{1}{Z}$.

$$r = \left(\frac{n^2}{Z}\right)a_0$$

where $a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2} = 0.53$ Å, is the radius of the electron orbit in the ground state of hydrogen atom and is called ***Bohr radius***.

The relation between v and r can be obtained as

$$r = \frac{Ze^2}{4\pi\,\varepsilon_0 mv^2} = \frac{nh}{2\pi mv}$$

Energy E is the sum of the kinetic energy and the potential energy. Kinetic energy $K = \frac{1}{2}mv^2$. Substituting for v,

$$K = \frac{mZ^2e^4}{8\varepsilon_0{}^2h^2}\cdot\left(\frac{1}{n}\right)^2$$

Potential energy $U = \frac{1}{4\pi\varepsilon_0}\frac{(Ze)(-e)}{r} = -\frac{mZ^2e^4}{4\varepsilon_0^2h^2}\left(\frac{1}{n}\right)^2$

Total energy $E = K + U \Rightarrow$

$$E = \frac{-mZ^2e^4}{8\varepsilon_0{}^2h^2}\left(\frac{1}{n^2}\right)$$

$\Rightarrow E \propto \dfrac{1}{n^2}$ and $E \propto Z^2$. We can write $E = -\left(\dfrac{Z^2}{n^2}\right)\left(\dfrac{me^4}{8\varepsilon_0{}^2ch^3}\right)ch$

$$E = -\left(\frac{Z^2}{n^2}\right)Rch$$

where R is called ***Rydberg constant*** given by

$$R = \frac{me^4}{8\varepsilon_0{}^2h^3c} = 1.097 \times 10^7\ m^{-1}$$

E = –Rch gives energy in joule, and is called *Rydberg* and has the magnitude 13.6 eV.

Hydrogen atom in the Bohr model

Energy levels

For hydrogen atom, Z = 1. Therefore, the radius of the innermost orbit is given by

$r_{n=1} = r_1 = \dfrac{\varepsilon_0 h^2}{\pi m e^2} = 0.053$ nm , (or 0.53 Å) and is called ***Bohr radius*** represented by the symbol a_0. Bohr radius is often used as a unit of distance in atomic physics.

The energy of the electron in this orbit is

$$E_{n=1} = E_1 = -Rch \text{ joule}$$

$$= -\frac{Rch}{e}eV = \frac{-1.09\times10^7\times3\times10^8\times6.625\times10^{-34}}{1.6\times10^{-19}}$$

$$= -13.6 \text{ eV}$$

For hydrogen atom, higher energy states are given by

$$E_n = \frac{-13.6}{n^2}eV$$

For hydrogen like ions, $E_n = -\left(\dfrac{Z^2}{n^2}\right)13.6$ eV

(Considering the same value of Rydberg constant for all ions).

The electron in hydrogen atom, therefore, has minimum energy in the innermost orbit defined by the quantum number n = 1 (maximum negative value of –13.6 eV)

The state of the atom with the electron revolving in the innermost orbit is called the ***ground state*** or ***normal state*** and it is the most stable state of the atom. Some energy should be given to the atom, so that the electron may jump to an outer orbit. The state of the atom when one of its electrons is forced to revolve in an outer orbit is called an ***excited state***.

An atom in an excited state is not stable; in due course it must jump to an inner orbit and emit the difference of energy in the form of a photon. If n_i and n_f are the initial and final states,

$$h\nu = E_{ni} - E_{nf}$$

$$\Rightarrow \frac{hc}{\lambda} = E_{ni} - E_{nf}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{hc}(E_{ni} - E_{nf}) = -\left(\frac{1}{hc}\right)Z^2Rch\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right]$$

$$\frac{1}{\lambda} = Z^2 \cdot R \cdot \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where $\dfrac{1}{\lambda}$ is the ***wave number*** (symbol $\bar{\nu}$; unit m^{-1}), which stands for number of waves per metre.

Hydrogen spectrum

By putting Z = 1 and n = 1 in the energy equation,

E_1 = –13.6 eV which is the ground state energy of the hydrogen atom.

The higher energy states are given by

$$\therefore E_n = \frac{-Z^2 \cdot (13.6)}{n^2}eV$$

$$\Rightarrow E_n = \frac{E_1}{n^2}$$

The energy levels of the hydrogen atom are shown schematically in Fig. 2.10. The arrows represent transitions of the electron between the energy levels. In each transition, the difference in energy is radiated in the form of electromagnetic waves.

The frequencies emitted by excited atoms can be observed by the following experimental arrangement: hydrogen gas enclosed in a sealed glass tube is heated to high temperature. Light coming from the heated gas (as in the case of the neon lamps) is collimated by a lens and allowed to pass through a narrow slit onto a screen. If the slit is a narrow line, the light that falls on the screen is in the form of parallel lines. It is then called a line spectrum.

Several series of spectral lines are observed.

These correspond to the formula

$$\frac{1}{\lambda} = \bar{\nu} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

predicted by the Bohr theory.

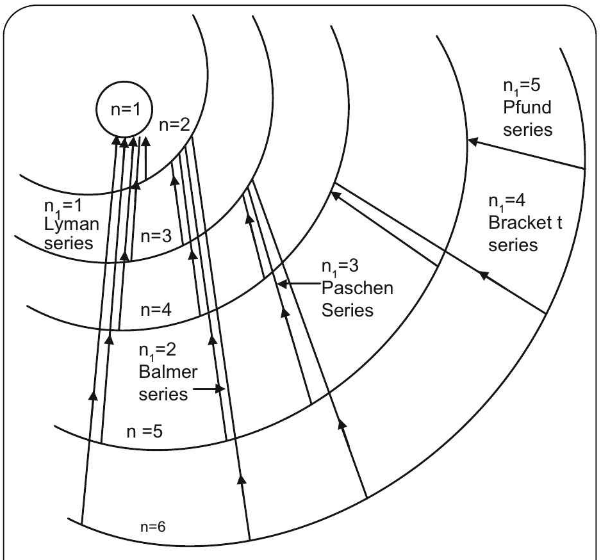


Fig. 2.10

Nomenclature of the spectral lines	Region of the electromagnetic spectrum	Wavelength
Lyman series: $\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$; n = 2, 3, 4,	Ultra violet	91.2 nm to 121.6 nm
Balmer series: $\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$; n = 3, 4, 5,	Partly uv and visible	365 nm to 656 nm
Paschen series: $\frac{1}{\lambda}=R\left(\frac{1}{3^2}-\frac{1}{n^2}\right)$; n = 4, 5, 6,	Iinfrared	820.8 nm to 1876.1 nm
Brackett series: $\frac{1}{\lambda}=R\left(\frac{1}{4^2}-\frac{1}{n^2}\right)$; n = 5, 6, 7,	Far infrared	1459.2 nm to 4053.3 nm
Pfund series: $\frac{1}{\lambda}=R\left(\frac{1}{5^2}-\frac{1}{n^2}\right)$; n = 6, 7, 8,	Far infrared	2280 nm to 7461.8 nm

It can be seen that only Balmer series lies in the visible region.

Another unit of distance frequently used in atomic physics is the Angstrom (symbol: Å) = 10^{-10} m = 0.1 nm.

(i) For Lyman series, λ_{min} occurs for transition from $n_i = \infty$ to $n_f = 1$ and λ_{max} occurs for transition from $n_i = 2$ to $n_f = 1$.

$$\therefore \ \frac{1}{\lambda_{min}} = R\left[\frac{1}{1^2}-\frac{1}{\infty^2}\right] \Rightarrow \lambda_{min} = \frac{1}{R}$$

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{1^2}-\frac{1}{2^2}\right] = \frac{3R}{4} \Rightarrow \lambda_{max} = \frac{4}{3R} = \frac{4}{3}\lambda_{min}$$

(ii) Similarly, for Balmer series, $\lambda_{min} = \frac{4}{R}$,

$$\lambda_{max} = \frac{36}{5R} = \frac{9}{5}\lambda_{min}$$

(iii) For Paschen series, $\lambda_{min} = \frac{9}{R}$, $\lambda_{max} = \frac{144}{7R} = \frac{16}{7}\lambda_{min}$

(iv) For Bracket series, $\lambda_{min} = \frac{16}{R}$, $\lambda_{max} = \frac{400}{9R} = \frac{25}{9}\lambda_{min}$

(v) For Pfund series, $\lambda_{min} = \frac{25}{R}$, $\lambda_{max} = \frac{900}{11R} = \frac{36}{11}\lambda_{min}$

(vi) For any series, $\lambda_{min} = \frac{n_f^{\,2}}{R}$,

$$\lambda_{max} = \frac{n_f^{\,2}(n_f+1)^2}{R(2n_f+1)} = \frac{(n_f+1)^2}{(2n_f+1)}\lambda_{min}$$

(vii) If excited atoms (in state n) fall to the ground state, the total number of spectral lines emitted (N) is given by

$$N = \frac{n(n-1)}{2}$$

Hence if atoms fall from n = 3 to n = 1, total number of spectral lines, $N = \frac{3(3-1)}{2} = 3$

These lines correspond to $3 \to 1$
$3 \to 2$ and
$2 \to 1$ respectively.

(viii) If excited atoms (in state n_i) fall to the state n_f, the total number of spectral lines emitted (N) is given by

$$N = \frac{(n_i - n_f + 1)(n_i - n_f)}{2}$$

Ionization potential

We have seen that the ground state energy of hydrogen atom is –13.6 eV. This is the energy with which the electron is bound to the nucleus. We know that E = 0 when electron and the nucleus are separated by infinite distance. Therefore, when energy of 13.6 eV is supplied to a hydrogen atom in its ground state, the electron is removed from the atom. The atom is now said to be ***ionized***. The electron after being removed from the nucleus moves independently with kinetic energy equal to the excess over E = 0.

The minimum energy needed to ionise an atom is called ***ionization energy***. It is measured in a unit called rydberg (Rd). **1 rydberg = 13.6 eV**, is the ionization energy of hydrogen atom in the ground state.

The potential difference through which an electron should be accelerated to acquire this energy is called ***ionization potential***.

For H atom in ground state, the ionization energy is 13.6 eV and the ionization potential is 13.6 V.

Excitation potential

Energy needed to take the atom from the ground state (n = 1) to an excited state (n >1) is called ***excitation energy*** of that excited state. The potential through which an electron is to be accelerated to gain that much energy is termed ***excitation potential***.

For example, the excitation energy of H atom in its first excited state (n = 2) is

$$-13.6\left(\frac{1}{2^2}-\frac{1}{1^2}\right)=10.2\text{ eV}$$ and the excitation potential is 10.2 V.

de Broglie wavelength of orbital electrons

(i) As per the assumption of Louis de Broglie, the circumference of the nth orbit of electron in the hydrogen atom (or hydrogen like ion) consists of n wavelengths. i.e., de-Broglie wavelength of electron in the nth orbit,

$$\lambda_n=\frac{h}{p}=\frac{h}{mv}=\frac{hr}{mvr}=\frac{hr}{\left(\frac{nh}{2\pi}\right)}=\frac{2\pi r}{n}$$

Hence, de Broglie wavelength of electron in ground state of hydrogen atom, $\lambda_1 = 2\pi a_0$ (a_0 = Bohr radius) and for hydrogen – like ion $(\lambda_1)_Z=\frac{2\pi a_0}{Z}\left[\because (r_1)_Z=\frac{a_0}{Z}\right]$

$$\therefore\ (\lambda_1)_Z\propto\frac{1}{Z}$$

(ii) de Broglie wavelength of electron in nth orbit of hydrogen atom is

$$\lambda_n=\frac{2\pi r_n}{n}=\frac{2\pi n^2a_0}{n}\left(\because r_n=n^2a_0\right)$$
$$=(2\pi a_0)\,n=n\lambda_1$$

$\therefore\ \lambda_n\propto n$; Also $(\lambda_n)_Z=n(\lambda_1)_Z$ for hydrogen like ion.

CONCEPT STRANDS

Concept Strand 7

(i) Calculate the three longest wavelengths in the Balmer series

(ii) The wavelength limits of the Balmer series.

Solution

(i) $\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{n_i^2}\right]$ for Balmer series ($\because n_f = 2$)

Put $R = 1.097\times10^7$, put n_i = 3, 4, 5, --------

We get $\lambda_1=\frac{36}{5R}=656\,\text{nm},\ \lambda_2=\frac{16}{3R}=486\,\text{nm}$

and $\lambda_3=\frac{100}{21\,R}=434$ nm

(ii) Put $n=\infty$

$\Rightarrow\ \lambda = 365$ nm

Balmer series lies between 365 nm and 656 nm

Concept Strand 8

For an electron in the ground state of the hydrogen atom, according to Bohr's theory, what is

(i) its angular momentum?

(ii) its linear momentum?

(iii) its linear speed?

(iv) its angular velocity?

(v) the force on the electron?

(vi) the acceleration of the electron?

Solution

(i) $L = \left.\dfrac{nh}{2\pi}\right|_{n=1} = \dfrac{1\times 6.625\times 10^{-34}}{2\pi} = 1.054 \times 10^{-34}$ J s

(ii) $p = mv = \dfrac{L}{r} = \dfrac{1.054\times 10^{-34}}{0.53\times 10^{-10}} = 1.99 \times 10^{-24}$ kg m s^{-1}

(iii) $v = \dfrac{p}{m} = \dfrac{1.99\times 10^{-24}}{9.1\times 10^{-31}} = 2.19 \times 10^{6}$ m s^{-1}

(iv) $\omega = \dfrac{v}{r}\left(= \dfrac{p}{mr}\right) = \dfrac{2.19\times 10^{6}}{0.53\times 10^{-10}} = 4.1 \times 10^{16}$ rad s^{-1}

(v) $F = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{e^2}{r^2}\left(\text{also } \dfrac{mv^2}{r} = p\omega\right) = \dfrac{9\times 10^{9}\times\left(1.6\times 10^{-19}\right)^2}{\left(0.53\times 10^{-10}\right)^2}$

$= 8.2 \times 10^{-8}$ N

(vi) $a = \dfrac{F}{m} = \dfrac{8.2\times 10^{-8}}{9.1\times 10^{-31}} = 9 \times 10^{22}$ m s^{-2}

X-RAYS

X-rays are produced when electrons with high kinetic energy are allowed to strike a metal target. Electrons that have been accelerated through potential differences of the order of 10^3 to 10^6 V have energies necessary for X-ray production. This phenomenon is the opposite of photoelectric emission.

X-rays are of the same nature as light or any other electromagnetic wave and therefore are governed by quantum relation for the energy of an X-ray photon, $E = h\nu$.

Wavelengths of X-rays range from 0.1 Å (0.01 nm) to 100Å. The energy of X-ray photons varies from 100 eV to 100,000 eV. The frequency of X-rays ranges from 10^{16} Hz to 10^{18} Hz.

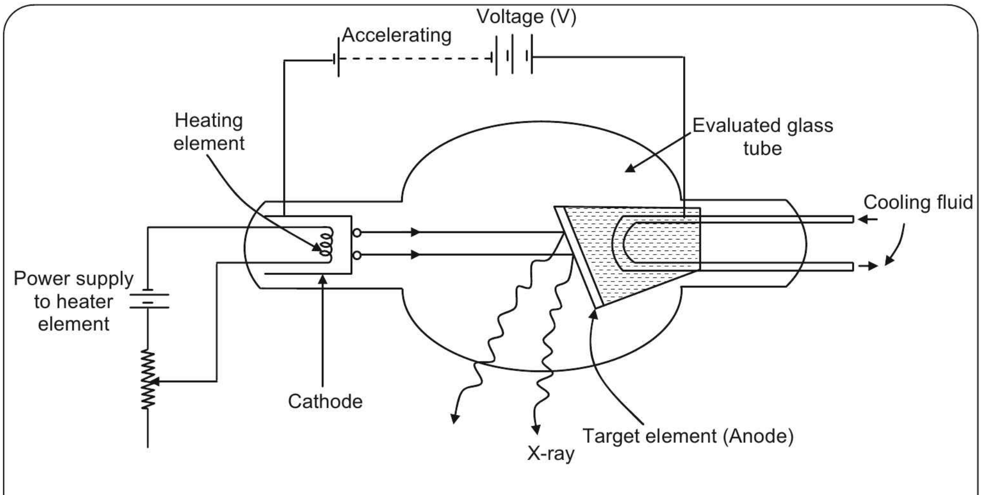


Fig. 2.11

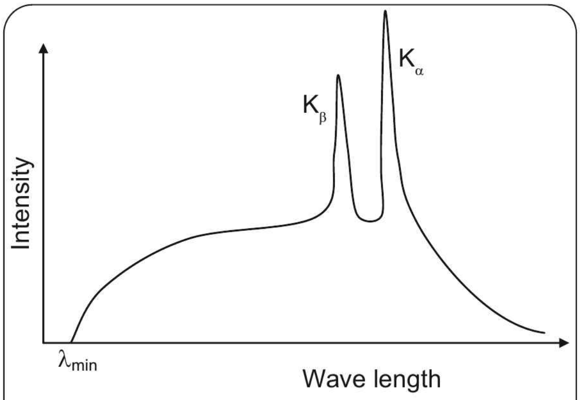
The usual arrangement to produce X-rays is a ***Coolidge tube***, which consists of a thermionic cathode and a high atomic number (heavy) metal anode. The electrodes are in a highly evacuated glass tube. High vacuum reduces the probability of collision of the electrons with gas molecules in the tube. Electrons liberated from the hot cathode and accelerated towards the anode maintained at a high potential bombard the metal anode causing the emission of X-radiation.

Two distinct processes are involved in X-ray emission.

(i) Some of the electrons are stopped by the target and their kinetic energy is converted directly into X- radiation.
(ii) Others transfer their energy in whole or part to the atoms of the target, which retain it temporarily as

'energy of excitation' but in a very short time emit it as X-radiation. These are known as ***characteristic X-rays***.

A typical X-ray spectrum consisting of continuous and characteristic spectra is shown in the Fig. 2.12.



Typical X-ray spectrum

Fig. 2.12

Process (ii) is characteristic of the material of the target whereas process (i) is not. Let us consider these processes in a little more detail. First we discuss the characteristic X-rays.

Accelerated electrons collide with the atoms of the anode or target, as it is called, which is usually a heavy element like tungsten. Electrons with sufficient energy will dislodge one of the inner electrons of a target atom, say, one of the K Shell electrons. This leaves a vacancy in K shell (n=1), which is immediately filled by the transition of an electron from one of the higher energy states in the L (n=2) or M (n=3) shell. The transition of the electron from the higher energy state to the lower energy state is accompanied by energy emission in the form of an X-ray photon which appears as a line in the spectrum; K_α line if the transition is from L to K or K_β if from M to K.

Since, in all these transitions, the electrons end up in the K shell, these transitions are known as the K series. In addition to K series there are also L series, M series, N series and so on. (These lines will be naturally of longer wavelengths and less energy).

In the first process, a bombarding electron may be brought to rest in a simple process if the electron happens to collide head on with an atom of the target, or if it makes several collisions before coming to rest, giving up part of its energy each time. The energy lost at each collision is radiated as a continuous spectrum of all possible wavelengths. These X- ray photons may have any energy from a minimum to a certain maximum, which is that of an electron that gives up all its energy in a single collision. Hence there is a short wave cut-off limit (high frequency limit) to this continuous spectrum. This frequency is ν_{max} where

$$h\nu_{max} = \frac{hc}{\lambda_{min}} = \frac{1}{2}mv^2 = eV$$

where V (in volt) is the accelerating potential. If V is increased, ν_{max} increases and the cut-of wavelength, λ_{min}. decreases.

$\therefore$ Cut off wavelength of X-ray tube,

$$\lambda_{min} = \frac{hc}{eV} = \frac{12400}{V} \text{ Å}$$

This is called **Duane Hunt law**.

The filament current in the Coolidge tube does not have any effect on λ_{min}. It serves only to increase the number of electrons and thereby the number of X-ray photons. Only the accelerating potential determines λ_{min}. The target element does not affect λ_{min}.

The terms ***hard X-rays*** for short wavelength (less than 4Å) and ***soft X-rays*** for long wavelength (greater than 4Å) are often used. Hard X-rays have more energy and large penetrating power than soft X-rays.

CONCEPT STRANDS

Concept Strand 9

Compute the potential difference through which an electron must be accelerated for the short wave limit of the X-ray spectrum to be 0.1 nm (1Å).

Solution

$$V = \frac{12400}{\lambda(\text{Å})} = \frac{12400}{1} = 12400 \text{ V}$$

Concept Strand 10

An X-ray tube operated at 10 kV produces electrons, which lose their KE to emit an X-ray photon at first collision. Find the wavelength corresponding to this photon.

Solution

$$\lambda_{min} = \frac{12400}{V}\text{Å} = \frac{12400}{10 \times 1000} = 1.24\ \text{Å}$$

$$\therefore \quad \lambda_{min} = 0.124\ \text{nm}$$

Intensity of X-rays

(i) The intensity of X-rays depends upon the number of electrons striking on the target element. Larger the number of electrons striking the target element, greater will be the number of X-ray photons emitted resulting in greater intensity of X-ray.

(ii) If the voltage or current of the heating filament is high, more electrons are available. Thus the intensity of X-ray depends on filament current.

(iii) The accelerating voltage (V) across the cathode and anode (target element) increase intensity as well as reduces minimum wavelength.

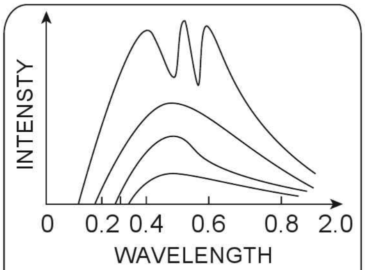


Fig. 2.13

Quality/Hardness of X-rays

(i) The quality/hardness of X-rays is related to the penetrating power of the X-rays. If the X-rays have high frequency (or high energy), they have smaller wavelengths. X-rays having wavelength 4Å and below have greater penetrating power and are called hard X-rays.

(ii) When the electrons are accelerated by a higher voltage across the X-ray tube (i.e., greater accelerating voltage across the cathode and target element), they acquire greater speed $\left(\because eV = \frac{1}{2}mv^2\right)$. If the colliding electrons have greater kinetic energy (or greater velocity), then the X-rays gain greater energy.

(iii) Thus, the quality/hardness of X-rays depend on the accelerating voltage applied across the X-ray tube but it is independent of the voltage/current of the heating filament.

Diffraction of X-rays

Diffraction occurs when the size of the slit is of the order of the wavelength of the radiation. Since the lattice space in crystals are of the order of the wavelength of X-rays, they can cause diffraction of X-rays. The diffraction of X-rays is governed by the relation

$$2d\sin\theta = n\lambda$$

where n = 1, 2, 3, ……, d = spacing of crystal planes and λ = wavelength of X-ray, θ = angle between the X-ray and lattice plane (called glancing angle). This is known as **Bragg's law**.

Efficiency of X-ray tube

In most of the X-ray tubes only 0.2% of the energy of the incident electrons is used to produce X-rays. The remaining energy is usually converted to heat. Hence a large cooling arrangement is required for the target element (or anode).

Absorption of X-rays

X-rays are absorbed by some materials, following the exponential relation

$$I = I_0 e^{-\mu x}$$

where

I_0 = intensity of incident X-rays (initial intensity)

I = final intensity of emergent X-rays

x = thickness of material

μ = absorption co-efficient of the material.

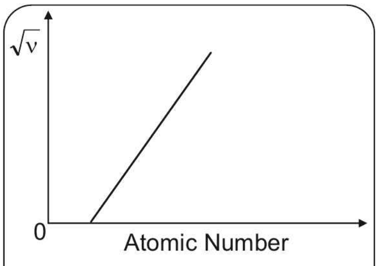


Fig. 2.14

Moseley's law

Numerous measurements of the K and L shell X-ray spectra by Moseley led to the law after his name: The graph of the atomic numbers of various target element versus $\sqrt{\nu}$, where ν is frequency of K_α lines of those elements, is a straight line.

According to Moseley's law, the frequency of K_α lines of several targets are given by

$$\sqrt{\nu} = a(Z-b)$$

where, a and b are constants. b = 1 for K_α line. b is a constant similar to ***shielding constant*** in chemistry.

The occurrence of b is explained as follows : Filled K shell means 2 electrons. When one of the electrons is excited to the L (n = 2) shell, a vacancy in the K shell (n = 1) arises. K_α line is due to the transition of the excited electron back into the K shell vacancy. The excited electron in the L shell would see an effective charge of (Z–1)e on the nucleus as if the lone second electron in the K shell was shielding the nuclear charge Ze.

Applying Bohr's model

$$\Delta E = h\nu = Rch(Z-b)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = Rch(Z-1)^2 \times \frac{3}{4}$$

$$\Rightarrow \nu = \frac{3}{4}Rc(Z-1)^2$$

$$\Rightarrow \sqrt{\nu} = \sqrt{\frac{3}{4}Rc}\,(Z-1)$$

$$\Rightarrow a = \sqrt{\frac{3}{4}Rc};\ \text{Hence for } K_\alpha \text{ lines, } a = \sqrt{\frac{3}{4}Rc} \text{ and } b = 1$$

$$\text{However, for } L_\alpha \text{ lines, } a = \sqrt{\frac{5}{36}Rc} \text{ and } b = 7.4$$

CONCEPT STRANDS

Concept Strand 11

Calculate the wavelength of K_α line for tungsten target (Z = 74).

Solution

$$\text{For } K_\alpha \text{ line, } \frac{1}{\lambda_\alpha} = \frac{3}{4}R(Z-1)^2$$

$$\Rightarrow \lambda_\alpha = \frac{4}{3R(Z-1)^2} = \frac{4}{3\times 1.096\times 10^7 \times 73^2} = 0.023 \text{ nm}$$

Concept Strand 12

An X-ray tube has a copper (Z = 29) target. Calculate the wavelength of K_α line. What is the minimum voltage required to generate this K_α line?

Solution

$$\frac{1}{\lambda} = R(Z-1)^2\frac{3}{4}$$

$$= 1.097\times 10^7 (28)^2 \times \frac{3}{4}$$

$$\Rightarrow \lambda = 0.16 \text{ nm}$$

To excite K_α, one K electron is to be removed to infinity. i.e., a vacancy has to be created in K shell

$$\Delta E = 13.6(Z-1)^2\left[\frac{1}{1^2}-\frac{1}{\infty}\right]$$

$13.6\,(29-1)^2 = 10662.4$ eV = 10.66 keV

∴ Minimum voltage required, V = 10.66 kV

NUCLEAR PHYSICS

Mass energy equivalence

Einstein proposed in 1905 that mass and energy are equivalent i.e., that mass can be converted to energy and vice versa according to the equation.

$$E = mc^2$$

where, E is the energy in joule produced by conversion of mass m (kg) and c is the speed of light (m s^{-1}). The energy equivalent of,

1 gram mass = 9×10^{13} J

= 2.5×10^7 kW h

= 5.625×10^{32} eV

Nucleus

The atomic nucleus was discovered by Rutherford in the α-particle scattering experiment. James Chadwick, while explaining the results of the scattering experiments of I. Curie and F. Joliot, in which beryllium atoms were bombarded with alpha particles, discovered certain uncharged particles of mass approximately that of the proton. He called this particle ***neutron***. He could write this reaction as

$$ {}_{2}^{4}He + {}_{4}^{9}Be \rightarrow {}_{6}^{12}C + {}_{0}^{1}n $$

Neutron (represented by n) has mass $m_n = 1.6749 \times 10^{-27}$ kg while proton mass is slightly smaller $m_p = 1.6726 \times 10^{-27}$ kg (1850 times that of electron). It is now known that the nucleus of all atoms consist of protons and neutrons. The protons and the neutrons together are known as ***nucleons.***

To represent the extremely small masses in nuclear physics, the atomic mass unit (amu: symbol u) is used. $1\ u = \frac{1}{12}$ (mass of ${}_{6}^{12}C$ atom). It has the magnitude1 amu $= 1.6606 \times 10^{-27}$ kg $= 931.49$ MeV (using $E = mc^2$ formula). On this scale, $m_n = 1.0087$ u (939.5 MeV) and $m_p = 1.0073$ u (938 MeV).

A free neutron is highly unstable outside the nucleus but stable inside.

Atomic number defines the number of protons in the nucleus (represented by the symbol **Z**) and ***neutron number*** specifies the number of neutrons (represented by N). ***Mass number*** (represented by **A**) gives the number of nucleons (both protons and neutrons together) and, therefore, A = Z + N.

Nuclei with same Z but different N and, therefore, different A are called ***isotopes***.

Nuclei with same N but different Z are known as ***Isotones.***

Nuclei with same A but different Z are known as ***Isobars***

Nuclide is the name for a nuclear species. The usual notation for nuclide is ${}_{Z}^{A}X$, where the letter X represents the chemical symbol of the atom. For example, ${}_{92}^{236}U$ and ${}_{92}^{235}U$, which are isotopes of uranium can be generally called uranium nuclides.

Nuclear stability

Figure 2.15 represents the neutron number plotted against the proton number. Nuclides with the same number of protons and neutrons fall on the straight line. For light stable nuclei, $\frac{N}{Z} = 1$. $\frac{N}{Z}$ Increases with Z for heavier nuclides and has the value 1.6 for the heaviest stable region. Beyond this value, the nuclides are unstable and are ***radioactive***. They decay into lighter stable nuclei.

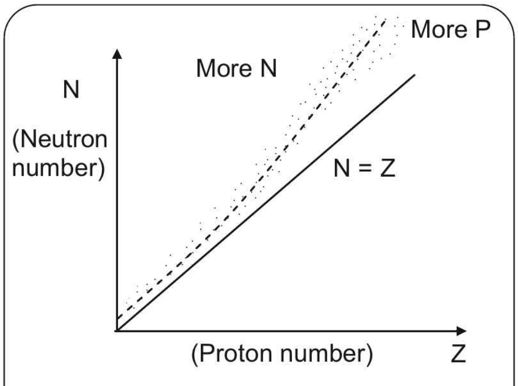


Fig. 2.15

Nuclear radius

Nucleus has no definite boundary. However, experiments show that we can define an average radius R of a nucleus

$$ R = R_0 A^{\frac{1}{3}}, $$

where, $R_0 = 1.2 \times 10^{-15}$m =1.2 fm. fm stands for the unit of distance ***Fermi*** $= 10^{-15}$ m . A is the mass number. Volume V of a nucleus is

$$ V = \frac{4}{3}\pi R_0^{\,3} A $$

The mass of the nucleus is due to the protons and neutrons in it.

Since $m_p \cong m_n$, mass of a nucleus is proportional to A and the nuclear density is independent of A. It has approximately the value 2.25×10^{17} kg m^{-3}

Nuclear charge

The electric charge of a nucleus is due to the protons in it and it is equal to q = Ze, where Z = atomic number of nucleus.

Nuclear forces

Inside the nucleus, the nucleons are subject to a force much stronger than gravitational or electromagnetic forces. The

strong interaction, as it is called, does not depend on charge. It has very small range ~ 10^{-15} m. In this region, it is much greater than the electromagnetic forces and is basically attractive. Neutrons are bound to other neutrons and protons, and the binding is the same for both. Nuclear forces are spin dependent. At very small distance (smaller than nuclear range), the nuclear forces can become repulsive. There are exchange forces as explained by Yukawa. The exchange particle is called a π meson.

Binding energy

In order to separate the nucleons from the nucleus, large amount of energy of the order of several MeV is required. This means that the rest mass energy of the nucleons in free state is larger than their total energy while they are inside the nucleus. The energy needed to separate the nucleons is called ***binding energy*** of the nucleus. Using the Einstein mass energy relation we can make an estimate of the binding energy of the nucleus. The rest mass of free proton, free neutron and the nucleus are, respectively, m_p, m_n and M. Then, binding energy of the nucleus is

$$B = [(Zm_p + Nm_n) - M]\, c^2 = \Delta m\, c^2$$

We can also use atomic masses instead of nuclear masses and write

$$B = [\, Zm_{\text{hydrogen atom}} + Nm_n - \text{mass of } {}^{A}_{z}x]\, c^2$$

because Z times m_e occurs both in the first term and the last term and hence cancels off.

The difference in total mass of the individual nucleons and the assembled nucleus, Δm, is known as the ***mass defect***.

CONCEPT STRAND

Concept Strand 13

Calculate the Binding energy of an alpha particle from the following data:

Mass of ${}^{1}_{1}$H atom = 1.007825 u

Mass of neutron = 1.008665 u

Mass of ${}^{4}_{2}$He atom = 4.00260 u

Solution

$$\Delta m = 2 \times 1.007825 + 2 \times 1.008665 - 4.00260$$
$$= 0.03038 \text{ u}$$
$$B = (0.03038 \text{ u})c^2 = 0.03038 \times 931\text{MeV}$$
$$= 28.3 \text{ MeV}$$

$\Rightarrow$ If two protons and two neutrons combine to form an alpha particle, 28.3 MeV of energy will be released.

Binding energy per nucleon

An important measure of how tightly a nucleus is bound is the ***binding energy per nucleon, B/A***. It gives a measure of the stability of the nucleus. The larger the binding energy per nucleon, the more stable is the nucleus. The plot of Binding Energy per nucleon $\left(\frac{B}{A}\right)$ versus A is shown in Fig. 2.15.

The stable nuclei are those with A ≈ 50 to 80, for which $\left(\frac{B}{A}\right)_{max}$ is 8 MeV or more. In the case of lighter and heavier nuclei, $\frac{B}{A}$ decreases. The most stable nucleus is ${}^{62}_{28}$Ni, which has $\left(\frac{B}{A}\right)_{max}$ = 8.795 MeV, followed by isotopes of iron 58_{Fe} and 56_{Fe} respectively.

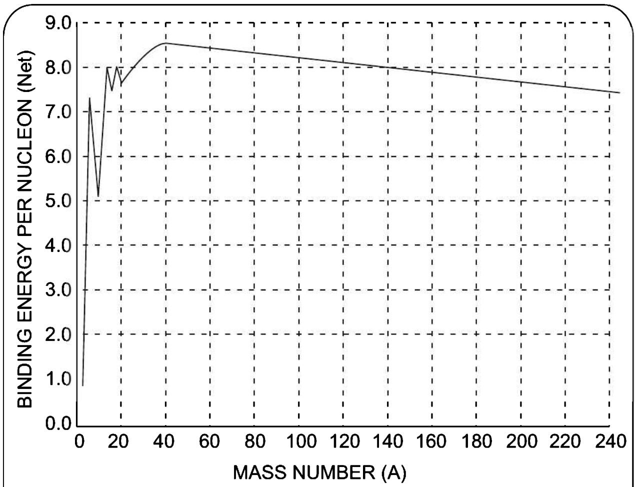


Fig. 2.16

CONCEPT STRAND

Concept Strand 14

Find the Mass defect and Binding energy of Lithium. (Mass of Li nucleus = 7.016005 u)

Solution

Mass of protons $= 3 \times 1.007277 = 3.021831$ u

Mass of neutrons $= 4 \times 1.00865 = 4.034660$ u

Total mass = 7.056491 u

$\therefore \quad \Delta m = (7.056491 - 7.016005)\ \text{u} = 0.040486$ u

$B = \Delta m \times c^2$

$= 0.040486 \times 931$ MeV

$= 37.7$ MeV

($\because$ 1 u $\approx$ 931 MeV)

Packing fraction (P)

The difference between the exact nuclear mass M of a nucleus and its mass number A, divided by mass number A, is called the packing fraction. $P = \dfrac{M - A}{A}$. It can be positive, negative or zero.

Nuclear fission

Nuclear fission was discovered by German scientists Otto Hahn and Fritz Strassmann in 1939.

Nuclear fission reaction is the process in which the nucleus of a very heavy atom splits into lighter nuclei, accompanied by the release of a large amount of energy. This happens because binding energy per nucleon of the heavy atom is less than that of lighter nuclei formed.

In a fission reaction, the sum of the mass of the products is always less than the sum of the masse of the reactants. The mass so lost is converted to energy according to Einstein's mass energy relation.

The material which can undergo nuclear fission is called a fissile material e.g., uranium, thorium etc. Nuclear fission can be explained using Bohr's liquid drop model of the nucleus.

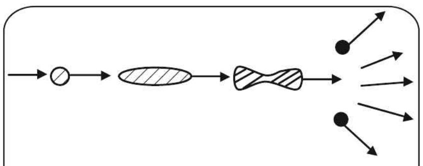


Fig. 2.17

By absorbing the fast moving neutron, the total energy of the uranium nucleus increases and it starts deforming (like a liquid drop deforming to have more surface area when its surface energy becomes more). First it takes an oval shape, then a dumb bell shape and then gets separated. The smaller shapes have less energy and the excess energy is released in the form of fast moving neutrons. (Fig. 2.16)

Example:
A fission reaction is

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_0n^1 + \text{Energy}$$

Here, a $_{92}U^{235}$ atom is bombarded by a thermal neutron (i.e., a slow neutron). The neutrons which are in thermal equilibrium with the molecules of the moderator are called thermal neutrons. They have a speed of about 2200 m s^{-1} and energy $\approx$ 0.025 eV. As a result, an unstable $_{92}U^{236}$ atom is formed. This splits into $_{56}Ba^{141}$ and $_{36}Kr^{92}$ releasing 3 neutrons. The mass lost in this process is about 0.223 u which is converted to an equivalent amount of energy equal to 207 MeV. The energy released appears in the form of kinetic energy of the fragments and as radiations.

Other modes of fission of U^{235} are given below:

$$_{92}U^{235} + {}_0n^1 \rightarrow {}_{92}U^{236} \rightarrow {}_{54}Xe^{140} + {}_{38}Sr^{94} + 2\,{}_0n^1 + \text{Energy}$$

$$_{92}U^{235} + {}_0n^1 + \rightarrow {}_{92}U^{236} \rightarrow {}_{52}Te^{135} + {}_{40}Zr^{98} + 3\,{}_0n^1 + \text{Energy}$$

The average number of neutrons per fission is about 2.5.

Chain reaction

If in a fission reaction, the neutrons released can cause further fission reactions on the remaining fissile material, the reaction will continue uninterrupted till the fuel is exhausted. This way of sustaining fission reaction is called chain reaction. As a result, large amount of energy will be released.

Chain reaction can be classified as controlled chain reaction and uncontrolled chain reaction.

In controlled type, the rate of chain reaction is controlled by some mechanism so that all the fissile material is not exhausted all of a sudden. In this case, the energy released can be employed for useful work. This is the principle of nuclear reactors.

In uncontrolled type chain reaction, the rate of reaction is not at all regulated. As a result, all the fissile material will be over in a small interval of time. The huge amount of energy released will be catastrophic. This is what is taking place in an atom bomb.

Critical size and critical mass

Chain reaction in a fissionable material is possible only if its size is greater than a minimum value known as critical size. If the size of the fissionable material is less than the critical size, good majority of the neutrons released at one stage will escape without causing further fission. As a result, fission process will be terminated.

The mass of the fissionable material corresponding to critical size is called critical mass.

Multiplication factor (K) of a chain reaction

Multiplication factor (K) of a fissile material is defined as the ratio of number of neutrons available for further fission at some instant to the number of neutrons used for fission at the previous instant.

$K = 1 \Rightarrow$ Chain reaction will sustain $\Rightarrow$ the size is critical.

$K > 1 \Rightarrow$ Rate of chain reaction will increase $\Rightarrow$ the size is super critical.

$K < 1 \Rightarrow$ Chain reaction will terminate $\Rightarrow$ the size is sub-critical.

'K' is also called as 'Reproduction constant'

Note:

(i) All the neutrons produced during fission reactions do not cause further fission since,
  (a) A few escape the bounding surface of the material.
  (b) A few are captured by nuclei of non-fissile materials present

(ii) Escape rate of neutrons from the surface will be minimum if the surface area is minimum. Of all shapes, sphere has the minimum surface area for a given volume. Hence escape rate of neutrons from surface will be minimum if the fissile material is of spherical shape.

(iii) For a spherical shape of radius r

$$\frac{\text{Escape rate of neutrons}}{\text{Production rate of neutrons}} \propto \frac{4\pi r^2}{\frac{4}{3}\pi r^3} \propto \frac{1}{r}.$$

The greater the radius of the sphere, the smaller the fraction of escape.

(iv) Natural uranium cannot be used for nuclear fission as it contains only a very small quantity of ${}_{92}U^{235}$ and more of ${}_{92}U^{238}$. Hence, through a process called 'uranium enrichment,' the fractional amount of ${}_{92}U^{235}$ in the natural uranium is increased and this enriched uranium is used for nuclear fission.

Nuclear Reactors

Nuclear reactor is a device in which nuclear fission chain reactions take place in a controlled, self-sustaining manner so that the energy released can be used for commercial purposes. Nuclear reactors are also known as atomic pile. The first nuclear reactor was designed by Fermi in 1942.

The general arrangements in a nuclear reactor are shown in Fig. 2.18.

The essential components of a nuclear reactor are:

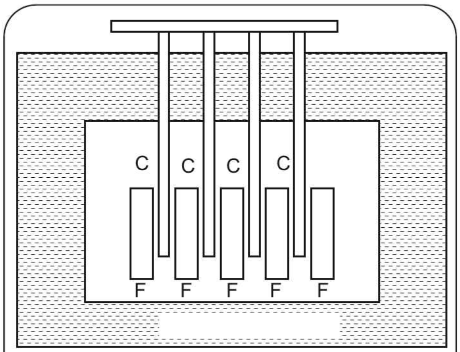


Fig. 2.18

(i) Nuclear fuel (ii) Moderator (iii) Control rods (iv) Cooling system (v) Neutron reflectors and (vi) Concrete shield.

(i) Nuclear fuel

The fissionable material is called nuclear fuel. The commonly used nuclear fuels are

(a) Natural uranium which is, 0.7 % of U-235 and rest U-238. Only U-235 atoms undergo fission reactions.
(b) Enriched urarium. This is prepared artificially by increasing the percentage of U-235 in the uranium obtained from natural resource.
(c) Pu-239 prepared from U-238
(d) U^{235} prepared from Th^{232}.

Usually the fissionable materials are arranged in the reactor in the form of rods F shown in the figure.

(ii) Moderators

The neutrons released in a fission process are called fast neutrons since their kinetic energy is very high (of the order MeV). Such fast neutrons are not suitable for further fission. Hence, certain substances are used in the reactor, known as moderators, to slow down these fast neutrons to thermal neutrons which are in thermal equilibrium with the molecules of the moderator. These thermal neutrons cause further fission and thereby sustain the chain reaction. Thermal neutrons have a speed of about 2200 m s^{-1} and 0.025 eV energy.

A substance used as a moderator should be of low atomic weight and it should not absorb the fast neutrons or thermal neutrons.

Substances usually used as moderators are (a) water (H_2O) (b) heavy water (D_2O) (c) graphite (d) berillium oxides and paraffin wax.

(iii) Control rods (C)

Control rods (shown in the figure as C) are usually made of cadmium. When they are pushed into the core, they absorb a lot of neutrons, thereby slowing down the fission reaction. When they are drawn out the fission reaction speeds up. Thus control rods control the rate of nuclear reaction.

(iv) Cooling system

Because of the fission reaction, large amount of heat is developed in the reactor. (All types of kinetic energy and radiant energy are finally converted to heat). A cooling material circulated around the core of the reactor removes this heat and assures safety of the device. The heat so collected is used to produce steam power, which finally is converted to electrical energy using steam turbines.

The usually used coolant liquids are water or molten sodium.

Note:

The desirable properties of a coolant are,

(a) high thermal conductivity
(b) high specific heat capacity
(c) non-reactive nature, and
(d) non-radioactive nature

(v) Neutron reflectors

The core of the reactor is surrounded by neutron reflectors. These will reflect back the escaping neutrons to the core and thereby reduce the critical size of the fissionable material.

Very often, graphite is used as a neutron reflector.

(vi) Concrete walls

Nuclear reactors are surrounded with thick concrete walls. This is to protect the surrounding from harmful radiations and leaking radioactive substances.

Classification of nuclear reactors

Depending on fuel-moderator assembly, nuclear reactors can be classified as homogeneous type and heterogeneous type.

In homogeneous type, fuel and moderator are mixed in a homogeneous way. For example, fuel uranium salt solution is mixed with moderator heavy water. The uranium salts used are either uranium oxide or uranyl sulphate.

In heterogeneous type, fuel and moderator exist separately in a heterogeneous fashion. For example, when graphite is used as moderator and uranium rods are used as fissile material, the reactor is a heterogeneous type.

Depending on utility, nuclear reactors can be classified as research reactors and power reactors.

The former type produces neutrons and radio isotopes for laboratory nuclear research. The latter type produces electric power for commercial purposes.

Breeder reactors

A material which can be converted to a fissionable material by bombarding with fast neutrons is called a fertile material. The above process of conversion is called breeding.

For example, U^{238} is a fertile material. It can be converted to the fissile material Pu^{239} as shown below:

$$_{92}U^{238} + {}_{0}n^{1} \rightarrow {}_{92}U^{239} + \gamma$$

$$_{92}U^{239} \rightarrow {}_{93}Np^{239} + \beta^{-}$$

$$_{93}Np^{239} \rightarrow {}_{94}Pu^{239} + \beta^{-} + \gamma$$

In a similar manner, the fertile material Th^{235} can be converted to fissile material U^{233} by breeding.

In breeder reactors, a fertile material is used as fuel. The fast neutrons inside convert it into fissionable materials.

The first Indian atomic power reactor APSARA was commissioned in 1956. Other Indian atomic reactors are CIRUS, ZERLINA, PURNIMA, DHRUVA and KAMINI.

Thermal neutron and fast neutrons

Slow neutrons of kinetic energy 0.025 eV are called thermal neutrons. They are so called because their energy is comparable to the kinetic energy of ideal gas molecules at NTP. These neutrons can cause fission reactions.

Neutrons of high energy (of the order MeV) are called fast neutrons. They cannot cause fission reactions directly. The neutrons generated by fission reactions are fast neutrons. They are slowed down by moderators so that they can cause further fission.

In Breeder type reactors, fast neutrons convert the fertile material into fissile material.

Nuclear waste

The material remaining in a nuclear reactor after fission process is complete is called nuclear waste. This contains highly toxic radioactive materials. The harmful nature of it will continue for thousands of years to come. Hence, nuclear waste is to be disposed of properly to avoid catastrophic environment pollution.

The method is to dump it very much below earth's surface after enclosing it in thick walled containers made of concrete and steel.

Another method is to embed the nuclear waste in borosilicate glass in the ratio 1 : 3, heat the mixture and cast it into suitable blocks. These blocks are burried deep inside earth's crust.

Principle of atom bomb

Atom bomb is a fission bomb. The material used is $_{92}U^{235}$ or $_{94}Pu^{239}$.

The method involves preparation of pieces of fissionable materials of sub critical size and keeping them separated. At the time of explosion, these pieces are brought together by remote control. As the size exceeds critical value, uncontrolled chain reaction takes place producing large amount of heat energy, harmful radiations and harmful radio isotopes.

Nuclear Fusion

Under suitable conditions, certain lighter nuclei will combine to form a heavier nucleus. This phenomenon is called nuclear fusion. The binding energy per nucleon is greater for the product nucleus than for the reacting nuclei. Hence, as a result of nuclear fusion, large amount of energy will be released. This energy (E) can be calculated using the relation,

$$E = (\Delta\, m)c^2$$

where, Δm is the mass annihilated during fusion reaction.

Fusion reactions are possible only at very high temperatures of the order 10^8 K. Hence, fusion reactions are also called thermonuclear reactions.

The high temperature required for fusion reactions are not available in Earth. But in Sun and stars the temperature is very high enough to have fusion reactions. In fact, the fusion reactions taking place there, are the source of heat energy radiated by them.

Examples of fusion reactions are:

(i) $_{1}H^{2} + {}_{1}H^{2} \rightarrow {}_{2}He^{4}$ + Energy
(ii) $_{6}C^{12} + {}_{1}H^{1} \rightarrow {}_{7}N^{13}$ + Energy
(iii) $_{7}N^{14} + {}_{1}H^{1} \rightarrow {}_{8}O^{15}$ + Energy

Advantages of fusion reactors over fission reactors

(i) The problem of nuclear waste does not arise
(ii) The raw materials required are cheap
(iii) Energy made available per unit mass is greater for fusion reactions than for fission reactions.

Fusion reactors

Researches are in progress for practical realization of fusion reactors.

The major difficulties encountered in realizing them are

(i) The high temperature required cannot be attained using standard materials.
(ii) If at all the temperature is attained, at such high temperature, matter will be in the plasma state. A container for plasma cannot be easily designed.

Principle of hydrogen bomb

Hydrogen bomb is a fusion bomb. The nuclear reaction used is,

$$_3Li^6 + {_1H^2} \rightarrow {_2He^4} + {_2He^4} + \text{Energy}$$

The very high temperature required for this thermonuclear reaction is attained by first exploding an atom bomb.

The destruction and damage caused by a hydrogen bomb is many times that due to an atom bomb.

Sources of stellar energy

The energy released by sun and stars is called stellar energy. Sun is radiating at the rate of 3.8 × 1016 J s^{-1}.

In 1939, H. Bethe suggested that the source of stellar energy is fusion reactions taking place in Sun and stars. The high temperature required for this reaction is available there. It is suggested that at that high temperature, four protons combine to form a helium nucleus given by the relation,

$$4\,({_1H^1}) \rightarrow {_2He^4} + 2\,({_1e^o}) + 24.7\text{ MeV}$$

Actually, this reaction does not proceed in a single step. Different procedures are suggested. The important among them is (i) Proton-proton cycle and (ii) Carbon-nitrogen cycle.

Proton–proton cycle

The individual steps in the reaction are,

$${_1H^1} + {_1H^1} \rightarrow {_1H^2} + {_1e^o} + 0.4\text{ MeV} \quad \ldots(1)$$
$${_1H^2} + {_1H^1} \rightarrow {_2He^3} + 5.5\text{ MeV} \quad \ldots(2)$$
$${_2He^3} + {_2He^3} \rightarrow {_2He^4} + 2({_1H^1}) + 12.9\text{ MeV} \quad \ldots(3)$$

The overall reaction is obtained by

$$2 \times (1) + 2 \times (2) + (3)$$

i.e., $4({_1H^1}) \rightarrow {_2He^4} + 2\,({_1e^o}) + 24.7\text{ MeV}$

Carbon–nitrogen cycle

The individual steps in the reaction are,

$${_1H^1} + {_6C^{12}} \rightarrow {_7N^{13}}$$
$${_7N^{13}} \rightarrow {_6C^{13}} + {_1e^o} + \gamma$$
$${_1H^1} + {_6C^{13}} \rightarrow {_7N^{14}} + \gamma$$
$${_1H^1} + {_7N^{14}} \rightarrow {_8O^{15}} + \gamma$$
$${_8O^{15}} \rightarrow {_7N^{15}} + {_{+1}e^o} + \gamma$$
$${_1H^1} + {_7N^{15}} \rightarrow {_6C^{12}} + {_2He^4}$$

Adding all the reactions, one gets the overall reaction as

$$4\,({_1H^1}) \rightarrow {_2He^4} + 2\,({_1e^o}) + 24.7\text{ MeV}$$

The carbon nucleus ${_6C^{12}}$ acts as a catalyst in this reaction.

CONCEPT STRANDS

Concept Strand 15

It is proposed to use the nuclear fusion reaction

$${}^2_1H + {}^2_1H \rightarrow {}^4_2He$$

in a nuclear reactor of 200 MW rating. If the energy from the above reaction is used with 25% efficiency of the reactor, how many gram of deuterium fuel will be needed per day?

$$m\left({}^2_1H\right) = 2.0141\text{ u}$$
$$m\left({}^4_2He\right) = 4.0026\text{ u}$$

Solution

$$\text{Energy required per day} = \frac{\text{Power(W)} \times \text{time(s)}}{\text{Efficiency}}$$

$$= \frac{200 \times 10^6 \times 24 \times 60 \times 60}{0.25} = 6912 \times 10^{10}\text{ J}$$

$$\Delta m = 2m\left({}^2_1H\right) - m\left({}^4_2He\right)$$

$$= 0.0256\text{ u} = 23.85\text{ MeV}$$

$$= 23.85 \times 1.6 \times 10^{-13}\text{ J}$$

∴ number of fusion reactions required per day.

$6912 \times 10^{10}/23.85 \times 1.6 \times 10^{-13} = 1.81 \times 10^{25}$

∴ Number of deuterium nuclei required per day

$= 2 \times 1.81 \times 10^{25}$

(∵ 2 deutrons per reaction)

In terms of gram,

$= 2 \times 1.81 \times 10^{25} \times 2.041 \times 1.66 \times 10^{-24}$

= 121 gram

[∵ 1 u = 1.66×10^{-24} gram]

Concept Strand 16

How much ^{235}U is required for a 3000 MW power plant at 100% efficiency. Each fission produces 200 MeV.

Solution

$200 \text{ MeV} = 200 \times 10^6 \times 1.6 \times 10^{-19}$ J

$= 3.2 \times 10^{-11}$ J

$\therefore$ Number of fission reactions per second

$$= \frac{3000 \times 10^6}{3.2 \times 10^{-11}} = 0.94 \times 10^{20}$$

Mass of U required per second = $(0.94 \times 10^{20}) \times 235 \times 1.67 \times 10^{-27}$ kg $= 37 \times 10^{-6}$ kg

$\therefore$ Requirement of ^{235}U per day = $37 \times 10^{-6} \times 86,400$

$= 3.2$ kg

RADIOACTIVITY

Among about 2500 known nuclides, fewer than 300 are stable. The others are unstable. The unstable heavier nuclides decay into lighter nuclei by emitting alpha particles (***alpha decay***) or ***beta particles*** (***beta decay***) or gamma rays ***(gamma decay)***. Some of them decay extremely fast in a matter of seconds or hours, whereas the others take long time (millions of years). Beta particle is the common name for electron (β^-) as well as the ***positron*** (β^+). The positron carries a charge +e, but otherwise identical to the electron. So, positron is known as the ***antiparticle*** of the electron.

Alpha decay

A typical example of alpha decay is given by the ***decay scheme***

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

where, X denotes the parent nucleus, Y, the daughter nucleus and $^{4}_{2}He$, the alpha particle. In alpha decay, the mass number decreases by 4 and the atomic number by 2. For example, consider the decay of the Bi isotope

$$^{212}_{83}Bi \rightarrow {}^{208}_{81}T\ell + {}^{4}_{2}He$$

In heavy nuclei, the coulomb repulsive force is large. Therefore, they are unstable. Alpha decay occurs in nuclei with A > 210.

The difference between the rest mass energies of initial constituents and the products is called the ***Q value*** of the decay process. The Q value is numerically equal to the kinetic energy of the products.

The α-decay can be represented by a general reaction

$$\underset{\text{(parent nucleus)}}{^{A}_{Z}X} \rightarrow \underset{\text{(Daughter nucleus)}}{^{A-4}_{Z-2}Y} + \underset{(\alpha\text{-particle})}{^{4}_{2}He} + Q$$

Here, $Q = \left[M\left({}^{A}_{Z}X\right) - M\left({}^{A-4}_{Z-2}Y\right) - M\left({}^{4}_{2}He\right)\right]c^2$ is the energy released in the reaction (Q-value) and it appears as the kinetic energy of the daughter nucleus and the alpha particle.

If Q > 0, then only α-decay is possible.

If we consider nucleus was at rest initially,

$\bar{p}_D = -\bar{p}_\alpha \left(\text{or} \left|\bar{p}_D\right| = \left|\bar{p}_\alpha\right|\right)$, where $\bar{p}_D$ = linear momentum of daughter nucleus and $\bar{p}_\alpha$ = linear momentum of α-particle.

$\therefore$ $KE_{\text{Daughter nucleus}} + KE_{\alpha\text{-particle}} = Q$

conservation of momentum

$$p_D = p_\alpha = \sqrt{2(KE_D)m_D} = \sqrt{2(KE_\alpha)m_\alpha}$$

$$\Rightarrow \frac{KE_D}{KE_\alpha} \cdot \frac{KE_\alpha}{KE_D} = \frac{KE_\alpha}{KE_D} = \frac{m_2}{m_\alpha}$$

$$\Rightarrow \frac{KE_\alpha}{KE_\alpha + KE_\alpha} = \frac{m_D}{m_D + m_\alpha} \Rightarrow \frac{KE_\alpha}{Q} = \frac{A-4}{A}$$

$$\Rightarrow KE_{\alpha\text{-particle}} = \frac{(A-4)Q}{A} \text{ and}$$

Similarly,

$$KE_{\text{Daughter nucleus}} = \frac{4Q}{A}$$

An alpha particle is a helium nucleus $^{4}_{2}He$, having two protons and two neutrons, having total effective spin zero.

The $KE_{\alpha \text{ particle}}$ in α-decay is usually about 5 MeV.

In the alpha decay of a particular nuclide, every emitted alpha particle has the same sharply defined kinetic energy.

Beta decay

In beta decay, either a neutron is converted into a proton or a proton is converted into a neutron.

Recall the N versus Z graph and the stable line/region. A nucleus to the left of the line has more neutrons. Such a nucleus will decay by $n \rightarrow p$ conversion for more stability.

A nucleus to the right of the stable line has more protons, and, therefore, $p \rightarrow n$ conversion takes place for stability.

To avoid violation of consumption of energy and linear and angular momenta existence of a particle neutrino (ν) and its anti particle antineutrino ($\bar{\nu}$) is required.

$$\underset{\text{neutron}}{n} \rightarrow \overset{\text{proton}}{p} + \underset{\text{electron}}{e^{-}} + \underset{\text{antineutrino}}{\bar{\nu}}$$

An example of this beta minus decay is

$$_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + e + \bar{\nu} + \text{energy (Q value)}.$$

$$_{19}^{40}K \rightarrow {}_{20}^{40}Ca + e^{-} + \bar{\nu}$$

Antineutrino has zero rest mass (like a photon) and is uncharged. The electron is also sometimes written as β^-. Here the Q value is shared by β^- and $\bar{\nu}$.

Similarly,

$$p \rightarrow n + \underset{\text{positron}}{e^{+}} + \overset{\text{neutrino}}{\nu}$$

An example of this beta plus decay is (Beta plus decay does not occur naturally. They are produced in artificial radioactivity only)

$$_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + e^{+} + \nu$$

The particle ν is called neutrino

$$_{11}^{22}Na \rightarrow {}_{10}^{22}Ne + e^{+} + \nu$$

Unlike in alpha decay, in the beta decay of a nuclide, the disintegration energy Q is shared between the decay products, the daughter nucleus, electron or positron and the antineutrino or neutrino.

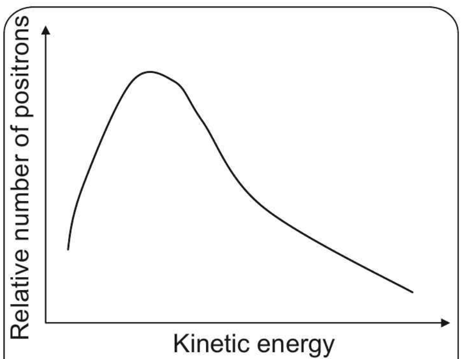


Fig. 2.19

Therefore, the kinetic energy of an electron or a positron in a beta decay process can vary from zero to a maximum value Q as shown in Fig. 2.19.

Note: There is also another method of β decay, called 'electron capture', which is beyond the scope of discussion of this book.

Gamma decay

The protons and neutrons in the nucleus exist in **ecquerel** energy states–ground and higher energy states.

Whenever an alpha or beta decay takes place the daughter nucleus is usually in a higher energy state. Therefore in a short time it comes to the ground state by emitting gamma radiation, of very short wavelength and very large energy, of the order of MeV. In γ emission, no change in A, Z or N occurs. Only the quantum states of nucleons change.

γ decay is always associated with α or β decay.

Law of radioactive decay

The number of radioactive nuclei in any sample of radioactive material decreases continuously as some of the nuclei disintegrate. The rate at which the number decreases, however, varies widely for different nuclei.

Let N represent the number of radioactive nuclei in a sample at time t and dN, the number that undergo transformation in a short interval of time dt.

Therefore, the change in N is –dN, the negative sign indicating decrease in the number N, and the rate of change of N is $-\frac{dN}{dt}$. The larger the number of nuclei in the sample, the larger will be the number that will undergo transformations, so that the rate of change of N is equal to a constant λ multiplied by N.

$$\therefore \frac{dN}{dt} = -\lambda N$$

$$\Rightarrow \frac{dN}{N} = -\lambda dt$$

Integrating from t = 0, at which time the number of nuclei was N_0, to the time t, when the number has reduced to N,

$$\Rightarrow \ell n N\Big|_{N_0}^{N} = -\lambda t\Big|_{0}^{t} \Rightarrow$$

$$N = N_0 e^{-\lambda t}$$

λ is called ***decay constant*** also called ***disintegration constant***, which is a measure of how fast the nuclide decays. The SI unit of λ is per second (s^{-1}).

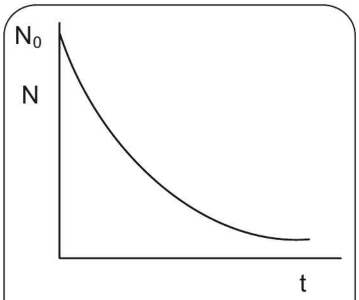


Fig. 2.20

The quantity $\left(-\dfrac{dN}{dt}\right)$ gives the number of decay events per second and is called the ***activity*** of the sample.

Thus activity

$$A = \lambda N = \lambda N_0 e^{-\lambda t}$$

$$(\lambda N_0)\, e^{-\lambda t} = A_0 e^{-\lambda t}$$

where,

A_0 = activity at t = 0

$$A = A_0 e^{-\lambda t}$$

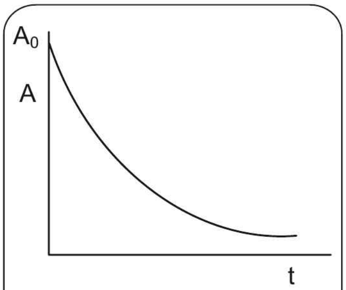


Fig. 2.21

SI unit of activity (disintegration per second) is called ***becquerel*** (symbol ***Bq***). A larger unit is the ***curie*** (***Ci***). 1 curie = 3.7×10^{10} disintegration/second.

Specific activity defines the activity per unit mass.

1 becquerel = 1 disintegration/second

1 rutherford = 10^6 disintegration/second = 10^6 becquerel

1 curie = 3.7×10^{10} disintegration/second = 3.7×10^{10} becquerel

Half life

The time elapsed during which half the active nuclei decay is called ***half life***

(Denoted by $t_{1/2}$)

$$\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda t_{\frac{1}{2}}}$$

$$\Rightarrow \quad t_{\frac{1}{2}} = \frac{\ell n 2}{\lambda} = \frac{0.693}{\lambda}$$

Since $\quad t_{\frac{1}{2}} = \dfrac{\ell n 2}{\lambda} \Rightarrow \lambda = \dfrac{\ell n 2}{t_{\frac{1}{2}}}$

$$N = N_0 e^{-\lambda t} = N_0 e^{-(\ell n 2)\frac{t}{t_{1/2}}}$$

$$\Rightarrow \frac{N}{N_0} = \left(e^{-(\ell n 2)}\right)^{\frac{t}{t_{\frac{1}{2}}}} = \frac{1}{\left(e^{\ell n 2}\right)^{\frac{t}{t_{\frac{1}{2}}}}}$$

$$= \frac{1}{2^{\left(t/t_{1/2}\right)}}$$

$$\Rightarrow N = \frac{N_0}{2^{\left(t/t_{1/2}\right)}}$$

i.e., $\quad \dfrac{N}{N_0} = \left(\dfrac{1}{2}\right)^{\frac{t}{t_{1/2}}}$

Similarly, activity

$$A = \frac{A_0}{2^{\left(t/t_{1/2}\right)}}$$

or

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

Mean life (Average life)

Consider a sample containing N_0 radioactive nuclides at t = 0. The number. of nuclei which decay between t and t + dt is $\lambda N dt$. The life of these nuclei is t and they decay between t and (t + dt).

∴ Sum of their lives is $(\lambda N\, dt)t$.

∴ Sum of the lives of all the N_0 nuclei is $\int_0^\infty t\lambda N dt$

$$= \int_0^\infty t \cdot \lambda \cdot N_0 e^{-\lambda t} dt$$

$$= \lambda N_0 \int_0^\infty t \cdot e^{-\lambda t} dt = -N_0 \int_0^\infty t\, d\left(e^{-\lambda t}\right) = \frac{N_0}{\lambda}$$

$\therefore$ Average life of the nuclei is

$$\frac{\left(\frac{N_0}{\lambda}\right)}{N_0}=\frac{1}{\lambda} \qquad \Rightarrow \tau = t_{ave}=\frac{1}{\lambda}$$

or

$$t_{ave}=\frac{t_{\frac{1}{2}}}{0.693}=1.44t_{1/2}$$

The decay constant can also be viewed as the probability per unit time for a radioactive nucleus to disintegrate, i.e., the probability that a nucleus will disintegrate in the next one second. This is because, by definition, probability is

$$\frac{\text{number off avourable events}}{\text{population}}$$

$$=\frac{\text{number of nuclei decaying}}{\text{total number of nuclei}}$$

Number. of nuclei decaying in the next dt seconds $= -dN = \lambda N dt$

$\therefore$ Probability of the nucleus decaying in the next dt seconds $= -\dfrac{dN}{N} = \lambda dt$

$\therefore$ Probability in the next one second $=\dfrac{\lambda dt}{dt}=\lambda$

Calculation of age of a specimen

Radioactive Dating

The relative abundance of a radioactive material can be used to establish the age of a sample

Dating by Carbon

Cosmic rays coming from outer space produce radioactive $^{14}C.\rightarrow^{14}_{7}N + ^{1}_{0}n \rightarrow^{14}_{6}C + ^{1}_{1}H$. In atmosphere as well as in living plants the ration of ^{12}C to ^{14}C is constant $\cong 10^{12}$: 1. But in a fossil as there is no fresh intake of carbon, intial ratio is changed, because ^{14}C continues to decay. $^{14}_{6}C = ^{14}_{7}N + \overline{\beta} + \overline{v}$. This is evidenced by reduced radioactivity due to ^{14}C is fossils. If its present activity in sample size of carbon is R its original activity R_0 nis established as the present activity in the sample size of carbon, from a living plant its age is established from relation $R = R_0 e^{-\lambda t}$

$$\Rightarrow t=\frac{1}{\lambda}\ell n\frac{R_0}{R}=\frac{T_{1/2}}{\ell_n 2},\ell n\frac{R_0}{R}.$$

In other words, if a carbon specimen of a fossil contains only $\frac{1}{n}$ as much ^{14}C as in an equal specimen of carbon from a presently living plant, then

$$N = N_0 e^{-\lambda t} \Rightarrow \frac{N}{N_0}=\frac{1}{n} \text{ and } t=\frac{T_{1/2}}{\ell n2},\ell n\frac{N_0}{N}.$$

CONCEPT STRANDS

Concept Strand 17

The disintegration rate of a certain radioactive sample at an instant is 4750 disintegrations per minute. After 5 minute, it is 2700 per min

Calculate

(i) decay-constant of the material.
(ii) half life of the sample ($\log_{10}1.760 = 0.2455$)

Solution

(i) $A = A_0 e^{-\lambda t}$

$2700 = 4750\ e^{-\lambda.5}$

$$\Rightarrow 5\lambda = \ell n\frac{4750}{2700} = \ell n1.76$$

$$= 2.303 \times 0.2455 = 0.565 \text{ min}^{-1}$$

$$\Rightarrow \lambda = \frac{0.565}{5} = 0.113 \text{ min}^{-1}$$

(ii) $t_{\frac{1}{2}} = \dfrac{0.693}{\lambda} = \dfrac{0.693}{0.113} = 6.13$ min

Concept Strand 18

Show that the disintegration constant is the reciprocal of time during which the number of atoms of a radioactive substance becomes $\frac{1}{e}$ of its original value.

Solution

Average life is $t_{ave} = \dfrac{1}{\lambda}$

$$N = N_0 e^{-\lambda t_{ave}} = N_0 e^{-\lambda\frac{1}{\lambda}} = N_0 e^{-1}$$

So average life is the time when N becomes $\left(\frac{1}{e}\right)$ times N_0

SUMMARY

$v = \dfrac{1}{\sqrt{\mu \in}}$	$v \rightarrow$ Velocity of electromagnetic wave through a medium $\mu \rightarrow$ Permeability of the medium $\in \rightarrow$ Permittivity of the medium
$U = \dfrac{1}{2} \in_0 E^2 + \dfrac{B^2}{2\mu_0}$	$U \rightarrow$ Energy density of electromagnetic waves
$E = vB$	$E \rightarrow$ Intensity of electric field $B \rightarrow$ Intensity of magnetic field $v \rightarrow$ Velocity of electromagnetic wave
$\dfrac{e}{m} = \dfrac{E^2}{2VB^2}$ (Thomsons experiment)	$e \rightarrow$ charge of electron $m \rightarrow$ mass of electron $E \rightarrow$ electric field applied $B \rightarrow$ magnetic field applied $V \rightarrow$ potential difference through which electron is accelerated
$q = \dfrac{mgd}{V}$ Millican's oil-drop experiment	$q \rightarrow$ charge of oil drop $m \rightarrow$ mass of oil drop $V \rightarrow$ potential difference applied across two plates of arrangement $d \rightarrow$ separation between plates
$E = h\nu = \dfrac{hc}{\lambda}$	$E \rightarrow$ Energy associated with a photon $h \rightarrow$ Planck's constant $\nu \rightarrow$ frequency of radiation $\lambda \rightarrow$ wave length of radiation
$(KE)max = \dfrac{1}{2}mv_{max}^2 = h(\nu - \nu_0)$	$(KE)_{max} \rightarrow$ maximum kinetic energy of emitted photo-electrons $\nu \rightarrow$ frequency of incident radiation $\nu_0 \rightarrow$ threshold frequency $m \rightarrow$ mass of electron
$\phi = h\nu_0$	$\nu_0 \rightarrow$ threshold frequency $\phi \rightarrow$ work function
$(KE)max = eV_s$	$e \rightarrow$ charge of electron $V_s \rightarrow$ stopping potential
$\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$	$\lambda \rightarrow$ wavelength of particle $p \rightarrow$ momentum of particle

$\lambda' - \lambda = \Delta\lambda = \frac{h}{m_0 c} 1 - (\cos\phi)$	$\Delta\lambda \rightarrow$ wavelength shift $\lambda' \rightarrow$ wavelength of scattered photon $\lambda \rightarrow$ wavelength of incident photon m $\rightarrow$ rest mass of electron $\phi \rightarrow$ angle of scattering
$\lambda = \frac{h}{\sqrt{2mqV}}$	$\lambda \rightarrow$ wavelength associated with a particle of charge 'q' and mass 'm' which is accelerated through a potential of 'V'
$r_0 = \frac{1}{4\pi \in_0} \frac{2Ze^2}{\left(\frac{1}{2}mv^2\right)}$	$r_0 \rightarrow$ Distance fo closest aprroach Z $\rightarrow$ atomic number E $\rightarrow$ charge of electron v $\rightarrow$ velocity of the scattering particle m $\rightarrow$ mass of particle
$E_k = \frac{1}{2}mv^2$	$E_k \rightarrow$ Kinetic energy of the scattering particle
$b = \frac{1}{4\pi \in_0} \frac{Ze^2}{\left(\frac{1}{2}mv^2\right)} \cot\left(\frac{\theta}{2}\right)$	b $\rightarrow$ impact paramter 0 $\rightarrow$ angle of scattering
$r_n \propto n^2$	$r_n \rightarrow$ radius of the nth orbit
$v_n = \frac{Zc\alpha}{n}$	$v_n \rightarrow$ velocity of electron in the nth orbit Z $\rightarrow$ atomic number $\propto \rightarrow$ fine structure constant
$\alpha = \frac{e^2}{2\varepsilon_0 ch} = \frac{1}{137}$	$\in_0 \rightarrow$ permittivity of free space
$E_n \propto \frac{1}{n^2}$	$E_n \rightarrow$ energy of electron in the nth orbit
$E_n = \frac{-13.6}{n^2} eV$	$E_n \rightarrow$ energy of electron in the nth orbit of Hydrogen atom
$E_n = \frac{-13.6}{n^2} Z^2 eV$	$E_n \rightarrow$ energy of electron in the nth orbit of Hydrogen like atom
$L_n = \frac{nh}{2\pi}$	$L_n \rightarrow$ angular momentum of an electron in the nth orbit
$\nu_n \propto \frac{1}{n}$	$\nu(nu)_{nu} \rightarrow$ frequency of an electron in the nth orbit

$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$	For Hydrogen $R \rightarrow 1.09 \times 10^7 m^{-1}$ -Rydberg constant
$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$	For Hydrogen like atoms n_1 and n_2 are initial and final states $R \rightarrow$ Rydberg Constant
$R = R_0 A^{1/3}$	$R \rightarrow$ Nuclear radius, R_0 – Radius constant (an empirical constant = 1.2×10^{-15}m $A \rightarrow$ Mass number
$\text{Packing fraction} = \frac{\text{Mass excess}}{\text{Mass number}} = \frac{M - A}{A}$	$M \rightarrow$ Mass of nucleus $A \rightarrow$ Mass number
$\frac{N}{N_0} = e^{-\lambda t}$	$N \rightarrow$ Number of atoms present at any instant $N_0 \rightarrow$ Initial number of atoms $t \rightarrow$ Time taken to reduce the number of atoms from N_0 to N
$T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$	$\lambda \rightarrow$ Decay constant $T_{1/2} \rightarrow$ Half life
$\tau = \frac{1}{\lambda}$	$\lambda \rightarrow$ Decay constant $\tau \rightarrow$ Mean life
$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$	$N \rightarrow$ Number of atoms remaining after n half lives $N_0 \rightarrow$ Total number of atoms at the start $n \rightarrow$ number of half lives $\Rightarrow n = \frac{t}{T_{1/2}}$

CONCEPT CONNECTORS

Connector 1: Find the wavelength that an electromagnetic radiation should have, if a photon in the beam is to have the same momentum as that of an electron moving with a speed of 2×10^5 m s^{-1}.
(Given $m_e = 9.1 \times 10^{-31}$ kg and $h = 6.63 \times 10^{-34}$ J s)

Solution: Momentum of electron = Momentum of photon;

But Momentum of photon = $\frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

i.e., $mv = \frac{h}{\lambda}$

$$\therefore \quad \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2 \times 10^5}$$

$= 3.64 \times 10^{-9}$ m

$= 3.64$ nm

Connector 2: Find the linear momentum and energy of a photon in a beam of light of wavelength 600 nm and intensity 100 W m^{-2}.

Solution: Energy $E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{600 \times 10^{-9}}$

$$= 3.315 \times 10^{-19}\text{J} = \frac{3.315 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.07 \text{ eV}$$

Linear momentum $p = \frac{E}{c} = \frac{3.315 \times 10^{-19}}{3 \times 10^8}$

$= 1.105 \times 10^{-27}$ kg m s^{-1}

Momentum and energy of a photon depend only on its wavelength.

Connector 3: Find the number of photons emitted per second by a 25 W source of monochromatic light of wavelength 6000Å.

Solution: No. of photons $= \frac{P}{hc/\lambda} \left(\because P = \frac{nhc}{\lambda}, n = \frac{P\lambda}{hc} \right)$

$$= \frac{25 \times 6000 \times 10^{-10}}{6.62 \times 10^{-34} \times 3 \times 10^8} = 7.55 \times 10^{19} \text{ per second}$$

Connector 4: What should be the stopping potential to stop the fastest electrons ejected by a Nickel surface (work function 5.0 eV) when it is irradiated with ultraviolet radiation of wavelength 200 nm?

Solution: $eV = \frac{1}{2}mv^2 = h\nu - w_0 = \frac{hc}{\lambda} - w_0$

$$= \left[\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{200 \times 10^{-9} \times 1.6 \times 10^{-19}} - 5 \right] = [6.215 - 5] = 1.215 \text{ eV}$$

Stopping potential = 1.215 V

Alternate method

$$E(eV) = \frac{1240}{\lambda(nm)} = \frac{1240}{200} = 6.2 \text{ eV}$$

$\Rightarrow$ Stopping potential = 6.2 – 5 = 1.2 V

Connector 5: If the wavelength of the incident radiation in a photoelectric cell is reduced from 4000Å to 3600Å, find the reduction in cut-off potential.

Solution: We have $eV_0 = \frac{1}{2}mv_{max}^2 = \frac{hc}{\lambda} - w_0$

$\therefore \quad V_0 = \frac{hc}{\lambda e} - \frac{w_0}{e}$

If the wavelength is reduced from λ_1 to λ_2 change in cut-off potential

$$\Delta V_0 = V_{0\lambda_1} - V_{0\lambda_2}$$

$$= \left(\frac{hc}{\lambda_1 e} - \frac{w_0}{e}\right) - \left[\frac{hc}{\lambda_2 e} - \frac{w_0}{e}\right]$$

$$= \frac{hc}{e}\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = \frac{hc}{e}\left[\frac{\lambda_2 - \lambda_1}{\lambda_1\lambda_2}\right]$$

$$= \frac{(6.63\times10^{-34}\times3\times10^{8})}{1.6\times10^{-19}\times10^{-10}}\left[\frac{4000-3600}{4000\times3600}\right] = 0.345 \text{ V}$$

Alternate method

Reduction in potential depends on change in energy of incident radiation

$$\Rightarrow \quad \Delta V = \frac{1240}{360} - \frac{1240}{400}$$

$$= 1240 \cdot \frac{400 - 360}{400 \cdot 360} = 0.34 \text{ V}$$

Connector 6: Monochromatic light of wavelength 310 nm is falling on an isolated metallic sphere of radius 1.8 cm. Find the number of photoelectrons ejected before the emission of photoelectrons is stopped.

The work function of the metal is 2 eV.

Solution: Since the sphere is isolated, it will become more and more positively charged when more and more electrons are ejected out of it. Due to this if the potential of the sphere is increased to V, the electrons ejected should have a minimum energy of (ϕ + eV)

$\therefore$ Emission will stop when $\frac{hc}{\lambda} = \phi + eV$

$$\text{Energy of photon} = \frac{1240}{\lambda(nm)} = \frac{1240}{310} = 4 \text{ eV}$$

The charge Q on the sphere to bring it to a potential V is given by

$Q = (4\pi\varepsilon_0 r)V$ where V = 4 – 2 = 2 V

$\therefore$ Number electrons emitted $n = \frac{Q}{e} = \frac{4\pi\varepsilon_0 rV}{e}$

$$= \frac{1.8\times10^{-2}\times2}{9\times10^{9}\times1.6\times10^{-19}} = 2.5 \times 10^{7}$$

Connector 7: Light of wavelength 500 nm falls on a metal of work function 1.9 eV. Find

(i) the energy of the photon in eV
(ii) the maximum kinetic energy of the photoelectron emitted and
(iii) the stopping potential

Solution:

(i) $\lambda = 500$ nm

$$E = \frac{1240}{500} = 2.48 \text{ eV}$$

(ii) $(K.E.)_{max} = h\nu - h\nu_0 = 2.48 - 1.9 = 0.58$ eV

(iii) $eV_0 = 0.58$ eV

$\Rightarrow V_0 = 0.58$ V

Connector 8: In a Millikan's oil drop experiment, a charged oil drop of density 0.8 g/cc is held stationary between two parallel plates 10 mm apart held at a potential difference of 100 V. When the electric field is switched off, the drop falls a distance of 1 mm in 25 s after attaining the terminal velocity. Calculate the radius of the drop and the charge on the drop.

Viscosity of air = 1.7×10^{-5} N s m^{-2} and g = 10 m s^{-2}

Solution: When the drop is stationary,

$$qE = mg = \frac{4}{3}\pi r^3 \rho g \quad \text{—(1)}$$

When the electric field is switched off, the terminal velocity, $v = \frac{1}{25} \times 10^{-3}$ m s^{-1}

$$6\pi r\eta v = \frac{4}{3}\pi r^3(\rho - \sigma)g \approx \frac{4}{3}\pi r^3 \rho g \quad \text{(density of air is much smaller than that of oil)}$$

$$r = \sqrt{\frac{9}{2} \times \frac{\eta v}{\rho g}} \quad \text{—(2)}$$

$$= \sqrt{\frac{9 \times 1.7 \times 10^{-5} \times 10^{-3}}{2 \times 800 \times 25 \times 10}} = 6.2 \times 10^{-7} \text{m}$$

To find the charge, $E = \frac{V}{d} = \frac{100}{10 \times 10^{-3}} = 10^4$ V m^{-1}

From equation (1)

$$q = \frac{4}{3}\pi r^3 \rho \frac{g}{E}$$

$$= \frac{4 \times 3.14 \times \left(6.2 \times 10^{-7}\right)^3 \times 800 \times 10}{3 \times 10^4}$$

$$= 7.98 \times 10^{-19} \text{ C}$$

$\Rightarrow$ Charge of the drop is 5e.

Connector 9: Find the wavelength of radiation emitted when He^+ makes a transition from n = 3 to n = 2.

Solution:

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right] = 4R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{9}R \qquad (\because Z = 2 \Rightarrow Z^2 = 4)$$

$$\therefore \quad \lambda = \frac{9}{5R} = \frac{9}{5 \times 1.09737 \times 10^7 \text{ m}^{-1}} = 164.0 \times 10^{-9} \text{ m} = 164 \text{ nm}$$

Connector 10: If the excitation energy of a hydrogen like ion in its first excited state is 40.8 eV, find the energy required to remove the electron from the ground state of the ion.

Solution:

$$E_n = \frac{-Z^2\, 13.6}{n^2}$$

$$E_{21} = Z^2 \times 13.6\,(1 - \frac{1}{2^2}) = \frac{3}{4} \times 13.6\, Z^2 = 40.8 \text{ eV}$$

$$E_1 = Z^2 \times 13.6\,(1 - \frac{1}{\infty}) \Rightarrow E_1 = \frac{4}{3} \times 40.8 = 54.4 \text{ eV}$$

Connector 11: The electron in a hydrogen atom makes a transition from a higher energy state to the state with n = 2. Find the longest wavelength of the radiation emitted.

Solution: The longest wavelength corresponds to the lowest energy of transition.

i.e., from $n = 3$ to $n = 2$

Energy $E_2 = -\frac{13.6}{2^2} = -3.4 \text{ eV}$

Energy $E_3 = -\frac{13.6}{3^2}\text{eV} = -1.5 \text{ eV}$

The longest wavelength

$$\lambda_{max} = \frac{hc}{\Delta E} = \frac{1242\,\text{eV nm}}{(3.4 \text{ eV} - 1.5 \text{ eV})} = 654 \text{ nm}$$

Connector 12: Find the magnetic field at the centre of the nucleus of a hydrogen atom in the ground state in terms of the fundamental constants.

Solution: We can write

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2} \quad \text{—(1)}$$

Based on Bohr's quantisation, in ground state $[L = mvr = \frac{h}{2\pi}]$

$$\Rightarrow \quad vr = \frac{h}{2\pi m} \quad \text{—(2)}$$

From (1) and (2)

$$v = \frac{e^2}{2\varepsilon_0 h} \quad \text{—(3) and}$$

$$r = \frac{\varepsilon_0 h^2}{\pi m e^2} \quad \text{—(4)}$$

Since the electron moves along a circular path, it crosses any point on the circle $\frac{v}{2\pi r}$ times per second.

$\therefore$ current, or charge crossing per unit time, $i = \frac{ev}{2\pi r}$

Also magnetic field at the centre of a circle of radius 'r' is given by

$$B = \frac{\mu_0 i}{2r} = \frac{\mu_0 ev}{2r \times 2\pi r} = \frac{\mu_0 ev}{4\pi r^2}$$

on substituting for v and r

We can write $B = \dfrac{\mu_0 e^7 \pi m^2}{8\varepsilon_0{}^3 h^5} = \dfrac{\pi \mu_0 m^2 e^7}{8\varepsilon_0^3 h^5}$

Connector 13: Electrons in a Li^{++} atom make transitions from 5th to 4th orbit and 4th to 3rd orbit. The resulting radiations are incident on a metal plate and eject photoelectrons. The stopping potential for the shorter wavelength is 3.95 V. Calculate the work function of the metal and the stopping potential for the longer wavelength.

Solution: $\Delta E = Z^2 .13.6\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right)$

For shorter wavelength $n_f = 3$, $n_i = 4$, $Z = 3$ for Li

$$\Delta E = 3^2 \times 13.6\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$= 5.95 \text{ eV} = h\nu$$

$$h\nu = eV_0 + \phi_0$$

$$\phi_0 = h\nu - eV_0$$

$$= 5.95 - 3.95 = 2 \text{ eV}$$

For the longer wavelength, $n_f = 4$, $n_i = 5$

$$\Delta E = 3^2 \times 13.6\left(\frac{1}{4^2} - \frac{1}{5^2}\right)$$

$$= 2.75 \text{ eV} = h\nu$$

$$eV_0 = h\nu - \phi_0 = 2.75 - 2 = 0.75 \text{ eV}$$

$$V_0 = 0.75 \text{ V}$$

Connector 14: The wavelength of the second member of Balmer series of hydrogen is 4861Å. Calculate the longest wavelength in Lyman series.

Solution: For Balmer series, for 2nd member

$$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = R\frac{3}{16}$$

For Lyman series, longest wavelength

$$\frac{1}{\lambda_2} = R\left(1 - \frac{1}{2^2}\right) = \frac{3}{4}R$$

$$\frac{\lambda_2}{\lambda_1} = R \times \frac{3}{16} \times \frac{4}{3R} \Rightarrow \lambda_2 = \frac{\lambda_1}{4} = 1215\text{Å}$$

Connector 15: A 1 MeV proton is fired towards a gold leaf (Z = 79). Calculate the distance of closest approach for a head on collision with a gold atom of the target.

Solution: The condition for closest approach is $\dfrac{1}{2}mv^2 = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Ze^2}{r_0} = e \times 10^6$

($\because$ 1 MeV $= 10^6 \times 1.6 \times 10^{-19}$ J $= e \times 10^6$ J)

$$\Rightarrow r_0 = \frac{9 \times 10^9 \times 79 \times 1.6 \times 10^{-19}}{10^6} = 1.14 \times 10^{-13} \text{ m}$$

Connector 16: Which state of the electron in a triply ionized beryllium atom has the same orbital radius as that of an electron in a hydrogen atom in the ground state?

Solution: $r = \frac{n^2}{Z}a_H \Rightarrow \quad r_{Be} = \frac{n^2}{4}a_H \qquad (\because Z = 4 \text{ for Be})$

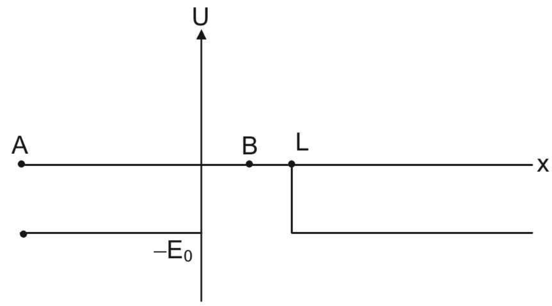
$r_{Be} = a_H$ for $n = 2$

Connector 17: A particle of mass m starting from position A with speed v_0 has potential energy as shown.

Find

(i) the time taken to move from x = 0 to x = L.

(ii) $\frac{\lambda_A}{\lambda_B}$.



Solution: (i) $\frac{p_A^2}{2m} - E_0 = E = \frac{p_B^2}{2m}$

$$p_B = \sqrt{p_A^2 - 2mE_0}$$

$$\Rightarrow \quad v_B = \frac{1}{m}\sqrt{p_A^2 - 2mE_0}$$

$$\Rightarrow \quad t = \frac{L}{v_B} = \frac{mL}{\sqrt{p_A^2 - 2mE_0}} = \frac{L}{\sqrt{v_0^2 - \frac{2E_0}{m}}} \qquad (\because p_A = mv_0)$$

(ii) $\lambda_A = \frac{h}{p_A}$

$$\lambda_B = \frac{h}{p_B}$$

$$\frac{\lambda_A}{\lambda_B} = \frac{p_B}{p_A} = \sqrt{1 - \frac{2mE_0}{p_A^2}} = \sqrt{1 - \frac{2E_0}{mv_0^2}}$$

Connector 18: An X-ray tube operated at 30 kV emits a continuous X-ray spectrum with a short wavelength limit 0.414Å. Calculate the value of Planck's constant.

Solution: $eV = h\nu = \frac{hc}{\lambda}$

$$h = \frac{eV\lambda}{c} = \frac{1.6\times10^{-19}\times30000\times0.414\times10^{-10}}{3\times10^{8}} = 6.62\times10^{-34}\text{ J s}$$

Connector 19: Metal A(Z = 79) has K_α X-ray of wavelength 0.2Å. An impurity element B added to metal A gives an additional faint K_α X-ray of wavelength 1.54Å. Calculate (i) Moseley's constant (ii) the atomic number of the impurity element.

Solution: Moseley's law states

$$\sqrt{f} = \sqrt{\frac{c}{\lambda}} = a\,(Z - b) = a\,(Z - 1) \text{ for } K_\alpha \text{ lines.}$$

where f : frequency and λ : wavelength of X-ray K_α line; c : velocity of light;

a = Moseley's constant and Z : atomic number of element.

(i) For metal A;

$$\sqrt{\frac{c}{\lambda}} = \sqrt{\frac{3\times10^{8}}{0.2\times10^{-10}}} = a(79-1)$$

$\Rightarrow$ $a = 4.96 \times 10^{7}\ s^{-1/2}$

(ii) for impurity $\frac{\lambda_{imp}}{\lambda_A} = \frac{1.54}{0.2} = 7.7 = \frac{(79-1)^2}{(Z_{imp}-1)^2} \Rightarrow Z_{imp} = 29$

Connector 20: A stationary nucleus A = 238 decays by emitting an α-particle. If the total energy released is 5.5 MeV, find the velocity of the α-particle.

Solution:

$$(K.E.)_\alpha = \frac{A-4}{A}\times Q \quad (\because Q = 5.5\text{ MeV} = 5.5 \times 10^{6} \times 1.6 \times 10^{-19}\text{ J})$$

$$= \frac{238-4}{238} \times 5.5 \times 10^{6} \times 1.6 \times 10^{-19}$$

$$= 8.652 \times 10^{-13}\text{ J} = \frac{1}{2}m_\alpha v_\alpha^{\ 2} = 2\ m_p\ v_\alpha^{\ 2} \quad (\because\ m_\alpha = 4\ m_p)$$

$$\Rightarrow \quad v_\alpha = \sqrt{\frac{8.652\times10^{-13}}{2\times1.66\times10^{-27}}} = 1.6 \times 10^{7}\text{ m s}^{-1}$$

Connector 21: The binding energy of ${}^{20}_{10}Ne$ is 160.6 MeV. Calculate the atomic mass. Given m_H = 1.007825 a.m.u. m_n = 1.008665 a.m.u

Solution:

$$B.E = \left[Zm_p + (A-Z)m_n - m_N\right]c^2 \quad [\text{where } m_N \rightarrow \text{mass of nucleus}]$$

$$= \left[Zm_p + (A-Z)m_n - (m - Zm_e)\right]c^2$$

$$= \left[Z(m_p + m_e) + (A-Z)m_n - m\right]c^2$$

$$= \left[Zm_H + (A-Z)m_n - m\right]c^2$$

$160.6 = [\ 10 \times 1.007825 + 10 \times 1.008665 - m]\ 931$

$$\Rightarrow m = 20.1649 - \frac{160.6}{931} = 19.992\text{ a.m.u.}$$

Connector 22: The binding energy per nucleon for deuterium and helium are respectively 1.1 MeV and 7.1 MeV. Calculate the energy released when two ${}^{2}_{1}H$ fuse to form ${}^{4}_{2}He$.

Solution: $2\,{}^{2}_{1}H \rightarrow {}^{4}_{2}He$

The energy released = 4(7.1) – 2(1.1 $\times$ 2) = 24.0 MeV

Connector 23: The half life of U^{238} for α-decay is 1.42×10^{17}s. How many disintegrations per second occur in 1 g of U^{238}?

Solution: $t_{1/2} = 1.42 \times 10^{17}$ s

$$\lambda = \frac{0.693}{1.42\times10^{17}} = 4.88 \times 10^{-18}\ s^{-1} \quad \left(\because \lambda = \frac{0.693}{t_{1/2}}\right)$$

$$\left|\frac{dN}{dt}\right| = \lambda N = 4.88 \times 10^{-18} \times \frac{6.023\times10^{23}}{238}$$

$$= 1.23 \times 10^{4}\text{ disintegrations/s}$$

Connector 24: The half life of a particular material is 2 years. It is to be used as fuel for a space probe. If the mission is to last 6 years, find the amount of energy available. Initial mass of fuel is 5 mg, energy released per reaction is 200 MeV and the atomic mass of element is 250.

Solution:: Number of atoms decayed = $N_0 (1 - e^{-\lambda t})$

6 years = 3 half lives

$$\Rightarrow \text{ Number of atoms decayed} = N_0\left[1-\left(\frac{1}{2}\right)^3\right] = \left[1-\frac{1}{8}\right]N_0 = \frac{7N_0}{8}$$

$$\therefore \quad 5 \times 10^{-3} \times \frac{7}{8}\times\frac{1}{250} \times 6.023 \times 10^{23} \text{ disintegrations}$$

$$\text{Energy} = 5 \times 10^{-3} \times \frac{7}{2000} \times 6.023 \times 10^{23} \times 200 \times 1.6 \times 10^{-13} = 3.37 \times 10^{8}\text{ J}$$

Connector 25: In an experiment on two radioactive isotopes A and B of an element, their mass ratio at a given instant is found to be 3. The half life of A is known to be 12 hours and that of B 16 hours. A has larger mass and it has an activity of 1 μCi initially. What would be the activity of each isotope and their mass ratio after two days?

Solution:

$$A = \left|\frac{dN}{dt}\right| = \lambda N$$

$A_1 = \lambda_1 N_1$,

$A_2 = \lambda_2 N_2$

$$\frac{N_1}{N_2} = 3,$$

$T_1 = 12$ hour, $T_2 = 16$ hour

$$\text{After two days, } N_1' = N_1\left(\frac{1}{2}\right)^{\frac{t}{T_1}} = N_1\left(\frac{1}{2}\right)^4$$

$$N_2' = N_2\left(\frac{1}{2}\right)^3$$

$$\frac{N_1'}{N_2'} = \frac{\frac{N_1}{16}}{\frac{N_2}{8}} = \frac{N_1}{N_2}\times\frac{8}{16} = 3\times\frac{1}{2} = \frac{3}{2}$$

$A_1 = 1$ μCi

$$A_1' = \lambda_1 N_1' = \lambda_1\frac{N_1}{16} = \frac{A_1}{16} = 0.0625\ \mu\text{Ci}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_2} = \frac{3}{4} \qquad \left[\because \lambda = \frac{0.6932}{T_{1/2}}\right]$$

$$A'_2 = \lambda_2 N_2' = \left(\frac{3}{4}\lambda_1\right)\left(\frac{2}{3}N_1'\right) = \frac{1}{2}\lambda_1 N'_1$$

$$= \frac{1}{2}A_1' = 0.03125\ \mu\text{Ci}$$

TOPIC GRIP

Subjective Questions

1. Light emitted by de-excitation of singly ionised He atoms falls on the surface of a metal whose threshold frequency is 5×10^{14} Hz. The stopping potential is found to be 0.57 V. Determine
   (i) the quantum numbers of the two levels involved in the emission of these photons.
   (ii) change in angular momentum of the electron.
   Ionisation potential of H = 13.6 V
   Planck's constant = 4.14×10^{-15} eV s = 6.6×10^{-34} Js

2. A charged particle accelerated by a potential difference of 200 V has a de Broglie wavelength of 0.002 nm. Find the mass of this particle if its charge is numerically twice that of an electron.

3. If the wavelength of the first member of Balmer series in hydrogen spectrum is 656 nm, what is the wavelength of the second member of Lyman series?

4. A fast moving neutron is slowed down to $\frac{1}{3}$ of its speed by head on collision with a stationary doubly ionised ^{7}Li atom which subsequently emits three spectral lines. What was the original speed of the neutron?

$$e = 1.6 \times 10^{-19}\ \text{C},\ m_p = m_n = \frac{5}{3} \times 10^{-27}\ \text{kg, Ionization potential of hydrogen} = 13.6\ \text{V}$$

(Ignore subsequent collisions)

5. Electrons accelerated by 0.82 V enter a region containing α particles. The electrons are captured by some alpha particles. Each capture results in the immediate emission of a photon and formation of a He$^+$ ion in the nth state. It subsequently emits radiation, which has 10 different wavelengths of photons. What is the wavelength of the first photon emitted? What are the minimum and maximum wavelengths of the photons?

6. A hydrogen like ion in a particular initial state collided with a fast neutron, received 10.2 eV energy and thereby the de-Broglie wavelength of its electron doubled. Its subsequent emission spectrum contained more than one line, with the longest wavelength in the visible region.
   Determine
   (i) the element,
   (ii) the number of lines in the spectrum,
   (iii) the shortest wavelength.

   (Ionization energy of H = 13.6 eV, $R = \frac{72}{7} \times 10^6\,\text{m}^{-1}$)

7. Determine the screening constant (factor b of Moseley's law) for the L-series of X-rays, given that X-rays of wavelength $\lambda = 0.14$ nm are emitted when an electron in a tungsten (Z = 74) atom is transferred from the M shell to the L shell. Also, determine the wavelength of the L_β line for Tungsten.

8. The following reaction is used in a nuclear reactor.

$$^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He}$$

Half life of ^{210}Po = 140 day
Assume that the efficiency of power generation is 20%.

Given ^{210}Po = 209.98264 amu

$^{206}Pb = 205.97440$ amu

$^{4}He = 4.00260$ amu

$\ell n\, 2 = 0.7$

Calculate

(i) the amount of ^{210}Po required to produce 6×10^6 joule of electrical energy per day.

(ii) the activity of the above material 70 days earlier.

9. A sample contains two radioactive isotopes decaying into two different daughter nuclei with half lives of 10 hour and 12 hour respectively. Initially their mass ratio is 2.2:1 and activity of the first one is 3.84×10^3 Bq. The ratio of the atomic masses of the two isotopes is 1.1:1. What would be their activities and mass ratio after 60 hour?

10. A decays into B with decay constant λ. A is produced at a constant rate of α nuclei per second. If, at $t = 0$, the population of A is N_0 and the population of B is zero, determine the populations N_A and N_B as functions of t.

Straight Objective Type Questions

Directions: **This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

11. Light of frequency x . 10^{15} H_Z falling on a metal ejects electrons that are fully stopped by a negative potential of 3.96 V. Photoemission begins in this metal when the frequency of incident light is $6.4 \times 10^{14}\, s^{-1}$ Then x is ($h = 6.6 \times 10^{-34}$ J s)

(a) 2.2 (b) 1.8 (c) 1.6 (d) 1.1

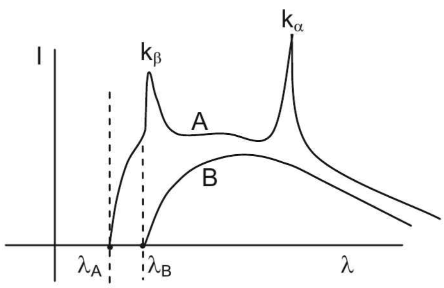
12. An α particle moves along a circle with radius 1.25 cm in a uniform magnetic field of intensity 0.1T. Its de Broglie wavelength in picometre is:

(a) 1.65 (b) 2.2 (c) 3.1 (d) 3.8

13. The maximum wavelength that will be emitted when electrons accelerated through a potential of 12.4 V collide with hydrogen atoms in the ground state, is (inÅ)

(a) 6545 (b) 4275 (c) 1212 (d) 1023

14.



The intensity of emitted X-ray versus wavelength for two different accelerating voltages for a particular target are as shown. In case of radiation pattern A, the accelerating voltage is just sufficient to establish characteristic k series of the element. Radiation pattern B starts at exactly $k_{\beta-}$ line wavelength of the element. Then the ratio of minimum wavelengths of the patterns $\frac{\lambda_B}{\lambda_A}$ is

(a) $\frac{9}{8}$ (b) $\frac{4}{3}$ (c) 2 (d) $\frac{5}{4}$

15. The binding energy per nucleon for $^{31}_{15}P$, given that its mass is 30.973763 amu.

($m_{proton} = 1.007825$ amu. $m_{neutron} = 1.008665$ amu) (in MeV) is:

(a) 6.24 (b) 7.35 (c) 9.51 (d) 8.47

Assertion–Reason Type Questions

Directions: **Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

16. Statement 1

Other conditions of the photoelectric experiment remaining same, increase in intensiy of incident light will increase photocurrent.

and

Statement 2

As incident energy per unit area per unit time increases, emitted photoelectrons will travel faster to the collector causing increased $\frac{dq}{dt}$.

17. Statement 1

If an atom absorbs a photon of certain wavelength and immediately emits another photon of a different wavelength, the net energy absorbed by the atom in the process is equal to the energy of a photon of wavelength equal to the difference in the two above mentioned wavelengths.

and

Statement 2

An atom gets to an excited state by absorbing energy and falls to a lower state by emitting energy.

18. Statement 1

In a typical X-ray spectrum, the cut off wavelength λ_{min}, is caused when an electron is completely stopped by a collision or a series of collisions with target atom/atoms.

and

Statement 2

The kinetic energy lost by an electron appears as X radiation.

19. Statement 1

If K_α line is present in the X-ray spectrum, all characteristic L series lines will also be present.

and

Statement 2

An X-ray spectrum will have all wavelengths more than λ_{min}.

20. Statement 1

The mass of a β-ray electron can be much greater than that of a cathode ray electron.

and

Statement 2

The maximum velocity of a β-ray electron is comparable to the velocity of light whereas that of a cathode ray electron is much less.

Linked Comprehension Type Questions

Directions: **This section contains a paragraph. Based upon the paragraph, multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

Passage I

X-Ray Absorption

X-rays have the capacity to penetrate materials that are opaque to visible or ultraviolet light. An X-ray photon loses its energy by its interaction with a deep electron in the K or L-shell and ejects it from the atom.

A high energy X-ray photon loses only a part of its energy in one such encounter and this accounts for the great penetrating ability of high energy (short wavelength) X-ray photons.

If a well-defined X-ray beam is made to fall upon a certain material, say a sheet of aluminium, the intensity of the X-ray beam must evidently decreases during its passage through the sheet.

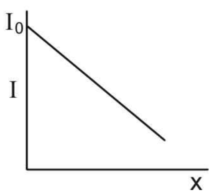
Let I_0 be the initial intensity of the beam incident normally upon the absorber and I its intensity after it has traversed a distance x through the absorber.

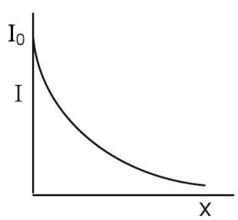
Let dI be the further small decrease in intensity across a small thickness dx. $\frac{dI}{dx}$ is found to be proportional to the Intensity I; or $\frac{dI}{dx} = -\mu I$, where μ is a constant called the linear absorption co-efficient of the material and depends upon the X-ray wavelength and the nature of the absorbing material. The negative sign indicates that I decreases as x increases.

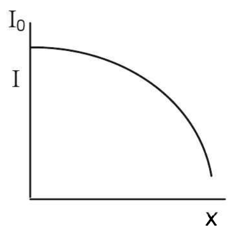
21. The unit of μ is
(a) metre (b) metre^{-1} (c) joule^{-1}metre^{-1} s (d) joule^{-1}metre^{-3} s

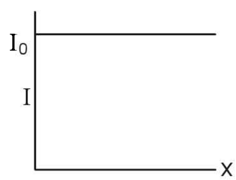
22. The linear absorption coefficient
(a) increases with increasing density.
(b) increases with decreasing density.
(c) first increases and then decreases with increasing density.
(d) is independent of density.

23. The variation of I with depth of penetration (distance x) is best represented by

(a) 

(b) 

(c) 

(d) 

Passage II

A sample of uranium ore contains $^{235}_{92}U$ and $^{238}_{92}U$. Analysis shows that the ore contains 0.80 g of $^{206}_{82}Pb$ for each gram of the relevant uranium isotope. By proper nucleon number analysis it can be proved that $^{238}_{92}U$ is the originator of the series that terminates at the stable product $^{206}_{82}Pb$. It is intended to find the age of the ore, the present quantity of $^{235}_{92}U$ and the present activity due to $^{235}_{92}U$. The decay constants of $^{235}_{92}U$ and $^{238}_{92}U$ are λ_1 and λ_2, where $\lambda_1 = 3 \times 10^{-17}$ s^{-1} and $\lambda_2 = 4.87 \times 10^{-18}$ s^{-1} respectively.

24. Let t be the time since the formation of the ore and N_0 the number of ^{238}U atoms. Then at time t, the number of ^{206}Pb atoms will be.

(a) $N_0\, e^{-\lambda_1 t}$ (b) $N_0 e^{-\lambda_2 t}$ (c) $N_0\left(1 - e^{-\lambda_1 t}\right)$ (d) $N_0\left(1 - e^{-\lambda_2 t}\right)$

25. t is (in year)

(a) 10^9 (b) 10^7 (c) 10^{11} (d) 10^{13}

26. If the sample initially contained 3 mg of ^{235}U, the present amount of mass is $\left(2^{\lambda_1/\lambda_2} = 71.5\right)$

(a) 15 μg (b) 35 μg (c) 42 μg (d) 68 μg

Multiple Correct Objective Type Questions

Directions: **Each question in this section has four suggested answers out of which ONE OR MORE answers is/are correct.**

27. Two hydrogen atoms A and B moving in opposite directions, A with velocity $v_A = 6.1 \times 10^4$ m s^{-1} and B with velocity v_B undergo an inelastic collision. After collision, A comes to halt and is in an excited state below excitation level 4. B is in normal state and proceeds with velocity v'_B. A gives out two radiations λ_1 and λ_2 in succession and comes to the ground state. If $\lambda_1 = 4870$Å, then

(a) $\lambda_2 = 1218$ Å (b) $\lambda_2 = 974$ Å (c) $v_B \simeq 2 \times 10^4$ m s^{-1} (d) $v_B' \simeq 4.1 \times 10^4$ m s^{-1}

28. For molybdenum(Z = 42) and cobalt(Z = 27) consider constant b = 1 as per Moseley's law, for K series and b = 7 for L series, and constant a is same for both series. For molybdenum λ_1 for K_α line of X-ray is 0.71 Å and λ_2 for K_β is 0.63 Å. Then

(a) λ for K_α of cobalt $\simeq$ 1.77Å (b) λ for K_α of cobalt $\simeq$ 1.2Å

(c) λ for L_α of molybdenum $\simeq$ 7.7Å (d) λ for L_α for molybdenum $\simeq$ 8.2Å

29. Two radioactive isotopes in a rock are in the mass ratio of 120 : 1 in the present age, while they were in the mole ratio 1 : 1 at the formation time of the rock. Their mean lives are 5×10^9 year and 1×10^9 year and their atomic masses are in the ratio 1.2 : 1, respectively $\left[\log_{10}^{e} = \dfrac{1}{2.3}\right]$

(a) Age of the rock is 5.75×10^9 year.

(b) Age of the rock is 6.21×10^8 year.

(c) Present activities of the two isotopes are in the ratio 20 : 1

(d) Present activities of the two isotopes are in the ratio 25 : 1

Matrix-Match Type Question

Directions: **Match the elements of Column I to elements of Column II. There can be single or multiple matches.**

30. Given are certain radioactive processes in column I and some of their properties in column II. Match them

Column I	**Column II**
(a) α decay	(p) Energy of particles determined uniquely
(b) β^- decay	(q) Energy of particles can vary
(c) Electron capture	(r) Particle emission is by tunneling across a potential barrier
(d) β^+ decay	(s) Accompanied by the emission of antineutrino, neutrino

IIT ASSIGNMENT EXERCISE

Straight Objective Type Questions

Directions: **This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

31. The dimensional formula for Planck's constant is
(a) ML^2T^{-1} (b) ML^2T^{-2} (c) MLT^{-2} (d) $ML^{-2}T^1$

32. The most efficient electromagnetic radiation for photoelectric emission from Zn surface is
(a) ultraviolet (b) visible (c) infrared (d) microwave

33. Two metals A and B have work functions of 2 eV and 5 eV respectively. When light of wavelength 400 nm is incident, electrons are emitted from
(a) A (b) B (c) Both A and B (d) Neither A nor B

34. In the photoelectric effect, if the intensity of incident light increases, the
(a) K.E. of emitted electrons decreases (b) K.E. of emitted electrons increases
(c) photo electric current increases (d) photo electric current decreases.

35. The speed of the fastest photoelectron emitted from a surface of threshold wavelength 600 nm, when the surface is irradiated by light of wavelength 400 nm is
(a) 9.3×10^5 m s^{-1} (b) 2×10^6 m s^{-1} (c) 2×10^7 m s^{-1} (d) 6×10^5 m s^{-1}

36. The stopping voltage in photoelectric effect depends
(a) only on the frequency of incident photon
(b) only on the intensity of the incident light
(c) only on the nature of the surface of metal
(d) on the frequency of incident light and the nature of the metal

37. For incident frequencies greater than threshold frequency, the number of photoelectrons emitted from a given emitter depends upon the
(a) intensity of incident light (b) frequency of incident light
(c) wavelength of incident light (d) velocity of incident light

38. A monochromatic source of light is placed at a distance d from a metal surface. Photoelectrons are ejected at a rate n and with maximum KE = E. If the source is brought to a distance d/2, the rate and maximum KE of photoelectrons become, respectively,
(a) 2n and E (b) 2n and 2E (c) 4n and E (d) n and 4E

39. Light of frequency 1.9 times the threshold frequency is incident on photo-sensitive material. If the frequency is halved, and intensity is doubled, the photoelectric current becomes
(a) double (b) zero (c) quadrupled (d) half

40. The experiment which established the quantized nature of electric charges is
(a) Thomson's experiment (b) Millikan's oil drop experiment
(c) Discharge tube experiment (d) Faraday's experiment

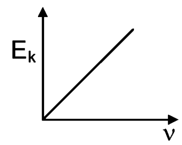
41. If 5% of the energy supplied to a bulb is radiated as visible light, the number of photons of visible light emitted per second by a 100 W bulb is (assume wavelength of the visible light to be 5.6×10^{-5} cm)
(a) 1.4×10^{19} (b) 1.4×10^{20} (c) 2×10^{19} (d) 2×10^{20}

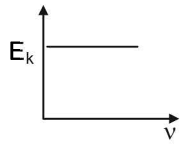
42. The quantum nature of light is required to explain photoelectric effect because.
(a) there is a minimum frequency of light below which photoelectrons are not emitted.
(b) the maximum K.E. of photoelectrons depends only on frequency of light and not on its intensity.
(c) a faint illumination of the metal surface at a wavelength below the cut-off wavelength is enough for the photoelectrons to leave the surface suddenly.
(d) All of the above

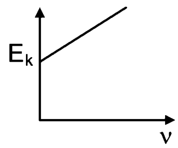
43. The photoelectric equation enunciated by Einstein is $E_k = h\nu - \varphi$, E_k indicates the
(a) mean kinetic energy of emitted electrons
(b) maximum kinetic energy of emitted electrons
(c) kinetic energy of all the emitted electrons
(d) minimum kinetic energy of emitted electrons

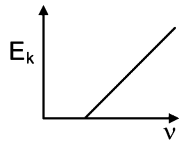
44. The critical or threshold wavelength for a photosensitive surface is 6000Å and the wavelength of the incident light is 5000Å. Then the maximum energy of the emitted electrons would be
(a) 0.041 eV (b) 0.41 eV (c) 41 eV (d) 4.1 eV

45. The maximum K.E.(E_k) of photoelectrons varies with the frequency (ν) of the incident radiation as

(a) 

(b) 

(c) 

(d) 

46. In order to increase the maximum K.E. of the ejected photoelectrons there should be an increase in the
(a) intensity of radiation
(b) wave length of radiation
(c) frequency of radiation
(d) both wavelength and intensity of radiation

47. If from the ground state level E_0, an atom is excited to an energy level E_1, the wavelength of the radiation absorbed is
(a) $\frac{h}{E_1 - E_0}$ (b) $\frac{E_0 - E_1}{hc}$ (c) $\frac{E_1 - E_0}{h}$ (d) $\frac{hc}{E_1 - E_0}$

48. Identify the incorrect statement
(a) Cathode rays are positively charged particles
(b) Cathode rays have momentum and energy
(c) Cathode rays travel in straight lines
(d) Cathode rays are deflected by electric and magnetic fields

49. Which of the experiments given below indicate the dual nature of matter?
(a) Thomson's experiment
(b) Millikan's oil drop experiment
(c) Davisson and Germer Electron diffraction experiment
(d) Rutherford's experiments on the scattering of α-particles by atoms

50. The de-Broglie wavelength associated with a He atom in helium gas at a temperature of 27 °C and pressure 1 atm is
(a) 7.3 Å (b) 73 Å (c) 0.73 Å (d) 0.073 Å

51. de-Broglie wavelength of the electrons accelerated through 54 V is
(a) 1.67 Å (b) 16.6 Å (c) 17.6 Å (d) 1.76 Å

52. The de Broglie wavelength associated with thermal neutron at room temperature (30 °C) is
(a) 0.24 nm (b) 0.42 nm (c) 4.2 Å (d) 1.47 Å

53. The amount of energy which should be added to an electron to reduce its de-Broglie wavelength from 100 pm to 50 pm is nearly

(a) 400 eV (b) 450 eV (c) 350 eV (d) 300 eV

54. The ionisation energy of a deuterium atom is (ionisation energy for H-atom is 13.6 eV)

(a) 13.6 eV (b) slightly less than 13.6 eV
(c) slightly more than 13.6 eV (d) 27.2 eV

55. If a hydrogen atom and a He^+ ion have the same size, their respective states are

(a) 1, 2 (b) 2, 2 (c) 2, 4 (d) not possible

56. The minimum energy to be supplied to a He^+ ion in the ground state so that its size is larger than an H-atom in the first excited state is

(a) 40.8 eV (b) 48 eV (c) 54.4 eV (d) 27.2 eV

57. Energy levels of an atom is evidenced by

(a) atomic numbers (b) atomic radii (c) spectral lines (d) mass defect

58. The ratio of K.E. to the total energy of the electron in the hydrogen atom according to Bohr's theory is

(a) 1 : –1 (b) 1 : 1 (c) 1 : 2 (d) 1 : –2

59. The linear momentum of an electron in the nth orbit of hydrogen atom is proportional to

(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$ (c) $\frac{1}{n^3}$ (d) n^2

60. The angular speed of the electrons in the nth Bohr orbit of the hydrogen atom is proportional to

(a) $\frac{1}{n}$ (b) n (c) n^3 (d) $\frac{1}{n^3}$

61. The radius of the hydrogen atom in its ground state is 5.3×10^{-11}m. After collision with an electron it is found to have a radius of 21.2×10^{-11}m. The principal quantum number of the final state of the atom is

(a) 16 (b) 4 (c) 3 (d) 2

62. The concept of nuclear atom was established from the experiment of the

(a) emission of electrons from metal surfaces
(b) scattering of α-particles by metal foils
(c) diffraction of electrons by crystals
(d) discharge of electricity through gas

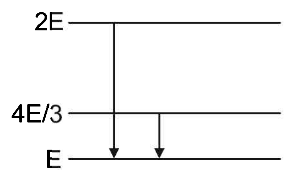
63. Find out which of the following is incorrect in Bohr model of the hydrogen atom.

(a) The radius of the nth orbit is proportional to n^2.
(b) The total energy of the electron in the nth orbit is inversely proportional to n.
(c) The magnitude of the potential energy of the electron in an orbit is greater than its kinetic energy.
(d) The speed of an electron is the nth orbit is inversely proportional to n.

64. The ionization potential of hydrogen is 13.6 V. The energy required to remove an electron from the second orbit of hydrogen is

(a) 6.8 eV (b) 3.4 eV (c) 27.2 eV (d) 1.7 eV

65. The given diagram shows the energy levels of an atom. When the system moves from 2E level to E level, a photon of wavelength λ is emitted. The wavelength of the photon emitted during its transition from 4E/3 level to E level is



(a) 3λ (b) $4\lambda/3$
(c) $3\lambda/4$ (d) $\lambda/3$

66. The accelerating voltage required for K_α emission from Z = 37 is V. The voltage required to produce L_α from Z = 74 is approximately (neglect screening constant)

(a) $\frac{V}{2}$ (b) 2V (c) $\frac{V}{4}$ (d) V

67. An X-ray tube operates at 50 kV. The shortest wavelength of X-rays is

(a) 2.5×10^{-11} m (b) 1.48×10^{-10} m (c) 24×10^{-6} m (d) 50×10^{-10} m

68. The wavelength that cannot be present in the X-ray spectrum, when an X-ray tube is operated at 40 kV is

(a) 0.2Å (b) 0.4Å (c) 1.0Å (d) 1.2Å

69. An electric field can deflect

(a) α-particles (b) γ-particles (c) X-rays (d) Neutrons

70. Decay of a free neutron results in the formation of

(a) Proton, Electron, Neutrino (b) Proton, Positron, Neutrino
(c) Proton, Electron, Antineutrino (d) Proton, Positron, Antineutrino

71. One atomic mass unit (u) is equivalent to

(a) 913.5 eV (b) 931.5 MeV (c) 951.3 eV (d) 935.1 eV

72. Mass defect of an atom refers to

(a) inaccurate measurement of mass of nucleons
(b) mass annihilated to produce energy to bind the nucleons
(c) packing fraction
(d) difference in number of neutrons and protons in the nucleus

73. In the uranium radioactive series, when uranium (238) decays to lead (206), the number of α and β particles emitted are

(a) 6, 8 (b) 8, 6 (c) 7, 9 (d) 9, 7

74. In the following nuclear reaction $^{11}C_6 \rightarrow {}^{11}B_5 + \beta^+ + X$
What does X stand for?

(a) Positron (b) Proton (c) Neutrino (d) Antineutrino

75. A sample contains 16 g of a radioactive material, the half life of which is 2 days. After 16 days the amount of radioactive material left in the sample is

(a) Greater than 1 g (b) $\frac{1}{4}$g (c) 1 g (d) Less than 0.1g

76. The half-life of a radioactive sample is 1600 years. After 800 years, if the number of radioactive nuclei changes by a factor f, then f is

(a) $\frac{1}{\ln 2}$ (b) $\frac{1}{\sqrt{2}}$ (c) ln2 (d) $\exp\left(-\frac{1}{2}\right)$

77. The nuclear matter density is of the order of

(a) 10^{24}kg/m^3 (b) 10^4kg/m^3 (c) 10^{10}kg/m^3 (d) 10^{17}kg/m^3

78. The nuclear force

(a) is equal to the electromagnetic force (b) is a short range force
(c) obeys inverse third power law of distance (d) obeys inverse square law of distance

79. The mass density of a nucleus varies with the mass number A as

(a) A^0 (b) 1/A (c) A (d) A^2

80. The control rod in a (nuclear) reactor is made of

(a) uranium (b) graphite (c) cadmium (d) lithium

81. Monochromatic radiation of frequency f and intensity I falls normally on a surface having area A. The number of photons striking the area per second is

(a) $\frac{IA}{hf}$ (b) $\frac{Ih}{fA}$ (c) $\frac{IA}{f}$ (d) $\frac{fA}{Ih}$

82. Photons of frequency f fall on a metal surface with work function ϕ. Then the photo electrons have kinetic energy

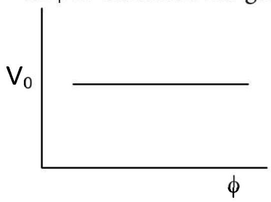
(a) of maximum value of hf (b) ϕ

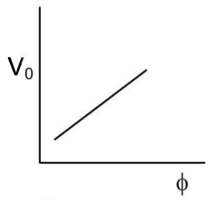
(c) $hf - \phi$ (d) between zero and $hf - \phi$.

83. For a metal surface of work function ϕ stopping potential is V for a given radiation. If a metal surface of work function 0.9ϕ is used, then the stopping potential (modulus value)

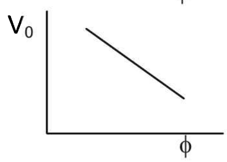
(a) decreases by 10% (b) increases by 10% (c) increases to $\frac{1.1\phi}{e}$ (d) increases to $\frac{0.1\phi}{e}$

84. In photoelectric effect experiment using same radiation wavelength, the stopping potential V_0 varies with the work function ϕ as shown in the graph,

(a) 

(b) 

(c) 

(d) 

85. Light impinges on a metal surface having a work function of 2.0 eV. The fastest electron emitted from the surface has a velocity 6.0×10^5 m s^{-1}. The frequency of light is

(a) 5.0×10^{14} Hz (b) 7.3×10^{14} Hz (c) 0.12×10^{14} Hz (d) 2.5×10^{14} Hz

86. A particle having kinetic energy K has de Broglie wavelength λ. If the kinetic energy is doubled the wavelength will be

(a) $\frac{\lambda}{4}$ (b) $\frac{\lambda}{2}$ (c) $\frac{\lambda}{\sqrt{2}}$ (d) $\sqrt{2}\,\lambda$

87. de Broglie wavelength of an electron of speed 1 km s^{-1} is

(a) 0.72×10^{-8} m (b) 0.75×10^{-10} m (c) 0.73×10^{-6} m (d) 0.75×10^{-12} m

88. Electrons are accelerated through a potential difference of V_0 kV. The de Broglie wavelength (in m) of the electrons is

(a) $\frac{h}{\sqrt{1000\, me\, V_0}}$ (b) $\frac{h}{\sqrt{2000\, me\, V_0}}$ (c) $\frac{h}{\sqrt{meV_0}}$ (d) $\frac{h}{2meV_0}$

89. de Broglie wavelength of an electron, of energy –0.54 eV, in the hydrogen atom is (a_0 = Bohr radius)

(a) $5\pi\, a_0$ (b) $10\pi a_0$ (c) $50\pi a_0$ (d) $\frac{\pi a_0}{25}$

90. Electrons with de Broglie wavelength 0.182Å are used to produce X-rays. The shortest wavelength of the X-rays is nearly

(a) 1.82Å (b) 3.64 Å (c) 2.73Å (d) 4Å

91. The ratio of the de Broglie wavelengths of the helium molecules at 127° C and hydrogen molecules at 27 °C is

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\sqrt{\frac{3}{4}}$ (d) $\sqrt{\frac{3}{8}}$

92. The magnetic moment due to the revolution of an electron in the nth orbit of hydrogen atom is proportional to

(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$ (c) n (d) n^2

93. The ratio of magnetic moment due to the revolution of an electron in the nth orbit of hydrogen atom, to the angular momentum of the same electron is proportional to

(a) n^0 (b) n (c) n^2 (d) $\frac{1}{n}$

94. The quantities having the same dimensions are

(a) momentum and impulse (b) torque and angular momentum
(c) angular momentum and Planck's constant (d) both (a) and (c)

95. The atomic number of hydrogen-like ion having ionization energy of E rydberg is

(a) E (b) E^2 (c) $\sqrt{E}$ (d) $\sqrt[3]{E}$

96. The ionization energy of hydrogen like ion is u rydberg. Its first orbit radius is ($a_0 = 0.53 \times 10^{-9}$ m)

(a) $\frac{a_0}{u}$ (b) $\frac{a_0}{u^2}$ (c) $\frac{a_0}{\sqrt{u}}$ (d) $a_0\sqrt{u}$

97. The modulus value of potential energy of the electron in the second orbit of a hydrogen like ion (Z = 3) is (in rydberg)

(a) $\frac{9}{2}$ (b) $\frac{9}{4}$ (c) $\frac{9}{8}$ (d) $\frac{3}{4}$

98. The excitation energy required for a hydrogen-like ion (Z = 2) to bring it to the second excited state from the ground state is (in rydberg)

(a) $\frac{2}{9}$ (b) $\frac{16}{9}$ (c) $-\frac{16}{9}$ (d) $\frac{32}{9}$

99. The SI unit of the Rydberg constant is

(a) $\frac{1}{\text{kg}}$ (b) $\frac{1}{\text{m}}$ (c) Js (d) $\frac{\text{kg}}{\text{J}}$

100. Photons of lowest frequency will be emitted by a hydrogen atom for the transition from

(a) n = 5 to n = 1 (b) n = 4 to n = 1 (c) n = 3 to n = 1 (d) n = 2 to n = 1

101. The characteristic K_α X ray from tungsten (Z = 74) has a wavelength λ. The characteristic wavelength of the K_α line from rhenium (Z = 75) will be

(a) $\frac{74}{75}\lambda$ (b) $\frac{73}{74}\lambda$ (c) $\left(\frac{73}{74}\right)^2\lambda$ (d) $\left(\frac{74}{73}\right)^2\lambda$

102. The nucleus ${}^{A}_{Z}X$ emits an α and a β particle. The resulting nucleus is

(a) ${}^{A-4}_{Z-1}Y$ (b) ${}^{A-4}_{Z-2}Y$ (c) ${}^{A-4}_{Z}Y$ (d) ${}^{A-2}_{Z-2}Y$

103. The energy required to remove a neutron from the nucleus of ${}^{42}_{20}Ca$ is (given mass of ${}^{41}_{20}Ca$ = 40.974599u, that of ${}^{42}_{20}Ca$ = 41.962278u, m_n = 1.008665 u)

(a) 10.2 MeV (b) 13.4 MeV (c) 11.4 MeV (d) 19.53 MeV

104. The binding energy of an α-particle is 28.24 MeV. The minimum frequency of a photon to split an α-particle into its nucleons is

(a) 1.48×10^{21} Hz (b) 6.82×10^{21} Hz (c) 6.82×10^{20} Hz (d) 3.41×10^{20} Hz

105. Consider a possible fusion reaction in which two neutrons and two protons combine to form an α-particle. The number of fusion reactions per second to generate a power of 100 MW is (m_n = 1.008665 u, m_p = 1.007825 u, m_α = 4.002604)

(a) 2.19×10^{19} (b) 21.9×10^{21} (c) 2.19×10^{21} (d) 2.19×10^{20}

106. Nuclei of a radioactive element are being produced at a constant rate α nuclei/s . The element has a decay constant λ. If N is the number of nuclei at any instant t, then the

(a) rate of decay = $\frac{dN}{dt}$ (b) rate of decay = $\lambda - \alpha N$

(c) rate of increase of N is $\frac{dN}{dt} = \alpha - \lambda N$ (d) rate of increase of N is $\frac{dN}{dt} = (\alpha - \lambda)N$

107. A radioactive element A decays into B with half life 4.5×10^9 year. B in turn decays with a half life of 5,000 year. If in the steady state number of nuclei of A is 2×10^{20}, the number of nuclei of B is

(a) 10^7 (b) 2.2×10^{14} (c) 0 (d) 2×10^{20}

108. The half-life of a radioactive material is x years. If it is stored for y year, the fraction decayed in this time is

(a) $\frac{1}{\sqrt[x]{2^y}}$ (b) $\frac{1}{\sqrt[y]{2^x}}$ (c) $1 - \frac{1}{\sqrt[x]{2^y}}$ (d) $1 - \frac{1}{\sqrt[y]{2^x}}$

109. The activity of a radioactive material decreases by x% in time t_0, the activity of the sample in time 2 t_0 will be

(a) $\frac{x}{100}$ of the original value (b) $\left(1 - \frac{x}{100}\right)$ of the original value

(c) $\left(\frac{x}{100}\right)^2$ of the original value (d) $\left(1 - \frac{x}{100}\right)^2$ of the original value

110. A radioactive sample has a mean life of t_0. The fraction of the nuclei left after time kt_0 is

(a) k (b) 1–k (c) $\frac{1}{e^k}$ (d) e^k

Assertion–Reason Type Questions

Directions: **Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

111. Statement 1

Even with a strictly monochromatic light source in the photoelectric experiment, the emitted photoelectrons will have a range of kinetic energies.

and

Statement 2

Different electrons suffer different number of collisions with atoms of metal before coming out of the surface.

112. **Statement 1**

When white light is passed through hydrogen gas at room temperature, there will be no absorption spectrum.

and

Statement 2

At room temperature hydrogen atoms will be in the ground state.

113. **Statement 1**

When a nucleus decays by α emission, the α particle emitted will have kinetic energy less than the Q value of the process.

and

Statement 2

Momentum and KE are shared between the α particle and the daughter nucleus as per laws of classical mechanics.

Linked Comprehension Type Questions

Directions**: This section contains a paragraph. Based upon the paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

Passage 1

A spectrometer is used to observe the Balmer series in the hydrogen spectrum (transitions from quantum number n > 2 to n = 2). The fractional change in wavelength depends upon the quantum number n. The maximum value of the fractional change in wavelength which the spectrometer can measure is the reciprocal of the resolving power R.

114. The fractional change in λ is given by (for a change in quantum number dn = 1)

(a) $\frac{d\lambda}{\lambda} = -\frac{2k\lambda^2}{n^2}$ (b) $\frac{d\lambda}{\lambda} = -\frac{2k\lambda^2}{n^3}$ (c) $\frac{d\lambda}{\lambda} = -\frac{2k}{n^3}$ (d) $\frac{d\lambda}{\lambda} = -\frac{2k\lambda}{n^3}$

115. The fractional change in wavelength $\left|\frac{d\lambda}{\lambda}\right|$ (for dn = 1) for Balmar series is equal to

(a) $\frac{2}{n^3}\left(\frac{1}{4} - \frac{1}{n^2}\right)$ (b) $\frac{8n^2}{n^3 - 4}$ (c) $\frac{2}{\frac{n^3}{4} - n}$ (d) $2\left(\frac{n^3}{4} - n\right)$

116. If R is the resolving power, then

(a) $\frac{n^3}{4} - n < R$ (b) $\frac{n^3}{4} - n > 2R$ (c) $\frac{n^3}{4} - n < 2R$ (d) $\frac{n^3}{4} - n < \frac{R}{2}$

Multiple Correct Objective Type Questions

Directions: **Each question in this section has four suggested answers of which ONE OR MORE answers is/are correct.**

117. A neutron moving with velocity v_n has an inelastic collision with a stationary singly ionized helium ion and both are deflected at 45° to the original path of the neutron. The helium ion gives out a radiation of 48.36 eV and comes to the ground state. If v_n' and v_H are the final velocities of neutron and helium,

(a) $v_n = \sqrt{2}\, v_n'$

(b) $v_H = \frac{v_n}{4}$

(c) $v_H = \dfrac{v_n}{4\sqrt{2}}$

(d) If the excited electron gives out more than one radiation instead of a single radiation, the largest possible wavelength is 1643Å.

118. Electrons with KE of 1.6 eV are colliding with α particles and some of them form singly ionized helium ion in 3rd excited state.

(a) The initial photon given out by the atom capturing the electron is of wavelength 2484Å.

(b) Subsequently the atoms can give out radiation of 6 different wavelength.

(c) The maximum wavelength is 4704Å.

(d) The maximum wavelength is 4502Å.

119. A radio-active element is being produced at a constant rate R nuclei per second. The decay constant of the element is λ. If N_0 is the initial value of the number of nuclei present, $T_{\frac{1}{2}}$ is the half-life period, and N is the number of nuclei at any instant t, and $R = 2N_0\lambda$ then,

(a) $\left|\dfrac{dN}{dt}\right| \neq \lambda N$

(b) At $t = T_{\frac{1}{2}}$ (half life), $N = \dfrac{3}{2}N_0$

(c) After long time $N = 2N_0$

(d) After long time $N = \dfrac{2N_0}{\ell n2}$

Matrix-Match Type Question

Directions: **Match the elements of Column I to elements of Column II. There can be single or multiple matches.**

120. Some characteristics of X-rays are given in column I and column II. Match them.

Column I	Column II
(a) Minimum wavelength of the X-ray spectrum	(p) depends on the screening constant (factor 'b' of Moseley's law)
(b) Intensity of the X-ray spectrum	(q) dependent on target material
(c) Wavelength of K_α line	(r) depends on the accelerating voltage
(d) Continuous spectrum	(s) depends on the atomic number of the target material

ADDITIONAL PRACTICE EXERCISE

Subjective Questions

121. A monochromatic light beam, incident on a metal surface whose work function is 2.1 eV, generates photoelectrons, which collide with hydrogen atoms in the ground state.
Determine the maximum wavelength of the incident light for which the
(i) hydrogen atoms are ionised.
(ii) hydrogen atoms emit visible light.

122. Find the frequency of photons required to excite the doubly ionised lithium atom to the third Bohr state.

123. Consider a hypothetical hydrogen atom in which the centripetal force $F = \dfrac{k}{r^{\frac{3}{2}}}$. Derive an expression for the radius of the nth Bohr orbit

124. de-Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one-dimensional array with nodes at each atomic site. It is found that one such standing wave is formed, if the distance d between the atoms of the array is 2 Å. A similar standing wave is formed when d increases to 3 $\overset{\circ}{A}$ but not in any intermediate value of d. Find the energy of the electron in electron volt.

125. The K absorption edge (minimum wavelength of the K series lines) of an element is 0.0185 nm. Determine
(i) the atomic number of the element.
(ii) the minimum potential difference through which an electron is to be accelerated to strike a target of this element so as to obtain the K_α line in the X-ray spectrum.

126. Find the mass of the atom of ${}^{208}_{82}Pb$ which has a binding energy per nucleon of 7.93 MeV. (m_p = 1.00727 u, m_n = 1.00866 u)

127. The decay of ${}^{238}_{92}U$ by emission of an α particle into thorium is represented by the equation ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$
(i) Determine the energy of the decay products.
(ii) Assuming uranium atom to be at rest before decay, find the velocity and the energy of the α particle.

m_U = 238.0508 u, m_{Th} = 234.0436 u, m_{He} = 4.0026 u

128. The equation for the fission reaction of ${}^{235}U$ is

${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3{}^{1}_{0}n$

(i) If the average binding energy per nucleon is 7.6 MeV for uranium and 8.5 MeV for the fission fragments, find the energy of the fission products.
(ii) How much uranium is split every second in order to generate energy at the rate of 100 MW?

129. In the chemical analysis of a rock, the mass ratio of two radioactive isotopes is found to be M_1:M_2 = M_R. Their atomic mass ratio is m_1:m_2 = a_{mr}. The mean lives of the two isotopes are T_{avg_1} and T_{avg_2} respectively. At the formation time of the rock, the number of atoms of these isotopes were equal. Express the age of the rock in terms of given data.

130. Determine the activity of 2.3×10^{-11} kg of ${}^{31}P$ when it decays by β emission with a half life of 14.3 days.

Straight Objective Type Questions

Directions: **This section contains multiple choice questions. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

131. In a photoelectric experiment using monochromatic radiation on two different metals, the stopping potentials are in the ratio $\frac{2}{5}$. The work functions are also in ratio $\frac{2}{5}$, one of the work functions is 0.67 eV. The source light can be

(a) red (b) yellow (c) blue (d) uv

132. In a photoelectric experiment using a monochromatic source, on two metals having work functions in ratio 1 : 2, the de Broglie wavelengths of the fastest photoelectrons in each case are in ratio 1 : 2. If the smaller stopping potential is 0.4 V, the photon energy in eV is

(a) 1.6 (b) 2.2 (c) 2.8 (d) 3.2

133. The ratio of de Broglie wavelengths of a proton and an α-particle is 1 : 2. Then the respective ratio of their

(a) velocities is 1 : 2 (b) kinetic energies is 4 : 1
(c) velocities is 8 : 1 (d) kinetic energies is 1 : 16

134. When a gas of monatomic hydrogen in the ground state is bombarded with a stream of electrons that have been accelerated from rest through a potential difference, the emission spectrum contains 10 lines. By what fraction the potential difference should have been increased so that the emission spectrum would have contained maximum possible number of lines?

(a) $\frac{1}{10}$ (b) 10 (c) $\frac{1}{24}$ (d) $\frac{15}{16}$

135. A hydrogen atom undergoes two transitions: from n_1 to n_2, then to n_3 so that the linear momentum of the electron changes from p to 4 p, then to $\frac{p}{9}$, then minimum possible n_3 is

(a) 1 (b) 4 (c) 9 (d) 36

136. The minimum energy of radiation (in eV) required to excite singly ionised helium gas in the ground state so that emission lines contain visible radiation is approximately

(a) 52.9 (b) 51.0 (c) 53.6 (d) 54.4

137. 71.4 eV energy is required to excite the electron of a hydrogen-like ion from 2nd orbit to 5th orbit. The atomic number is

(a) 2 (b) 3 (c) 4 (d) 5

138. Which among the following wavelengths in Å will be absent in the X-ray spectrum of an X-ray tube operated at 62 kV with a target of atomic number 62?

(a) 0.062 (b) 0.62 (c) 6.2 (d) 62

139. Wavelength of the first two lines of Paschen series are λ_1, and λ_2 respectively. Then the wavelength of the first line of Brackett series is

(a) $\lambda_1 + \lambda_2$ (b) $\frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$ (c) $\lambda_1 - \lambda_2$ (d) $\frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}$

140. Characteristic Kα X-ray of an element is approximately 2Å (2 Å > $\lambda_{K\alpha}$> 1.9 Å). Atomic number of the target element is

(a) 21 (b) 26 (c) 31 (d) 36

141. An α particle moves along a circle of radius 1.25 cm inside a uniform magnetic field of intensity 0.1T. Its de Broglie wavelength in pico-metre is

(a) 1.65 (b) 2.2 (c) 3.1 (d) 3.8

142. The ratio of the excitation energy of a hydrogen- like ion to take it to the fourth excited state from ground state to that required to take it to the second excited state from ground state is

(a) $\frac{27}{25}$ (b) $\frac{9}{25}$ (c) $\frac{25}{9}$ (d) $\frac{6}{5}$

143. In a hydrogen atom, when the electron jumps from second excited state to the ground state, the energy of the photon emitted is

(a) 13.6 eV (b) 10.2 eV (c) 12.1 eV (d) 1.5 eV

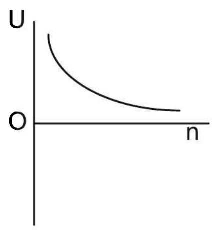
144. The angular momentum of an electron in the hydrogen atom having energy –0.85 eV is

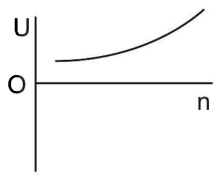
(a) $\frac{h}{\pi}$ (b) $\frac{4h}{\pi}$ (c) $\frac{2h}{\pi}$ (d) $\frac{3h}{2\pi}$

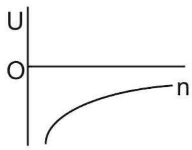
145. The frequency of revolutions of an electron in the nth orbit of hydrogen atom is proportional to

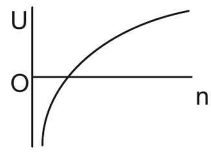
(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$ (c) $\frac{1}{n^3}$ (d) n^2

146. In a hydrogen atom, the variation of potential energy of the electron U with quantum number n is represented by the graph

(a) 

(b) 

(c) 

(d) 

147. A hydrogen atom makes a transition from n = 6 to n = 2 energy level. The photon emitted in this process is in the range

(a) infrared (b) ultraviolet (c) visible (d) X-ray

148. The electron in a hydrogen atom makes a transition $m \rightarrow n$. The frequency of revolution of the electron in the initial state is $\frac{1}{8}$ of that in the final state. The possible values of m and n are

(a) 2, 1 (b) 4, 1 (c) 4, 2 (d) (a) and (c)

149. A nucleus (mass number A) splits into two nuclei with equal mass numbers. The ratio of the radius of each daughter nuclei to that of the parent nucleus is

(a) $\frac{1}{2}$ (b) $\frac{2}{1}$ (c) $\frac{\sqrt[3]{2}}{1}$ (d) $\frac{1}{\sqrt[3]{2}}$

150. A proton and an antiproton follow an annihilation process producing two photons of equal frequency. The wavelength of the photons is

(a) 1.1×10^{-4} Å (b) 0.13×10^{-4} Å (c) 1.7×10^{-7} Å (d) 1.9×10^{-9} Å

151. Identify the pair of "isobars" from the following

(a) ${}^{1}_{1}H$ and ${}^{2}_{1}H$ (b) ${}^{2}_{1}H$ and ${}^{3}_{1}H$ (c) ${}^{12}_{6}C$ and ${}^{13}_{6}C$ (d) ${}^{30}_{15}P$ and ${}^{30}_{14}Sc$

152. High speed neutrons are slowed down by
(a) applying a strong magnetic field
(b) passing them through heavy water
(c) elastic collision with heavy nuclei
(d) applying a strong electric field

153. One microgram of matter is changed into energy. The amount of energy released is
(a) 9×10^8J
(b) 9×10^4J
(c) 9×10^7J
(d) 9×10^{10}J

154. A radioactive nucleus y undergoes a few decays according to the scheme.

$$y \xrightarrow{\alpha} y_1 \xrightarrow{\beta} y_2 \xrightarrow{\alpha} y_3 \xrightarrow{\gamma} y_4$$

The mass number and atomic number of y are 180 and 72 respectively, the corresponding number for y_4 are
(a) 172, 71
(b) 176, 71
(c) 172, 69
(d) 176, 69

155. A radioactive isotope X with a half life of 1.37×10^9 year decays to Y which is stable. A sample of rock from the Moon was found to contain both X and Y atoms in the ratio 1 : 7. The age of the rock is
(a) 9.59×10^9 year
(b) 4.11×10^9 year
(c) 3.85×10^9 year
(d) 1.96×10^9 year

156. A nuclide with mass number m and atomic number n disintegrates by emitting an α-particle and a β-particle. The resulting nuclide has mass number and atomic number
(a) m + 4, n – 1
(b) m – 4, n + 1
(c) m – 2, n
(d) m – 4, n – 1

157. One eighth of the initial mass of a certain radioactive isotope remains undecayed after one hour. The half-life of the isotope in minute is
(a) 8
(b) 20
(c) 30
(d) 45

158. The binding energy per nucleon for $^{31}_{15}P$ (in MeV), given that its mass is 30.973763 u.

(m_{proton} = 1.007825 u, $m_{neutron}$ = 1.008665 u)

(a) 10.9
(b) 10.12
(c) 9.51
(d) 8.47

159. For an X-ray target, K_α line is 0.66Å, K_β is 0.55Å; Then L_α line can be estimated (in Å) as approximately
(a) 0.11
(b) 22
(c) 3.3
(d) 0.44

160. Referring to the above question, the actual value of wavelength of L_α line will be
(a) more than estimated
(b) less than estimated
(c) equal to estimated
(d) cannot be concluded unless atomic number is known

161. The ratio of nuclear radii of $^{27}_{13}A\ell$ and $^{125}_{52}Te$ is approximately
(a) 27 : 125
(b) 14 : 73
(c) 3 : 5
(d) 13 : 52

162. In the process indicated below:

$^9_4Be + ^4_2He \rightarrow ^{12}_6C + x$

x is
(a) neutron
(b) proton
(c) electron
(d) photon

163. When an electron is emitted by a nucleus, its neutron : proton ratio
(a) decreases
(b) increases
(c) remains same
(d) can be (a) or (b)

164. To calculate the age of a fossil by carbon dating process, it is required to determine
(a) amount of ^{14}C in fossil
(b) amount of ^{12}C
(c) ratio of ^{14}C to ^{12}C in fossil and living plant
(d) half life of ^{14}C

165. Work function of a material is 3 eV. This material will give photocurrent if illuminated by a

(a) 0.1 W uv lamp (b) 1 W sodium lamp (c) 10 W red neon lamp (d) 100 W ir lamp

166. If the wavelength of a photon is same as the de Broglie wavelength of an electron moving at 10^8 m s^{-1}, the ratio of the energy of the photon to the kinetic energy of the electron is

(a) 3 (b) 6 (c) 9 (d) 18

167. ${}^{220}_{86}Rn \rightarrow {}^{216}_{84}Po + {}^{4}_{2}He$, $t_{1/2} = 51.5$ s

${}^{216}_{84}Po \rightarrow {}^{212}_{82}Pb + {}^{4}_{2}He$, $t_{1/2} = 0.16$ s

${}^{212}_{82}Pb \rightarrow {}^{212}_{83}Bi + {}^{0}_{-1}e$, $t_{1/2} = 10.6$ h

10 minute after taking an initial sample of Rn, the element with the greatest mass will be

(a) Rn (b) Po (c) Pb (d) Bi

168. If a free neutron decays and if one of the products is a proton, another product will be

(a) positron (b) neutrino (c) antineutrino (d) none of the above

169. If the total energy required to completely ionize a hydrogen atom and a helium atom is E electron volt, then the energy required to singly ionize a helium atom is (in eV)

(a) E – 13.6 (b) E – 40.8 (c) E – 54.4 (d) E – 68

170. Heavy water used in the reactor

(a) cools the reactor (b) slows down the neutron

(c) absorbs the neutron (d) absorbs the fission material

Assertion–Reason Type Questions

***Directions:* Each question contains Statement-1 and Statement-2 and has the following choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

171. Statement 1

In an atom, total energy of an orbiting electron is negative.

and

Statement 2

Electron has negative charge

172. Statement 1

Energy of a photon does not change with medium.

and

Statement 2

Frequency depends only on source.

173. Statement 1

Even if all the electrons colliding with the target of an X-ray tube have identical kinetic energy, the spectrum will still be continuous.

and

Statement 2

The collision of an electron with a target atom is inelastic.

174. **Statement 1**

Each of fission products will have more binding energy per nucleon than original nucleus.

and

Statement 2

Fission releases energy.

175. **Statement 1**

Nuclear density is independent of the number of nucleons.

and

Statement 2

Nuclear radius R is proportional to $A^{1/3.}$

176. **Statement 1**

In a photoelectric experiment, decreasing the frequency of source, while keeping intensity same, will result in higher saturation current.

and

Statement 2

Saturation current is proportional to the number of incident photons.

177. **Statement 1**

Half life is less than mean life.

and

Statement 2

Average life for the whole population is mean life while that for half the population is half-life.

178. **Statement 1**

In fission reactions, the products have larger binding energy per nucleon than the parent.

and

Statement 2

As atomic number increases, the binding energy per nucleon first increases and then decreases.

179. **Statement 1**

If an X-ray spectrum contains characteristic K_α-line of the target element, the cut-off wavelength of the spectrum should be less than $\frac{8}{9}$ of the $\lambda_{k\beta}$ ($\lambda_{k\beta} \rightarrow$ characteristic K_β line wavelength).

and

Statement 2

In an X-ray spectrum if the characteristic K_α line of the element is present, then its characteristic K_β line also will be present.

180. **Statement 1**

A photon of wavelength λ on colliding with a hydrogen atom can result in radiation of $\lambda_2 > \lambda$.

and

Statement 2

The only reason is that part of the energy of the photon is absorbed by the atom and balance energy of the photon comes out as a radiation of larger wavelength.

Linked Comprehension Type Questions

Directions: **This section contains 3 paragraphs. Based upon the paragraph, 3 multiple choice questions have to be answered. Each question has 4 choices (a), (b), (c) and (d), out of which ONLY ONE is correct.**

Passage 1

The target of an X-ray tube is an alloy of 3 elements, A, B, C whose K-shell ionization energies are in the ratio 7:8:9. The one with highest atomic number is Rhodium Z= 45. The operating voltage produces characteristic X-ray of all the elements.

181. λ_{min} of the X-ray spectrum possible is

(a) .81Å (b) 0.74Å (c) 0.65Å (d) 0.47Å

182. The wavelength of the characteristic k_β line for element A is

(a) 0.68Å (b) 0.73Å (c) 0.81Å (d) 0.93Å

183. The wavelength of k_α line for element B is

(a) 0.5Å (b) 0.6Å (c) 0.7Å (d) 0.81Å

Passage II

Activity A (disintegration per unit time) vs time t is plotted on a graph paper for 3 radioactive samples having initial masses equal. The plots are 3 straight lines concurrent at (t_0, A_0). At $2t_0$ the ratio $\ell n\frac{A_0}{A_1} = 0.2$ for the least radio-active element (A_0, A_1 are activities at time t_0 and $2t_0$ respectively). At t = $2t_0$ their activities are in the ratio 3:2:1 then

184. The graph paper used for the plots is

(a) standard graph sheet (b) semi – log graph sheet (c) $\log_{10} - \log_{10}$ graph sheet (d) ℓn. – ℓn graph sheet

185. The half life of the most radio active element is

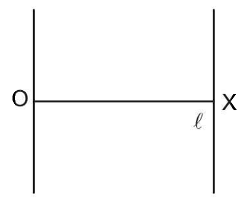
(a) 0.54 t_0 (b) 0.63 t_0 (c) 0.81 t_0 (d)1.1 t_0

186. Their initial activities were in the ratio

(a) 3 : 2 : 1 (b) 2 : 3 : 6 (c) 1 : 2 : 3 (d) 1 : 4 : 9

Passage III

The theory of orbiting electrons in an atom can only be logically explained by quantum mechanics. The mathematical theory of quantum mechanics is dealt by Schrodinger equations. Here the elecron is not treated as a particle with a precise location in the orbit at a particular instant in time, but as a wave function which predicts the probability of locating an electron at a particular location at that instant. According to this view, it is not possible to locate the exact position of electron around the nucleus at any instant. There is however a region with high density of distribution of electronic charge, which is referred to as electronic cloud around the nucleus. The density of the cloud at any point represents the probability of finding an election at that point.



The Schrodinger equation in one dimension is given by $\frac{-h^2}{8\pi^2 m}.\frac{d^2\psi}{dx^2} + V\psi = E\psi$

Where ψ = probability amplitude function,

V = potential and E = energy of the particle of mass m. This equation is used to study the motion of a particle, say an electron, confined in a one dimensional well of width ℓ

$(0 \le x \le \ell)$. The potential V is described by V = 0

For $0 < x < \ell$ and $V = \infty$ elsewhere. In other words the function $\psi = 0$ at $x = 0$ and at $x \ge \ell$.

187. The variation of ψ with x inside the well is

(a) linear
(b) periodic and simple harmonic
(c) non-linear but not periodic
(d) periodic but not simple harmonic

188. If $\psi = A\cos kx + B\sin kx$, then (n = a positive integer)

(a) $A = 0, k = \dfrac{n\pi}{\ell}$
(b) $B = 0, k = \dfrac{2n\pi}{\ell}$
(c) $B = 0, k = \dfrac{n\pi}{2\ell}$
(d) $A = 0, k = \dfrac{(2n+1)\pi}{\ell}$

189. Consider the electron to be in the first and second energy levels, the difference being 0.05 eV. The value of ℓ is

(a) 4.8 nm
(b) 6 nm
(c) 8 nm
(d) 12 nm

Multiple Correct Objective Type Questions

Directions: **Each question in this section has four suggested answers out of which ONE OR MORE answers is/are correct.**

190. de Broglie wavelength of the electron in a hydrogen-like ion of atomic number Z in an excited state n is

(a) directly proportional to n
(b) directly proportional to Z
(c) inversely proportional to n
(d) inversely proportional to Z

191. The energy of the most energetic photoelectron emitted from a metal target depends on

(a) intensity of incident radiation
(b) wavelength of incident radiation
(c) photoelectric efficiency
(d) threshold frequency

192. Which among the following make a nucleus unstable?

(a) Low binding energy
(b) High packing fraction
(c) High $\dfrac{n}{p}$ ratio
(d) High binding energy per nucleon

193. The decay constant of a radioactive element is 0.693 per year. Then,

(a) there is a 30.7% chance that a given nucleus will disintegrate within the next one year
(b) There is a 30.7% chance that a given nucleus will not disintegrate within next year
(c) half life of the element is 1 year
(d) 63% of a sample of the element will decay in $\dfrac{1}{0.693}$ year

194. For nuclei with A > 100,

(a) binding energy per nucleon decreases with increase in A
(b) binding energy per nucleon increases with increase in A
(c) energy is released if the nucleus breaks into two parts
(d) energy is released if two nuclei fuse together

195. For radioactive decay process:

(a) On α-decay, all emitted α-particles will have same kinetic energy
(b) On β^+ decay, all emitted β^+ particles will have same kinetic energy

(c) On β^- decay, all emitted β^- particles will have same kinetic energy
(d) γ-decay follows α, β^+, β^{-1} decay

196. Whenever a hydrogen atom emits a photon in Paschen series, subsequently
(a) it may emit a photon in Balmer series
(b) it may emit a photon in Lyman series
(c) it may emit two photons
(d) it must emit two photons

197. The mean life of a radioactive sample is $\dfrac{300}{\ell n(2)}$day $\left[\ell n2 = 0.7\right]$

(a) In 900 days 1 gram of the original sample becomes $\frac{1}{8}$ gram
(b) Decay constant $\lambda = 2.33 \times 10^{-3}$ per day
(c) The probability that a nucleus in original sample will decay in 300 days is $\frac{1}{2}$
(d) The probability that a nucleus in original sample will not decay in 300 days is $\frac{1}{2}$

Matrix-Match Type Questions

Directions: **Match the elements of Column I to elements of Column II. There can be single or multiple matches.**

198. In the Bohr model of the atom, the electron orbiting about the nucleus is characterized by the properties shown in column I and their dependencies in column II. Match them (n – principal quantum number, e – charge of the electron)

Column I	Column II
(a) Angular momentum	(p) $\propto n^2$
(b) Speed	(q) $\propto e$
(c) Radius of the orbit	(r) $\propto e^2$
(d) Magnetic moment of the revolving electron	(s) $\propto n$

199. Monochromatic radiation of photon energy x eV falls on substance A, causing emission of particles/packets of several energy values, y_i eV

Column I	Column II
(a) If minimum $y_i = 1.9$	(p) A can be hydrogen atoms at ground state
(b) If maximum $y_i = 1.9$	(q) A can be a photoelectric metal
(c) If some $y_i = 1.9$ but neither maximum nor minimum	(r) x can be 12.1
(d) If no $y_i = 1.9$	(s) x can be > 12.85

200. Match the columns:

Column I	Column II
(a) alpha decay	(p) mass of products is less than mass of reactants
(b) beta decay	(q) binding energy per nucleon increases
(c) fission	(r) mass number is conserved
(d) fusion	(s) charge is conserved

SOLUTIONS

ANSWERS KEYS

Topic Grip

1. (i) 4 to 3
 (ii) 1.05×10^{-34} J s
2. 8.5×10^{-28} kg
3. 102.5 nm
4. 1.59×10^{5} m s^{-1}
5. $\lambda_{initial} = 414$ nm,
 $\lambda_{min} = 23.79$ nm
 $\lambda_{max} = 1018$ nm
6. (i) He
 (ii) 6
 (iii) 26 nm
7. b = 5.6; $\lambda_{\beta} = 0.104$ nm
8. (i) 2.5 gram
 (ii) A = 5×10^{19} per day
9. $A_1' = 60$ s^{-1}; $A_2' = 50$ s^{-1};
 mass ratio = 1.1
10. $N_A = N_0 e^{-\lambda t} + \frac{\alpha}{\lambda}(1 - e^{-\lambda t})$

 $$N_B = \left(N_0 - \frac{\alpha}{\lambda}\right)\left(1 - e^{-\lambda t}\right) + \alpha t$$

11. (c)	**12.** (a)	**13.** (a)
14. (a)	**15.** (d)	**16.** (c)
17. (d)	**18.** (d)	**19.** (b)
20. (a)	**21.** (b)	**22.** (a)
23. (b)	**24.** (d)	**25.** (a)

26. (c)
27. (a), (c), (d)
28. (a), (c)
29. (a), (c)
30. (a) → (p), (r)
 (b) → (q), (s)
 (c) → (s)
 (d) → (q), (s)

IIT Assignment Exercise

31. (a)	**32.** (a)	**33.** (a)
34. (c)	**35.** (d)	**36.** (d)
37. (a)	**38.** (c)	**39.** (b)
40. (b)	**41.** (a)	**42.** (d)
43. (b)	**44.** (b)	**45.** (d)
46. (c)	**47.** (d)	**48.** (a)
49. (c)	**50.** (c)	**51.** (a)
52. (d)	**53.** (b)	**54.** (c)
55. (d)	**56.** (b)	**57.** (c)
58. (a)	**59.** (a)	**60.** (d)
61. (d)	**62.** (b)	**63.** (b)
64. (b)	**65.** (a)	**66.** (d)
67. (a)	**68.** (a)	**69.** (a)
70. (c)	**71.** (b)	**72.** (b)
73. (b)	**74.** (c)	**75.** (d)
76. (b)	**77.** (d)	**78.** (b)
79. (a)	**80.** (c)	**81.** (a)
82. (d)	**83.** (d)	**84.** (d)
85. (b)	**86.** (c)	**87.** (c)
88. (b)	**89.** (b)	**90.** (c)
91. (d)	**92.** (c)	**93.** (a)
94. (d)	**95.** (c)	**96.** (c)
97. (a)	**98.** (d)	**99.** (b)
100. (d)	**101.** (c)	**102.** (a)
103. (d)	**104.** (b)	**105.** (a)
106. (c)	**107.** (b)	**108.** (c)
109. (d)	**110.** (c)	**111.** (a)
112. (b)	**113.** (a)	**114.** (d)

115. (c)
116. (c)
117. (a), (c), (d)
118. (a), (b), (c)
119. (a), (b), (c)
120. (a) → (r)
 (b) → (r)
 (c) → (p), (s)
 (d) → (q)

Additional Practice Exercise

121. (i) 79 nm
 (ii) 87 nm
122. 26.25×10^{15} Hz
123. $r = \left[\frac{n^2h^2}{4\pi^2 mk}\right]^{2/3}$
124. 37.7 eV
125. (i) 71
 (ii) 67 kV
126. m = 207.916 u
127. (i) E = 4.3 MeV
 (ii) v = 1.42×10^{7} m s^{-1}
 E = 4.22 MeV
128. (i) 194.5 MeV
 (ii) 1.25×10^{-6} kg
129. $t = \dfrac{\lambda n\left(\dfrac{M_R}{a_{mr}}\right)}{\dfrac{1}{T_{avg2}} - \dfrac{1}{T_{avg1}}}$
130. A = 6.76×10^{-3} Ci

131. (b)	**132.** (c)	**133.** (c)
134. (c)	**135.** (d)	**136.** (b)
137. (d)	**138.** (a)	**139.** (d)
140. (b)	**141.** (a)	**142.** (a)
143. (c)	**144.** (c)	**145.** (c)
146. (c)	**147.** (c)	**148.** (d)

149. (d) **150.** (b) **151.** (d)
152. (b) **153.** (c) **154.** (c)
155. (b) **156.** (d) **157.** (b)
158. (d) **159.** (c) **160.** (a)
161. (c) **162.** (a) **163.** (a)
164. (c) **165.** (a) **166.** (b)
167. (c) **168.** (c) **169.** (d)
170. (b) **171.** (b) **172.** (a)
173. (c) **174.** (a) **175.** (a)
176. (a) **177.** (c) **178.** (a)
179. (b) **180.** (c) **181.** (d)
182. (a) **183.** (c) **184.** (b)
185. (a) **186.** (b) **187.** (b)
188. (a) **189.** (a)
190. (a), (d)
191. (b), (d)
192. (a), (b), (c)
193. (b), (c), (d)
194. (a), (c)
195. (a), (d)
196. (a), (b), (c)
197. (a), (b), (c), (d)
198. (a) → (s)
(b) → (r)
(c) → (p)
(d) → (q), (s)
199. (a) → (p), (r)
(b) → (q), (r), (s)
(c) → (p), (q), (r), (s)
(d) → (p), (q), (r), (s)
200. (a) → (p), (q), (r), (s)
(b) → (p), (q), (r), (s)
(c) → (p), (q), (r), (s)
(d) → (p), (q), (r), (s)

HINTS AND EXPLANATIONS

Topic Grip

1.

(i) $\phi = h\nu_0 = 4.14 \times 10^{-15} \times 5 \times 10^{14} = 2.07$ eV

$E_{photon} = 2.07 + 0.57 = 2.64$ eV

ΔE for H: (all in eV)

$E_1 = -13.6$, $E_2 = -3.4$, $E_3 = -1.51$, $E_4 = -0.85$

ΔE_{2-1}, ΔE_{3-1}, ΔE_{4-1} are all > 2.64 eV

If ΔE is the energy of transition between various states for hydrogen, then the corresponding $\Delta E'$ for helium is $Z^2\,\Delta E = 4\Delta E \Rightarrow \therefore\ 4\Delta E = 2.64$ ev.

$\Delta E = 0.66$ eV $\Rightarrow$ which corresponds to transition of $E_4 \to E_3$ ie. n_4 to n_3

transition 4 to 3

(ii) $\Delta L = \Delta n \dfrac{h}{2\pi} = \dfrac{1 \times 6.6 \times 10^{-34}}{2 \times \pi} = 1.05 \times 10^{-34}$ J s

2. $mv = \dfrac{h}{\lambda}$

$$V2e = \frac{1}{2}mv^2 \Rightarrow mv^2 = 4Ve$$

$$\Rightarrow \frac{m^2v^2}{m} = 4Ve \Rightarrow \frac{h^2}{\lambda^2 m} = 4Ve$$

$$\Rightarrow m = \frac{h^2}{4Ve\lambda^2}$$

$$= \frac{\left(6.6 \times 10^{-34}\right)^2}{4 \times 200 \times 1.6 \times 10^{-19} \times (0.002)^2 \times 10^{-18}}$$

$$= 8.5 \times 10^{-28}\text{ kg}$$

3. $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$\Rightarrow$ Balmer series: $n_1 = 2$ and first member: $n_2 = 3$

$$\Rightarrow \frac{1}{656 \times 10^{-9}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36} \qquad \text{— (1)}$$

Lyman series: $n_1 = 1$ & $n_2 = 3$ (2nd member)

$$\frac{1}{\lambda'} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8R}{9} \qquad \text{— (2)}$$

$$\frac{(1)}{(2)} = \lambda' = 656 \times 10^{-9} \times \frac{5}{32} = 102.5\text{ nm}$$

4. 3 lines $\Rightarrow n_1 = 1$ to $n_2 = 3$; $Z = 3$

$$\therefore\ \Delta E = 9 \times 13.6\left(\frac{1}{1} - \frac{1}{9}\right) = 9 \times 13.6 \times \frac{8}{9}\text{ eV}$$

$$= 13.6 \times 8 \times 1.6 \times 10^{-19}\text{ J} \qquad \text{— (1)}$$

Collision

Let initial velocity of neutron = u.

Let velocity of Li^{++} after collision = v

Momentum eqn:

$$mu = \frac{mu}{3} + 7mv$$

$$\Rightarrow \frac{2u}{3} = 7v$$

$$\Rightarrow v = \frac{2u}{21} \qquad \text{— (2)}$$

Energy equation

$$\frac{1}{2}mu^2 = \frac{1}{2}\frac{mu^2}{9} + \frac{1}{2}7m \cdot 4\frac{u^2}{21^2} + \Delta E$$

$$\Rightarrow \frac{26}{63}u^2 = \frac{\Delta E}{m}$$

$$u = \sqrt{\frac{63\Delta E}{26m}} = \sqrt{\frac{63 \times 13.6 \times 8 \times 1.6 \times 10^{-19}}{26 \times \frac{5}{3} \times 10^{-27}}}$$

$$= 1.59 \times 10^5\text{ m s}^{-1}$$

5. Initial energy of electron = 0.82 eV

Since the He+ give 10 radiation ${}^nC_2 = 10$; $n = 5$

So the electron will have energy

$$= \frac{-13.6Z^2}{n^2} = \frac{-13.6 \times 4}{25} = -2.18\text{ eV}$$

$\Delta E = 0.82 - (-2.18) = 3$ eV

$$\therefore\ \lambda_{initial} = \frac{hc}{\Delta E} = \frac{1242}{3} = 414\text{ nm}$$

Minimum corresponds to $E_5 - E_1$

$$E_1 = \frac{-13.6 \times (2)^2}{(1)^2} = -54.4\text{ eV}$$

$$\Delta E = E_5 - E_1 = 54.4 - 2.18 = 52.22\text{ eV}$$

$$\lambda_{min} = \frac{1242}{52.2(\text{eV})} = 23.79\text{ nm}$$

Maximum λ corresponds to $E_5 - E_4$

$$E_4 = \frac{-13.6\times(2)^2}{(4)^2} = -3.4\,eV$$

$$\Delta E = E_5 - E_4 = 3.4 - 2.18 = 1.22\,eV$$

$$\lambda = \frac{1242}{1.22} = 1018 \text{ nm}$$

6. (i) 10.2 eV $\Rightarrow$ 13.6 – 3.4

$\Rightarrow$ If hydrogen atom shifts from

n = 2 to n = 1

But since more than one line H is not the element

$$\Rightarrow \quad 10.2 = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Energy levels are (in eV)

n =	1	2	3	4	5	6
H	–13.6	–3.4	–1.511	–0.85	–0.544	–0.378
He^+	–54.4	–13.6	–6.04	–3.4	–2.176	–1.511
Li^{++}	–122.4	–30.6	–13.6	–7.65	–4.896	–3.4

Possibilities : He^+ and 4 to 2

or Li^{++} and 6 to 3

both cases $\lambda_{debroglie}$ will double [$\lambda = \frac{h}{mv}$ $\Rightarrow$ If λ doubles $v \to \frac{v}{2}$.

But $v \propto \frac{1}{n}$, $v' \to \frac{v}{2}$ means $n' = 2n$]

$$\therefore \quad \lambda_{deB} \propto n \Rightarrow \frac{6}{3} = \frac{4}{2} = 2$$

Hence double

If λ_1 is largest wavelength,

If He^+, Z = 2 $\Rightarrow$

$$\frac{1}{\lambda_1} = 4R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = 4\times\frac{72}{7}\times\frac{10^6\times 7}{9\times 16}$$

$\Rightarrow$ λ_1 = 500 nm (visible)

if Li^{++}, Z = 3

$$\Rightarrow \quad \frac{1}{\lambda_1} = 9R\left[\frac{1}{25} - \frac{1}{36}\right]$$

$$= 9\times\frac{72}{7}\times 10^6\times\frac{(36-25)}{36\times 25} = \frac{9\times 72\times 11}{7\times 36\times 25}\times 10^6$$

$$\Rightarrow \quad \lambda_1 = \frac{7}{792}\times 10^{-4} > 800\,\text{nm (not visible)}$$

$\therefore$ element is He

(ii) Number of line = ${}^4C_2 = \frac{4\times 3}{1\times 2} = 6$

(iii) $\frac{1}{\lambda_{min}} = 4R\left[\frac{1}{1} - \frac{1}{16}\right] \Rightarrow \lambda_{min} = 26$ nm

7. $$\nu = \frac{c}{\lambda} = Rc(Z-b)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

b $\to$ screening constant for the series

$$\frac{c}{0.14\times 10^{-9}} = \left\{c\times 1.1\times 10^7\times(74-b)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)\right\}$$

$\Rightarrow$ b = 5.6

L_β line $\Rightarrow$ n = 4 to n = 2

$$\frac{1}{\lambda_\beta} = R(Z-b)^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) \Rightarrow \left(\frac{\lambda_\beta}{\lambda_\alpha}\right)$$

$$= \frac{\left(1/4 - 1/9\right)}{\left(1/4 - 1/16\right)} = \frac{5}{36}\times\frac{64}{12} \Rightarrow \lambda_\beta = 0.104 \text{ nm}$$

8. (i) Δm = 209.98264 – 205.97440 – 4.00260

= 0.00564 amu

= 0.00564 × 931 MeV

= 5.25 MeV = 8.4 × 10^{-13} J

$$\lambda = \frac{0.7}{t_{1/2}} = \frac{0.7}{140 \text{ day}} = 0.005\text{ day}^{-1}$$

Let M gram be required to produce 6 × 10^6 J/day.

$\therefore$ Number of nuclei $N = \frac{M}{210}\times 6.023\times 10^{23}$; Since $t_{1/2} >> 1$ day,

$$-\frac{dN}{dt} = \lambda N$$

$$= 0.005\times\frac{M}{210}\times 6.023\times 10^{23} \text{ per day}$$

= 0.143 × 10^{20} M per day and this value does not change much in one day.

$N = N_0e^{-\lambda t}$. For a day, λt = 0.005

and $e^{-0.005} \simeq 0.995$

Change in N in a day is negligible

$\therefore$ Energy produced per day

$= 0.143 \times 10^{20}\ M \times 8.4 \times 10^{-13}$ J/day

$= (1.2 \times 10^7\ M)$ J/day

$\therefore$ Required $M = \dfrac{6 \times 10^6}{1.2 \times 10^7} = 0.5$ gram

Efficiency = 20% $\Rightarrow$ 2.5 gram of material is required to produce the given amount.

(ii) $\dfrac{A_0}{A} = 2^{\frac{t}{t_{1/2}}} = 2^{\frac{70}{140}} = \sqrt{2} \Rightarrow A_0 = \sqrt{2}A$

But $A = 0.143 \times 10^{20}M$

$= 0.143 \times 10^{20} \times 2.5 = 3.575 \times 10^{19}$

$\therefore A_0 = \sqrt{2} \times 3.575 \times 10^{19}$

$= 5 \times 10^{19}$ per day

9. Initially, $A_1 = \lambda_1 N_1$; $A_2 = \lambda_2 N_2$

Let m_1 and m_2 be the atomic masses respectively

$\therefore$ Given $\dfrac{m_1}{m_2}\dfrac{N_1}{N_2} = \dfrac{2.2}{1}$ But $\dfrac{m_1}{m_2} = \dfrac{1.1}{1}$

$\therefore \dfrac{N_1}{N_2} = 2,\ T_1 = 10\text{h},\ T_2 = 12\text{h}$

But $N = \dfrac{N_0}{2^{\frac{t}{T_{1/2}}}}$

After 60 h $\Rightarrow N_1' = \dfrac{N_1}{2^{(60/10)}}$

$$N_1' = N_1\left(\frac{1}{2}\right)^6 = \frac{N_1}{64}$$

Similarly, $N_2' = N_2\left(\dfrac{1}{2}\right)^5 = \dfrac{N_2}{32}$

$$\therefore \frac{N_1'}{N_2'} = \frac{1}{2}\frac{N_1}{N_2} = 1:1$$

$\Rightarrow$ mass ratio$= \dfrac{m_1 N'_1}{m_2 N'_2} = 1.1$

$$A_1' = \lambda_1 N_1' = \lambda_1 \frac{N_1}{64} = \frac{A_1}{64} = \frac{3.84 \times 10^3}{64} s^{-1}$$

$$= 60\ s^{-1}$$

$$A_2' = \lambda_2 N_2'$$

$$\frac{\lambda_2}{\lambda_1} = \frac{T_1}{T_2} = \frac{10}{12}$$

$$\frac{A_1'}{A_2'} = \frac{\lambda_1}{\lambda_2}\cdot\frac{N_1'}{N_2'} = \frac{12}{10}\cdot 1$$

$$A_2' = \frac{10}{12}\cdot A_1' = \frac{10}{12}\cdot 60 = 50\ s^{-1}$$

10. $\dfrac{dN_A}{dt} = -\lambda N_A + \alpha$

$$\frac{dN_A}{\lambda N_A - \alpha} = -dt \Rightarrow \frac{dN_A}{\left(N_A - \frac{\alpha}{\lambda}\right)} = -\lambda dt$$

On integration

$$\Rightarrow \ell n\left(N_A - \frac{\alpha}{\lambda}\right)\Bigg|_{N_0}^{N_A} = -\lambda t\Bigg|_0^t$$

$$\Rightarrow \ell n\left[\frac{N_A - \frac{\alpha}{\lambda}}{N_0 - \frac{\alpha}{\lambda}}\right] = -\lambda t$$

$$\Rightarrow \frac{\left(N_A - \frac{\alpha}{\lambda}\right)}{\left(N_0 - \frac{\alpha}{\lambda}\right)} = e^{-\lambda t}$$

$$\Rightarrow \left(N_A - \frac{\alpha}{\lambda}\right) = \left(N_0 - \frac{\alpha}{\lambda}\right)e^{-\lambda t}$$

$$\Rightarrow N_A = \frac{\alpha}{\lambda} + \left(N_0 - \frac{\alpha}{\lambda}\right)e^{-\lambda t}$$

$$\Rightarrow N_A = N_0 e^{-\lambda t} + \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$$

$$\& N_B = N_0 + \alpha t - N_A$$

$$= N_0 + \alpha t - N_0 e^{-\lambda t} - \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$$

$$\Rightarrow N_B = N_0(1 - e^{-\lambda t}) + \alpha t - \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$$

$$\Rightarrow N_B = \left(N_0 - \frac{\alpha}{\lambda}\right)\left(1 - e^{-\lambda t}\right) + \alpha t$$

11. $\phi = \dfrac{h\nu_0}{e} = \dfrac{6.6 \times 10^{-34} \times 6.4 \times 10^{14}}{1.6 \times 10^{-19}} = 2.64$ eV

Stopping potential = 3.96 eV (given)

$\therefore$ $h\nu = 2.64 + 3.96 = 6.6$ eV

$\Rightarrow \nu = \dfrac{6.6 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 1.6 \times 10^{15}$ Hz

$x = 1.6$

12. $\dfrac{mv^2}{r} = qvB \Rightarrow mv = qBr$

$$\Rightarrow \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{qBr}$$

$$= \frac{6.6 \times 10^{-34}}{2 \times 1.6 \times 10^{-19} \times 0.1 \times 1.25 \times 10^{-2}}$$

$= 1.65 \times 10^{-12}$ m = 1.65 pm

13. $E_1 = -13.6$ eV
$E_2 = -3.4$ eV
$E_3 = -1.51$ eV
$E_4 = -0.85$ eV
$E_{2-1} = 10.2$ eV
$E_{3-1} = 12.09$ eV
$E_{4-1} = 12.75$ eV
$\therefore$ It will be excited from n = 1 to n = 3. result will be 3 wavelengths.
$\lambda_{3-2}, \lambda_{3-1}, \lambda_{2-1}$

$$\frac{1}{\lambda_{3-2}} = R\left(\frac{1}{4} - \frac{1}{9}\right) = 1.1 \times 10^7 \times \frac{5}{36}$$

$\Rightarrow \lambda_{3-2} = 654.5$ nm= 6545 A°

$$\frac{1}{\lambda_{3-1}} = R\left(\frac{1}{1} - \frac{1}{9}\right) = \lambda_{3-1} = 102.3 \text{ nm}$$

= 1023 A°

$$\frac{1}{\lambda_{2-1}} = R\left(\frac{1}{1} - \frac{1}{4}\right) = \lambda_{2-1} = 121.2 \text{ nm}$$

= 1212 A°

14. λ_B is the K_β emission of pattern A and is the minimum wavelength for pattern B

In case of radiation pattern A, accelerating voltage gives as much energy for the electron just enough for it to knock-out a k-shell electron from the element. Hence its energy E = Rhc $(Z-1)^2$. If any of these bombarding electrons are fully stopped at first collision in the target material, then the corresponding photon emitted has a wavelength equal to the cut-off wavelength λ_A given by $\frac{1}{\lambda_A} = R(Z-1)^2$. As per the given data, cut off wavelength for pattern B is given by the K_β wavelength of pattern A

$$\frac{1}{\lambda_B} = R(Z-1)^2\left(1 - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda_B} = R(Z-1)^2 . \frac{8}{9}$$

$$\Rightarrow \quad \therefore \frac{\lambda_B}{\lambda_A} = \frac{9}{8}$$

15. 15 protons, 16 neutrons
Mass defect

= 15 × 1.007825 + 16 × 1.008665 – 30.973763

= 0.282252 amu

$\Rightarrow$ Binding energy per nucleon

$$\frac{0.282252 \times 931}{31} = 8.47 \text{ MeV}$$

16. Statement 1 is true but not 2. The number of photoelectrons emitted per unit time will increase causing increased $\frac{d(ne)}{dt}$.

17. Net energy absorbed = $\frac{hc}{\lambda_1}$ = and not $\frac{hc}{\lambda_2 - \lambda_1}$

18. cut off wavelength $\Rightarrow$ complete KE lost in a single collision only. When lost in a number of collisions, continuous spectrum of different wavelengths is produced. Hence statement 1 is false, but 2 is true.

19. Electrons capable of knocking off K-shell electron can also knock-out L-shell on M-shell electrons.

20. Velocity of β-ray electron (near to that of light) > velocity of cathode ray electron, mass (m) increases with velocity (v) according to the relation $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$ (theory of relativity). Hence statement 1 and statement 2 are true.

21. $\frac{dI}{I} = -\mu dx$

$\Rightarrow$ L. H. S. has obviously no dimension

$\Rightarrow$ Hence for dimensional integrity, μ has dimension of m^{-1}

22. Higher density

$\Rightarrow$ higher probability of collision

$\Rightarrow$ lower penetrating ability $\Rightarrow$ higher μ

23. $\frac{dI}{dx} = -\mu I \Rightarrow \frac{dI}{I} = -\mu dx \quad \Rightarrow$ integrating;

$$\ell n I \Big]_{I_0}^{I} = -\mu x$$

$$\ell n \frac{I}{I_0} = -\mu x \Rightarrow \frac{I}{I_0} = e^{-\mu x} \Rightarrow I = I_0 e^{-\mu x}$$

$\Rightarrow$ exponentially decaying

24. $N = N_0\, e^{-\lambda_2 t}$

$\therefore$ ^{238}U decayed $= N_0\,(1 - e^{-\lambda_2 t})$

$= \text{Number of } {}^{206}Pb \text{ atoms}$

25. $$\frac{m_{Pb}}{m_{U_{238}}} = \frac{206\left(1 - e^{-\lambda_2 t}\right)}{238\ e^{-\lambda_2 t}} = 0.80$$

$$e^{\lambda_2 t} - 1 = 0.8 \times \frac{238}{206} = 0.92 \Rightarrow e^{\lambda_2 t} = 1.92 \simeq 2$$

$\therefore\ \lambda_2 t \approx \ell n\, 2$

$$t(\text{year}) = \frac{\ell n\, 2}{86400 \times 365 \times 4.87 \times 10^{-18}} \approx 10^9$$

26. $e^{\lambda_2 t} = 2$

$$e^{\lambda_1 t} = \left(e^{\lambda_2 t}\right)^{\frac{\lambda_1}{\lambda_2}}$$

$$\therefore\ e^{\lambda_1 t} = 2^{\frac{\lambda_1}{\lambda_2}} \approx 71.5$$

Present mass of $^{235}U = 3\, e^{-\lambda_1 t} = \dfrac{3}{e^{\lambda_1 t}}$

$$\frac{3}{71.5} \cong 42\mu g$$

27. The energy levels of hydrogen atom are (in eV)

$-13.6, -3.4, -1.5, -0.85$ (up to 3rd excitation)

$\lambda_1 = 4870Å \Rightarrow$ corresponding

$$eV = \frac{1242\,(eV - nm)}{487\,(nm)} = 2.55eV$$

Obviously the only possibility is, the atom is in 3[rd] excitation level and it comes to I[st] excitation level [$\because 3.4 - 0.85 = 2.55$ eV]

Now $\lambda_2 = (13.6 - 3.4)eV = \dfrac{1242}{10.2} nm \simeq 121.8nm$

$= 1218Å$

For conservation of momentum

$mv_A - mv_B = mv_B' \Rightarrow v_B' = (v_A - v_B)$

Energy equation

$$\Rightarrow\ \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2$$

$$= \frac{1}{2}m\left(v_A - v_B\right)^2 + 12.75 \times 1.6 \times 10^{-19}$$

[$\because \Delta K = 13.6 - 0.85 \simeq 12.75$ eV

$= 12.75 \times 1.6 \times 10^{-19}$ J]

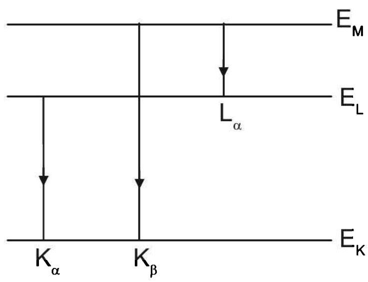
$$v_A^2 + v_B^2 = (v_A - v_B)^2 + \frac{12.75 \times 1.6 \times 10^{-19} \times 2}{1.67 \times 10^{-27}}$$

$2v_A v_B \simeq 24.4 \times 10^8 \Rightarrow v_B$

$$= \frac{24.4 \times 10^8}{2 \times 6.1 \times 10^4} = 2 \times 10^4\ m\ s^{-1}$$

$v_B' = v_A - v_B = (6.1 - 2) \times 10^4 = 4.1 \times 10^4\ m\ s^{-1}$

28.



$K\alpha$ for cobalt:

$$\sqrt{\frac{v_{Mo}}{v_{Co}}} = \frac{a\left(Z_{Mo} - b\right)}{a\left(Z_{Co} - b\right)} = \frac{42 - 1}{27 - 1} \Rightarrow \frac{\lambda_{Co}}{\lambda_{Mo}} = \frac{41^2}{26^2}$$

$$\lambda_{Co} = 0.71 \times \frac{(41)^2}{(26)^2} \simeq 1.77Å$$

$L\alpha$ for molybdenum

From energy diagram

$$\Rightarrow\ E_{L_\alpha} = E_{K_\beta} - E_{K_\alpha}$$

$$\frac{hc}{\lambda_{L\alpha}} = \frac{hc}{\lambda_{K\beta}} - \frac{hc}{\lambda_{K\alpha}}$$

$$\frac{1}{\lambda_{L\alpha}} = \frac{\lambda_{K\alpha} - \lambda_{K\beta}}{\lambda_{K\alpha}.\lambda_{K\beta}}$$

$$\Rightarrow\ \lambda_{L\alpha} \frac{0.71 \times 0.63}{0.71 - 0.63} Å \simeq 5.6Å$$

But now we have taken $b = 1$

$$\Rightarrow\ \therefore\ \frac{1}{\lambda'_{L\alpha}} \propto (z-1)^2.$$

But actually $\dfrac{1}{\lambda_{L\alpha}} \propto (z - 7)^2$

$$\therefore\ \lambda_{L\alpha} = \frac{(42-1)^2}{(42-7)^2} \times 5.6 \simeq 7.7Å$$

29. Initially, both isotopes were in the same ratio. Hence

at present: $\dfrac{N_1}{N_2} = \dfrac{Ne^{-t/T_{avg\,1}}}{Ne^{-t/T_{avg\,2}}}$

$$= e^{t\left[\frac{1}{T_{avg\,2}} - \frac{1}{T_{avg\,1}}\right]}$$

$$\text{Mass ratio} = \frac{M_1}{M_2} = \frac{N_1 m_1}{N_2 m_2}$$

$$\Rightarrow 120 = \frac{N_1}{N_2} \times 1.2 \Rightarrow \frac{N_1}{N_2} = 100$$

$$\therefore\ e^{t\left[\frac{1}{T_{avg\,2}} - \frac{1}{T_{avg\,1}}\right]} = 100$$

$$\Rightarrow \qquad t = \frac{\ell n\, 100}{\dfrac{1}{T_{avg\,2}} - \dfrac{1}{T_{avg\,1}}}$$

$$= \frac{2\,\ell n\, 10}{\dfrac{1}{10^9} - \dfrac{1}{5 \times 10^9}}$$

$$t = \frac{10^9 \times 2 \dfrac{1}{\log_{10}^{e}}}{0.8} = 10^9 \times 2.5 \times 2.3$$

$$= 5.75 \times 10^9 \text{ year}$$

The ratio of their present activities

$$\Rightarrow \qquad \frac{A_1}{A_2} = \frac{N_1 \lambda_1}{N_2 \lambda_2} = \frac{N_1}{N_2}\frac{T_{avg\,2}}{T_{avg\,1}}$$

$$\frac{A_1}{A_2} = 100 \times \frac{1}{5} = 20$$

30. In α-decay, there are just two reaction products. Therefore, the energies of recoil of the particles are uniquely determined. Also, α-particle is emitted by tunneling across the coulomb potential in the nucleus.

β^- and β^+ decay are accompanied by the emission of antineutrino and neutrino, respectively. Since there are more than three particles in the reaction, the recoil velocity of the particles can vary. Hence β^- and β^+ particles can have a spectrum of energy values

Electron capture is done usually from K-shell

IIT Assignment Exercise

31. $[h] = \dfrac{[E]}{[v]} = ML^2T^{-1}$

32. Energy of photon increases with frequency. More energy is better for emission

33. The threshold wave lengths are:

$$A = \frac{1242}{2} = 621 \text{ nm}$$

$$B = \frac{1242}{5} = 248 \text{ nm}$$

$\therefore$ Only A emits when $\lambda = 400$ nm

34. Intensity increases the number of emitted photoelectrons.

35. $\text{KE (eV)} = \dfrac{hc}{\lambda_0} - \dfrac{hc}{\lambda} = 1240 \times \left(\dfrac{1}{400} - \dfrac{1}{600}\right)$

$$= 1.03 \text{ eV}$$

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 1.03 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$$= 6 \times 10^5 \text{ m s}^{-1}$$

36. Stopping voltage depends on both frequency of the incident light and nature of the metal.

37. The intensity of incident light is proportional to number of photoelectrons.

38. Rate of emission $\propto$ intensity

When distance is halved, the intensity becomes four times (inverse square law). The K.E. depends on frequency of incident photon. It is independent of distance.

39. Below the threshold frequency, the number of emitted electron is zero.

40. Millikan's oil drop experiment showed that charges are integral multiples of e, the electronic charge.

41. $E = nh\nu = n\dfrac{hc}{\lambda}$

$$\frac{100 \times 5}{100} = \frac{n \times 6.6 \times 10^{-34} \times 3.0 \times 10^8}{5600 \times 10^{-10}}$$

$$\Rightarrow \qquad n = 1.4 \times 10^{19} \text{ s}^{-1}$$

42. (a), (b), (c) Laws of photoelectric emission

43. Laws of photoelectric emission

44. $KE = 1240\left(\frac{1}{500} - \frac{1}{600}\right) = 0.41\ eV$

45. $E_k = h\nu - \phi$ (Straight line with negative intercept on the Y axis)

46. $E_k = h\nu - \phi \Rightarrow$ to increase E_k, υ should be increased.

47. $\Delta E = \frac{hc}{\lambda}$

$\therefore \quad \lambda = \frac{hc}{\Delta E}$

Difference in energy of the levels correspond to wavelength

$$\lambda = \frac{hc}{(E_1 - E_0)}$$

48. Cathode rays are electrons.

49. In electron diffraction experiments electrons (particles) exhibit wave phenomena (diffraction). Hence dual nature.

50. $\frac{1}{2}mv^2 = \frac{3}{2}kT$

$m^2v^2 = 3\ mkT$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{3mkT}}$$

$$m = \frac{M}{N} = \frac{4}{6\times10^{26}}\ kg$$

$k = 1.38 \times 10^{-23}$ J/K, T = 300 K

$\lambda = 0.73 \times 10^{-10}\ m = 0.73Å$

51. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 54}}$$

$= 1.67Å$

Aliter:

$$\lambda = \frac{12.27}{\sqrt{V}}Å = \frac{12.27}{\sqrt{54}}Å = 1.67\ Å$$

52. Slow neutrons which are in thermal equilibrium with the medium through which they pass are called thermal neutrons.

K.E of thermal neutrons $= \frac{3}{2}kT$

$$\lambda = \frac{h}{\sqrt{3.m.k.T}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{3 \times 1.6 \times 10^{-27} \times 1.38 \times 10^{-23} \times 303}}$$

$= 1.47Å$

53. $\lambda = \frac{h}{\sqrt{2mE}}$

$$100 \times 10^{-12} = \frac{h}{\sqrt{2mE_1}}$$

$$50 \times 10^{-12} = \frac{h}{\sqrt{2mE_2}}$$

$\therefore \quad E_2 = 4E_1$

so energy added $= 4E_1 - E_1 = 3E_1$

$\therefore \quad \frac{h}{\sqrt{2mE_1}} = 100 \times 10^{-12}$

$$\rightarrow E_1 = \frac{(6.6\times10^{-34})^2}{2\times9.1\times10^{-31}\times10^{-20}}\ J$$

$$= \frac{(6.6\times10^{-34})^2}{2\times9.1\times10^{-31}\times10^{-20}\times1.6\times10^{-19}}\ eV$$

$\therefore \quad E_1 = 150\ eV$

Energy added $= 3E_1 = 450\ eV$

Aliter:

Accelerating potencial $V_1 = \left(\frac{12.27}{\lambda_1}\right)^2$, λ_1 in Å

$$= \left(\frac{12.27}{1}\right)^2 = 150\ V$$

Accelerating potencial $V_2 = \left(\frac{12.27}{\lambda_2}\right)^2 = 602.2\ V$

$\therefore \quad \Delta V = (V_2 - V_1) = 602.2 - 150.0 = 452.2\ V$

$\therefore \quad \Delta U = e\,\Delta V = 452.2\ eV \simeq 450\ eV$

54. $E_n \propto m$
where m is the reduced mass. When mass of the nucleus increases, reduced mass increases.

55. $r \propto \dfrac{n^2}{Z} \Rightarrow \dfrac{n_H^2}{1} = \dfrac{n_{He}^2}{2}$

$\Rightarrow$ n_H and n_{He} cannot simultaneously be integers.

56. $r = \dfrac{n^2}{Z} a_0$

$r_H = 4a_0$

$r_{He} = \dfrac{n^2 . a_0}{2}$

$\dfrac{n^2}{2} a_0 > 4a_0$

$n^2 > 8$

$n = 3$

$E = 4 \times 13.6 \left(1 - \dfrac{1}{3^2}\right) = 48\,eV$

57. Spectral lines indicate that energy is quantized and not continuous.

58. $E = K + V = \dfrac{1}{2}mv^2 - \dfrac{e^2}{4\pi \varepsilon_0 r}$

where $\dfrac{mv^2}{r} = \dfrac{e^2}{4\pi \varepsilon_0 r^2}$

$\Rightarrow \dfrac{1}{2}mv^2 = \dfrac{e^2}{8\pi\varepsilon_0 r}$

$\Rightarrow E = \dfrac{e^2}{8\pi \varepsilon_0 r} - \dfrac{e^2}{4\pi \varepsilon_0 r} = \dfrac{-e^2}{8\pi \varepsilon_0 r} = -K$

$\therefore \dfrac{K}{E} = -1$

59. $\dfrac{e^2}{4\pi\varepsilon_0 r^2} = \dfrac{mv^2}{r}$ (centripetal force) —(1)

$mvr = \dfrac{nh}{2\pi}$ —(2)

(quantization of angular momentum)

$v = \dfrac{nh}{2\pi mr}$ —(3) substitute in (1)

$\dfrac{e^2}{4\pi\varepsilon_0 r^2} = m\dfrac{n^2h^2}{4\pi^2 m^2 r^3} \Rightarrow r = \dfrac{n^2h^2\varepsilon_0}{\pi me^2}$ —(4)

Use (4) in (2)

$$mv = \frac{nh}{2\pi} \times \frac{\pi me^2}{n^2h^2\varepsilon_0} = \frac{me^2}{2nh\varepsilon_0}$$

Aliter:

$L = mvr = \dfrac{nh}{2\pi}$

$r \propto n^2$

$\Rightarrow mv = \dfrac{L}{r} \propto \dfrac{n}{n^2}$

$\Rightarrow mv \propto \dfrac{1}{n}$

60. $r \propto n^2$

$v \propto \dfrac{1}{n}$

$\omega = \left(\dfrac{v}{r}\right) \propto \dfrac{1}{n^3}$

61. $r \propto n^2$

$\dfrac{r_1}{r_2} = \left(\dfrac{n_1}{n_2}\right)^2 = \dfrac{5.3 \times 10^{-11}}{21.2 \times 10^{-11}} = \left(\dfrac{1}{n}\right)^2$

$\Rightarrow n = 2$

62. Scattering showed that there was a massive, small centrally located nucleus in the atom.

63. $E \propto \dfrac{1}{r} \propto \dfrac{1}{n^2}$ [see solution of Q 58]

64. The ionisation potential = 13.6 V = E_1
According to Bohr's theory

$E_2 = \dfrac{E_1}{n_2^2} = \dfrac{13.6}{2^2} = \dfrac{13.6}{4} = 3.4\text{ eV}$

65. $E_m - E_n = h\nu = \dfrac{hc}{\lambda}$

Given: $2E - E = \dfrac{hc}{\lambda}$ — (1)a

Or $E = \dfrac{hc}{\lambda}$ — (1)b

Sub: $4E/3 - E = \dfrac{hc}{\lambda'}$ — (2)

or $\dfrac{1}{3}E = \dfrac{hc}{\lambda'}$ — (2)b

$\dfrac{(1)\,b}{(2)\,b}$, $\Rightarrow \dfrac{\lambda'}{\lambda} = \dfrac{E}{E} \times 3 \Rightarrow \lambda' = 3\lambda$

66. The energy required to knock-off an electron in the nth orbit:

$$E \propto \frac{Z^2}{n^2}$$

For K_α, $n = 1 \Rightarrow \quad E \propto \frac{(37)^2}{(1)^2} \propto 37^2$

For L_α, $n = 2 \Rightarrow \quad E \propto \frac{(74)^2}{(2)^2} \propto 37^2$

$\therefore$ Voltage ratio 1:1

67. $\frac{hc}{\lambda} = h\nu = eV$

$$\lambda = \frac{hc}{eV} = \frac{1242\,\text{ev nm}}{50\times10^3\,\text{ev}} = 24.84\times10^{-3}\,\text{nm}$$

$= 2.5 \times 10^{-11}$m

68. $\lambda_{min} = \left(\frac{12400}{V}\right)Å = \left(\frac{12400}{40\times10^3}\right) = 0.3Å$

69. Only charged particles are affected by an electric field.

70. $n \to p + \overline{e} + \overline{\nu}$

71. 1 u = 931.5 MeV

72. The energy equivalent to mass defect is used to keep the nucleons bound together in the nucleus.

73. $\#\,\alpha = \frac{238 - 206}{4} = 8 \Rightarrow$ This corresponds to loss of: $(8 \times 2) = 16$+ve charges. Actual difference in a atomic No = 92 – 82 = 10

Hence $6\beta^-$ should have been emitted for electric neutrality

$\#\,\beta = -92 + 82 + 8 \times 2 = 6$

74. Neutrino

$p \to n + \beta^+ + \nu$

In $^{11}B_5$ original mass number is conserved. But atomic number and hence charge is reduced. Hence proton converted to neutron

75. $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{T}}$

$N_0 = 16$ g, $t = 16$ and $T = 2$

$N = 16\left(\frac{1}{2}\right)^{\frac{16}{2}}$ i.e., $N < 0.1$g

$\Rightarrow$ **Aliter:** The quantities in gm after successive half lives can easily be written as : 8, 4, 2, 1, 0.5, 0.25, 0.125, 0.0625 (16 days) obviously the result is 0.0625 < 0.1 g.

76. $\frac{N}{N_0} = f = \left(\frac{1}{2}\right)^{\frac{800}{1600}} = \frac{1}{\sqrt{2}}$

77. $\rho = \frac{Am}{\frac{4}{3}\pi r^3} = \frac{Am}{\frac{4}{3}\pi r_0^3 A} = \frac{1.67\times10^{-27}}{\frac{4}{3}\pi(1.2\times10^{-15}\,m)^3}$

$\approx 2 \times 10^{17}$ kg m^{-3}

78. Are very short range forces

79. $\rho \propto \frac{A}{r^3} = \frac{A}{\left(r_0 A^{\frac{1}{3}}\right)^3} = \frac{1}{r_0^3} \Rightarrow \rho \propto A°$

80. Cadmium has several stable isotopes, so that it can absorb neutrons, and hence control the nuclear reaction.

81. $\frac{(hf)n}{A} = \text{intensity } I \Rightarrow n = \frac{IA}{hf}$

82. $hf - \phi$ is the maximum KE of photo electron

83. $eV_0 = hf - \phi$,
$eV_0' = hf - 0.9\phi$

$\Rightarrow$ percentage change $= \frac{100\times0.1\phi}{hf-\phi} = \frac{10\phi}{hf-\phi}$

cannot conclude

But $V_0' - V_0 = \frac{0.1\phi}{e}$

84. $eV_0 = h\nu - \phi \Rightarrow V_0 = \frac{h\nu}{e} - \frac{\phi}{e} \Rightarrow V_0$

V_0 decreases linearly with ϕ.

85. $\frac{1}{2}m_e v^2_{max} = hf - \phi$

$m_e = 9.1 \times 10^{-31}$ kg

$\phi = 2$ eV $= 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19}$ J

$v = 6 \times 10^5$ m s^{-1} (given)

$\therefore \quad \frac{1}{2} \times 9.1 \times 10^{-31} \times (6 \times 10^5)^2$

$= 6.6 \times 10^{-34} \times f - 3.2 \times 10^{-19}$

$f = 7.3 \times 10^{14}$ Hz

86. $\lambda = \dfrac{h}{\sqrt{2mK}} \qquad \lambda' = \dfrac{\lambda}{\sqrt{2}}$

87. $\lambda = \dfrac{h}{mv}$

$= \dfrac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{3}} = 0.727 \times 10^{-6}\,m$

88. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}} = \dfrac{h}{\sqrt{2m \times e \times 1000\,V_0}}$

$= \dfrac{h}{\sqrt{2000\,meV_0}}$

89. $\lambda_b = \dfrac{h}{mv} = \dfrac{2\pi r}{n} = \dfrac{2\pi}{n}.\dfrac{n^2 a_0}{Z} = \dfrac{2\pi n a_0}{Z}$

$Z = 1,$

$n = \sqrt{\dfrac{13.6}{0.54}} = 5$

$\therefore \quad \lambda_b = 10\pi a_0$

Aliter

$\lambda_n = n\lambda_1 = n\,(2\pi a)$

$= 5 \times 2\pi a_0 = 10\pi a_0$

90. $\lambda_b = \dfrac{h}{\sqrt{2mK}} \qquad K = \dfrac{h^2}{2m\lambda_b^2} = \dfrac{hc}{\lambda}$

$\Rightarrow \quad \lambda = \dfrac{2\times 9.11\times 10^{-31}\times (0.182)^2 \times 10^{-20}\times 3\times 10^{8}}{6.6\times 10^{-34}}$

$\therefore \quad \lambda = 2.73 \times 10^{-10} m$

91. $\lambda = \dfrac{h}{p} \Rightarrow \dfrac{\lambda_1}{\lambda_2} = \dfrac{p_2}{p1}$

$\dfrac{p^2}{2m} = E$

$E \propto T$

$\Rightarrow \quad \dfrac{p_2}{p_1} = \sqrt{\dfrac{m_2 T_2}{m_1 T_1}}$

$\dfrac{\lambda_1}{\lambda_2} = \sqrt{\dfrac{m_2 T_2}{m_1 T_1}} = \sqrt{\dfrac{2 \times 300}{4 \times 400}} = \sqrt{\dfrac{3}{8}}$

92. Magnetic moment $\mu = \pi r^2 i$

where r is the radius of the orbit and i is the equivalent current. If v is the speed of the electron and t, the time for one revolution,

$\therefore \quad \mu = \pi r^2 \dfrac{e}{t} = \dfrac{\pi r^2 ev}{2\pi r} = \dfrac{rev}{2} = \dfrac{e}{2m}\,mvr = \dfrac{e}{2m}.L$

where L is the angular momentum.

$\therefore \quad \mu = \dfrac{e}{2m}\dfrac{nh}{2\pi} = \dfrac{eh}{4\pi m}.n \Rightarrow \mu \propto n$

93. Refer solution for Q- 92

$\dfrac{\text{Ratio of magentic moment}}{\text{angular momentum}} = \dfrac{nhe}{4\pi m.\dfrac{nh}{2\pi}} = \dfrac{e}{2m} \propto n^0$

94. $J = \Delta p$

$L = n\left(\dfrac{h}{2\pi}\right)$

95. $E = \dfrac{Z^2}{n^2}$ rydberg. n =1 ground state.

Ionization energy, $E = \dfrac{Z^2}{(1)^2}$ rydberg

$Z = \sqrt{E}$

96. $u = \dfrac{Z^2}{1^2}, \quad Z = \sqrt{u} \qquad r_1 = \dfrac{(1)^2 a_0}{Z} = \dfrac{a_0}{\sqrt{u}}$

97. Potential energy (modulus value)

$= \dfrac{2Z^2}{n^2}\text{ rydberg} = \dfrac{2(9)}{2^2} = \dfrac{9}{2}\text{rydberg.}$

98. $E_{ex} = -\dfrac{Z^2}{n^2} + Z^2 = Z^2\left(1 - \dfrac{1}{n^2}\right)\text{rydberg}$

$= 4\left(1 - \dfrac{1}{3^2}\right) = \dfrac{32}{9}\text{rydberg}$

99. Rch = energy (J)

$R = \dfrac{J}{\dfrac{m}{s}.(Js)} = \dfrac{1}{m}$

100. $\Delta E = hf$. Obviously ΔE will be lowest for n = 2 to n = 1

101. $\lambda \propto (Z-1)^{-2} \Rightarrow \dfrac{\lambda_T}{\lambda_R} = \dfrac{(74-1)^{-2}}{(75-1)^{-2}}$

$\therefore \quad \lambda_R = \lambda_T \left(\dfrac{74}{73}\right)^{-2} = \lambda \times \left(\dfrac{73}{74}\right)^2$

102. ${}^{A}_{Z}X \rightarrow {}^{A_1}_{Z_1}Y + {}^{4}_{2}He + {}^{0}_{-1}\beta$

$Z_1 + 1 = Z$

$Z_1 = Z - 1$

$A_1 + 4 = A$

$A_1 = A - 4$

103. Mass of ${}^{41}_{20}Ca$ + neutron

$= 40.974599 + 1.008665$ u

Mass of ${}^{42}_{20}Ca = 41.962278$ u

$\therefore$ Difference = 0.021u

$\therefore$ Energy required = 0.021(931) = 19.53 MeV

104. $hf = 28.24 \text{ MeV} \Rightarrow f = \dfrac{28.24 \times 10^6}{(4.13 \times 10^{-15} \text{ eV s})}$

$f = 6.82 \times 10^{21}$ Hz

105. Mass defect $= (2m_n + 2m_p - m_{He}) = 0.030377$ u

$= 28.3 \text{ MeV} = 28.3 \times 10^6 \times 1.6 \times 10^{-19}$ J

$\therefore$ Energy required per second $= 100 \times 10^6$ J

$\therefore$ Number of reactions per second

$$\frac{100 \times 10^6}{28.3 \times 10^6 \times 1.6 \times 10^{-19}} = 2.2 \times 10^{19} \text{ s}^{-1}$$

106. Decay constant = λ

$\Rightarrow$ Rate of decay = λN, creation = α

$\Rightarrow \dfrac{dN}{dt} = \alpha - \lambda N$

107. $\dfrac{dN_B}{dt} = \alpha - \lambda_B N_B$

where $\alpha = \lambda_A N_A$

Steady state

$\Rightarrow \dfrac{dN_B}{dt} = 0$

$\lambda_A N_A = \lambda_B N_B$

$N_B = \dfrac{\lambda_A N_A}{\lambda_B}$

But half life $T_{\frac{1}{2}} \propto \dfrac{1}{\lambda} \quad \therefore N_B = \dfrac{T_B}{T_A} N_A$

$\Rightarrow \quad N_B = \dfrac{5 \times 10^3}{4.5 \times 10^9} \times 2 \times 10^{20} = 2.2 \times 10^{14}$

108. $N = \dfrac{N_0}{2^P}$ where $p = \dfrac{t}{t_{\frac{1}{2}}} = \dfrac{y}{x}$

Nuclei decayed $= N_0 - N = N_0 \left[1 - \dfrac{1}{2^P}\right]$

Fraction decayed

$$= \frac{N_0 - N}{N_0} = 1 - \frac{1}{2^P} = 1 - \frac{1}{\sqrt[x]{2^y}}$$

109. $A = A_0 e^{-\lambda t}$

$\left(1 - \dfrac{x}{100}\right) A_0 = A_0 e^{-\lambda t_0}$

activity in time $2t_0$ will be

$A = A_0 \, e^{-\lambda(2t_0)}$

$\therefore \quad \dfrac{A}{A_0} = \left(e^{-\lambda t_0}\right)^2 = \left(1 - \dfrac{x}{100}\right)^2$

110. $N = N_0 e^{-\lambda t} \quad t_0 = \dfrac{1}{\lambda}, \qquad t = k t_0 = \dfrac{k}{\lambda}$

$\therefore \quad N = N_0 e^{-k}, \quad \dfrac{N}{N_0} = e^{-k} = \dfrac{1}{e^k}$

111. Self explanatory

112. Statement 1 and 2 are true, but statement 2 does not fully explain statement I

When electrons fall to first orbit from outer orbits Lyman series, which consists of ultraviolet rays, is produced. Conversely only ultraviolet rays (not white light, which is visible) can excite electrons from first orbit to outer orbits.

113. $mv + MV = 0$

$\dfrac{1}{2}mv^2 + \dfrac{1}{2}MV^2 = Q$

Solving gives v for each α particle and $\dfrac{1}{2}mv^2 < Q$

114. For Balmer series the transition is from n>2 to n = 2

$\therefore \quad \dfrac{1}{\lambda} = k\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right) = k\left(\dfrac{1}{4} - \dfrac{1}{n^2}\right)$

$$\left(\frac{1}{\lambda^2}\right)\frac{d\lambda}{dn} = -k(-2)n^{-3} = \frac{2k}{n^3}$$

$$\therefore \quad \frac{d\lambda}{dn} = -\frac{2k\lambda^2}{n^3}$$

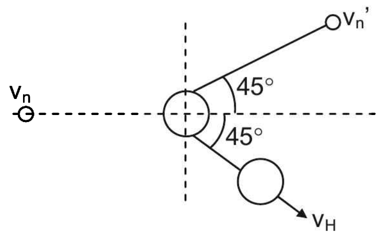
$$\therefore \quad \frac{d\lambda}{\lambda} = -\frac{2k\lambda}{n^3} \quad [\because (dn) = 1, \text{ n being an integer.}]$$

115. $$\left|\frac{d\lambda}{\lambda}\right| = \frac{2k}{n^3\left(\frac{1}{\lambda}\right)} = \frac{2k}{n^3\left[k\left(\frac{1}{4}-\frac{1}{n^2}\right)\right]} = \frac{2}{\frac{n^3}{4}-n}$$

116. $$\frac{\lambda}{d\lambda} = R = \frac{\frac{n^3}{4}-n}{2}$$

$$\frac{n^3}{4} - n < 2R$$

117.



For the helium atom the energy levels are: 13.6 $\frac{z^2}{n^2}$ eV $= \frac{-13.6\times 4}{n^2}$ eV

$\Rightarrow$ −54.4 eV, −13.6 eV, −6.044 eV etc. Since the radiation given out is 48.36 eV $\Rightarrow$ corresponds to (54.4 – 6.044) $\Rightarrow$ hence it is at 3rd excitation level. $\therefore$ If the atom gives out more than one radiation, the maximum wavelength possible, corresponds to minimum energy difference = 13.6 – 6.044 = 7.56 eV

$$\lambda_{max} = \frac{1242}{7.56} \simeq 164.3 \text{ nm} = 1643 \text{ Å}$$

conservation of momentum perpendicular to initial path: (m $\rightarrow$ mass of neutron)

$$4m\frac{v_H}{\sqrt{2}} = m\frac{v_n'}{\sqrt{2}} \Rightarrow v_H = \frac{v_n'}{4}$$

conservation of momentum along the initial path: substitute $v_H = \frac{v_n'}{4}$

$$4m\frac{v_n'}{4}\frac{1}{\sqrt{2}} + mv_n'\frac{1}{\sqrt{2}} = mv_n \Rightarrow v_n = \sqrt{2}\, v_n'$$

$$\therefore \quad v_H = \frac{v_n'}{4} = \frac{v_n}{\sqrt{2}\,4}$$

118. The energy of the electron in the 3rd excited state is (n = 4, Z = 2)

$$eV = \frac{-13.6Z^2}{n^2} = \frac{-13.6\times 4}{4\times 4} = -3.4 \text{ eV}$$

$$\Delta E = 1.6 - (-3.4) = 5 \text{ eV}$$

$$\Rightarrow \lambda = \frac{1242}{5} = 248.4 \text{ nm}$$

$$= 2484 \text{Å}$$

The number of radiations possible from the 3rd excited state i.e., n = 4 $\Rightarrow$ $4C_2 = 6$

The various energy levels are, −54.4, −13.6, −6.04, −3.4 (in eV)

$\therefore$ Max $\lambda \Rightarrow$ (6.04 – 3.4) eV = 2.64 eV

$$\frac{1242(\text{eV nm})}{2.64 \text{ eV}} = 470.4 \text{ nm} = 4704 \text{ Å}$$

119. The element is being produced at a rate R. Each second it is decaying at a rate λN (where N is the number of nuclei at that instant)

$$\therefore \quad \frac{dN}{dt} = R - \lambda N \Rightarrow \frac{dN}{R-\lambda N} = dt$$

$$\int_{N_0}^{N}\frac{dN}{R-\lambda N} = \int dt \Rightarrow -\frac{1}{\lambda}\ell n(R-\lambda N)\Big]_{N_0}^{N} = t$$

$$\Rightarrow \quad -\frac{1}{\lambda}\ell n\frac{R-\lambda N}{R-\lambda N_0} = t$$

$$R - \lambda N = (R - \lambda N_0)e^{-\lambda t}$$

$$\Rightarrow \quad N = \frac{R - \{(R-\lambda N_0)e^{-\lambda t}\}}{\lambda}$$

After long time t the term $(R - \lambda N_0)e^{-\lambda t}$ becomes insignificant

$$(\text{since } e^{-\lambda t} \rightarrow \frac{1}{\infty} = 0)$$

$$\therefore \quad N = \frac{R}{\lambda} = \frac{2N_0\lambda}{\lambda} = 2N_0$$

$$\text{At } t = T_{\frac{1}{2}};\ \lambda t = \frac{\ell n2}{T_{\frac{1}{2}}}T_{\frac{1}{2}} = \ell n2$$

$$\Rightarrow e^{-\lambda t} = \frac{1}{e^{\ell n 2}} = \frac{1}{2}$$

$$\therefore \quad N = \frac{R - (R - \lambda N_0)\frac{1}{2}}{\lambda} \Rightarrow \frac{R}{2\lambda} + \frac{N_0}{2}$$

$$= \frac{2N_0\lambda}{2\lambda} + \frac{N_0}{2} = \frac{3N_0}{2}$$

120. The continuous spectrum of X-rays is due to emission by electron decelerating in the target, and therefore, it is only weakly dependent on the nature of the target material.

(d) → (q)

The minimum wavelength of the continuous spectrum is given by the condition that the accelerated electron loses its kinetic energy completely in a single collision inside the target emitting X- radiation.

$$\therefore \quad eV = \frac{hc}{\lambda_{min}}$$

(a) → (r)

The intensity of the spectrum depends on the accelerating voltage

(b) → (r)

The k_α – line in the spectrum is produced by the transition of the L electron to the K shell where a vacancy is created by the bombarding electron.

Moseleys law for k_α frequency → $\sqrt{v} = a(z - b)$

∴ (c) → (p), (s)

Additional Practice Exercise

121. (i) Requires 13.6 eV

$$\Rightarrow \frac{hc}{\lambda} \geq 2.1 + 13.6 \geq 15.7\text{eV}$$

$$\Rightarrow \lambda \leq \frac{1242}{15.7} = 79 \text{ nm}$$

(ii) Electrons excited to n = 2 produce only 1st Lyman line ⇒ ultraviolet.

Electron excited to n = 3 produce n = 3 to n = 1 (Lyman) and n = 3 to n = 2 (Balmer)

$$\Delta E_{3-2} = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = 1.89\text{eV}$$

⇒ visible spectrum photon.

∴ Energy of electron required =

$$13.6\left(1 - \frac{1}{9}\right) = 12.09\text{eV}$$

∴ Energy of beam = 2.1 + 12.09 = 14.19 eV

⇒ Maximum wavelength

$$= \frac{1242}{14.19} = 87\,\text{nm}$$

122. $E_n = \frac{-Z^2}{n^2}$ryderg

$$E_1 = -\frac{3^2}{1^2}\ (13.6)\text{ eV} = -122.4\text{ eV}$$

$$E_3 = -\frac{3^2}{3^2}\ (13.6\text{ eV}) = -13.6\text{ eV}$$

$$\therefore \quad E_3 - E_1 = 108.8\text{ eV}$$

$$hf = 108.8\text{ eV}$$

$$f = \frac{108.8 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}\text{ Hz} = 26.25 \times 10^{15}\text{ Hz}$$

123. $\frac{mv^2}{r} = \frac{k}{r^{3/2}}$

$$v^2 = \frac{k}{m\sqrt{r}} \quad \text{— (1)}$$

$$mvr = \frac{nh}{2\pi}$$

$$m^2r^2 \,.\, v^2 = \left(\frac{nh}{2\pi}\right)^2 \text{. Substitute from eqn (1)}$$

$$\Rightarrow r^2 = \frac{n^2h^2}{4\pi^2} \cdot \frac{m\sqrt{r}}{km^2}$$

$$\Rightarrow r^{\frac{3}{2}} = \frac{n^2h^2}{4\pi^2 mk}$$

$$r = \left(\frac{n^2\,h^2}{4\pi^2\,mk}\right)^{\frac{2}{3}}$$

124. $\frac{n\lambda}{2} = 2\text{Å} \Rightarrow n\lambda = 4\text{ Å}$ —(1)

In the nth mode, $\frac{d}{n}$ = distance between consecutive modes = $\frac{\lambda}{2}$

$$\Rightarrow d = n\frac{\lambda}{2};\ d = 2\text{ Å} \Rightarrow m\lambda = 4\text{Å}$$

In the (n + 1)th mode, $\frac{d}{(n+1)} = \frac{\lambda}{2}$

$$\Rightarrow d = (n+1)\frac{\lambda}{2};\ d = 3\text{ Å}$$

$$(n+1)\frac{\lambda}{2} = 3\text{ Å} \Rightarrow (n+1)\lambda = 6\text{ Å} \qquad \text{—(2)}$$

$$(2)\text{—}(1) \Rightarrow \lambda = 2\text{ Å}$$

$$\Rightarrow \quad \therefore \frac{h}{mv} = 2\text{Å}$$

$$v = \frac{h}{m\lambda} \Rightarrow \therefore KE = \frac{1}{2}mv^2 = \frac{1}{2}\times m \times \frac{h^2}{m^2\lambda^2}$$

$$KE = \frac{1}{2}\frac{h^2}{m\lambda^2} = \frac{1}{2}\times\frac{\left(6.63\times10^{-34}\right)^2}{\left(9.1\times10^{-31}\right)\times4\times10^{-20}}$$

$$= 0.604 \times 10^{-17}\text{J}$$

$$= 37.7 \text{ eV}$$

125. (i) K absorption edge means minimum wavelength of characteristic K series lines

$\Rightarrow$ $n = \infty$ to $n = 1$

$$\therefore \quad \frac{1}{\lambda_{min}} = R(Z-1)^2$$

$$\Rightarrow \quad \frac{1}{0.0185\times10^{-9}}$$

$$= 1.1\times10^7 (Z-1)^2 \left(\frac{1}{1^2} - \frac{1}{\infty}\right)^2$$

$$\Rightarrow \quad (Z-1)^2 = 4900 \Rightarrow Z = 71$$

(ii) The minimum value is the energy to ionize the K electron

$$\Rightarrow \quad 13.6\times(Z-1)^2\left(\frac{1}{1^2}\right)$$

$$\Rightarrow \quad 13.6\times70^2 = 67 \text{ kV}$$

126. 208(7.93) = Binding energy BE

$$\therefore \quad \text{mass defect } (\Delta m) = \frac{208(7.93)}{931} = 1.77169\text{u}$$

$\Delta m = Z(m_p) + (A-Z)m_n - m_x$. We get

$$m_x = 82\times1.00727 + 126\times1.00866 - 1.77169$$

$$m_x = 207.916 \text{ u}$$

127. (i) $E = \Delta m \times 931.5$

$$= (238.0508 - 234.0436 - 4.0026)\ 931.5$$

$$= 4.3 \text{ MeV}$$

(ii) Let the common momentum be p

$$\Rightarrow \quad \frac{p^2}{2m_\alpha} + \frac{p^2}{2m_{Th}} = \Delta E$$

$$E_\alpha\left(1 + \frac{m_\alpha}{m_{Th}}\right) = \Delta E$$

$$E_\alpha = \left(\frac{m_{Th}}{m_{Th} + m_\alpha}\right)\Delta E$$

$$= \frac{234.0436}{234.0436 + 4.0026}\times4.3$$

$$= 4.22 \text{ MeV}$$

$$\Rightarrow \quad \frac{1}{2}m_\alpha v^2 = 4.22 \text{ MeV} = 6.752\times10^{-13}\text{ J}$$

$$\Rightarrow \quad v = 1.42\times10^7 \text{ m s}^{-1}$$

$$(\because m_\alpha = 4\times1.67\times10^{-27}\text{ kg})$$

128. (i) Energy released

$$= BE_{Ba} + BE_{kr} - BE_U$$

$$= 8.5(141+92) - 7.6\ (235) = 194.5 \text{ MeV.}$$

(ii) $100 \text{ MW} = \dfrac{100\times10^6}{1.6\times10^{-19}}\text{ eV s}^{-1}$

$$= 6.25\times10^{26}\text{ eV s}^{-1} = 6.25\times10^{20}\text{ MeV s}^{-1}$$

Each fission produces 194.5 MeV

Number of fissions per second

$$= \frac{6.25\times10^{20}}{194.5}$$

Binding energy of 7.93 MeV will correspond to a mass defect of $\dfrac{7.93}{931} \simeq 0.00854 \simeq 0.014$

$\therefore$ Mass of one ^{235}U nucleus

$$= 235\ (1.66\times10^{-27})\text{ kg}$$

Mass of ^{235}U per second

$$= 235\ (1.66\times10^{-27})\times\frac{6.25\times10^{20}}{194.5} = 1.25\times10^{-6}\text{ kg}$$

129. $\dfrac{M_1}{M_2} = \dfrac{N_1 m_1}{N_2 m_2}$ (Where the symbols have the usual meaning)

$$\frac{N_1}{N_2} = \frac{M_1 m_2}{M_2 m_1} = \frac{M_R}{a_{mr}} \qquad \text{—(1)}$$

$$N_1 = N_{01}\, e^{\frac{-t}{T_{avg1}}};\ N_2 = N_{02}\, e^{\frac{-t}{T_{avg2}}}$$

Given $\quad N_{01} = N_{02}$

$$\Rightarrow \quad \text{Hence } \frac{N_1}{N_2} = \frac{e^{\frac{-t}{T_{avg1}}}}{e^{\frac{-t}{T_{avg2}}}} = e^{t\left(\frac{1}{T_{avg2}} - \frac{1}{T_{avg1}}\right)}$$

Substitute from eqn — (1)

$$\frac{M_R}{a_{mr}} = e^{t\left(\frac{1}{T_{avg2}} - \frac{1}{T_{avg1}}\right)}$$

$$t = \frac{\ell n\left(\frac{M_R}{a_{mr}}\right)}{\frac{1}{T_{avg2}} - \frac{1}{T_{avg1}}}$$

130. $\frac{m}{M} = \frac{N}{N_0} \Rightarrow N = N_0\left(\frac{m}{M}\right)$

$$\begin{bmatrix} m = 2.3 \times 10^{-11}\,\text{kg} \\ N_0 = \text{Avogadro's number} \\ M = \text{molar mass of } ^{31}\text{P} \end{bmatrix}$$

$$= \frac{6.02 \times 10^{23} \times 2.3 \times 10^{-11}}{31 \times 10^{-3}} = 4.46 \times 10^{14} \text{ atoms}$$

Activity $A = \lambda N$

$$t_{\frac{1}{2}} = \frac{\ell n 2}{\lambda} \Rightarrow 14.3 \times 24 \times 3600\text{ s}$$

$$= \frac{0.693}{\lambda} \quad (\therefore \ell n2 = 0.693)$$

$\lambda = 5.6 \times 10^{-7}\,\text{s}^{-1}$

$$\Rightarrow \quad A = \frac{4.46 \times 10^{14} \times 5.6 \times 10^{-7}}{3.7 \times 10^{10}} \text{ Ci}$$

$[\because 1\text{ Ci} = 3.7 \times 10^{10} \text{ disintegration s}^{-1}]$

$A = 6.76 \times 10^{-3}$ Ci

131. In eV, $E = \phi + V_s$

If $\frac{\phi_1}{\phi_2} = \frac{2}{5}$, then $\frac{V_2}{V_1} = \frac{2}{5}$,

(Since $\phi_1 + V_1 = \phi_2 + V_2$)

$\Rightarrow \phi_2 - \phi_1 = V_1 - V_2$

$\phi_2 = 2.5\,\phi_1$, $V_1 = 2.5\,V_2$ (data)

$\therefore\ \phi_2 - \phi_1 = 1.5\,\phi_1$; $V_1 - V_2 = 1.5\,V_2$

$\Rightarrow \phi_1 = V_2$ and $\therefore\ \phi_2 = V_1$

Case (i)

If $\phi_1 = 0.67$ eV, then $\phi_2 = 2.5 \times 0.67$ eV,

$\therefore\ E = \phi_1 + V_1 = \phi_1 + \phi_2 = 3.5 \times 0.67$

$$\Rightarrow \lambda = \frac{1240}{3.5 \times 0.67} \cong \frac{1240 \times 3}{7} \cong \frac{3720}{7} \cong 531 \text{ nm}$$

$\Rightarrow$ visible light (yellow)

Case (ii)

If $\phi_2 = 0.67$ eV, then $\phi_1 = \frac{0.67}{2.5}$ eV $= 0.268$ eV

$\Rightarrow E = \phi_2 + V_2 = \phi_2 + \phi_1$

$= 0.67 + 0.268 = 0.938$ eV

$$\Rightarrow \lambda = \frac{1240}{0.938} \cong 1322 \text{ nm}$$

$\Rightarrow$ infrared.

132. $\lambda = \frac{h}{p}$, $K.E = \frac{p^2}{2m} \Rightarrow \frac{KE_1}{KE_2} = \frac{p_1^2}{p_2^2} = \frac{\lambda_2^2}{\lambda_1^2} = 4.$

$$\Rightarrow KE_2 = \frac{KE_1}{4}$$

$h\nu = \phi_1 + K_1 = \phi_2 + K_2$

$$= \phi_1 + K_1 = 2\phi_1 + \frac{K_1}{4}$$

Given $\frac{K_1}{4} = 0.4$ eV

$\Rightarrow K_1 = 1.6$ eV

$\phi_1 = \frac{3}{4}K_1 = 1.2$ eV

$h\nu = \phi_1 + K_1 = 2.8$ eV

133. $\frac{\lambda_1}{\lambda_2} = \frac{1}{2} \Rightarrow \frac{p_1}{p_2} = \frac{2}{1}\left(\because \lambda = \frac{h}{p}\right)$

kinetic energy, $E = \frac{p^2}{2m}$

$$\therefore \quad \frac{E_1}{E_2} = \frac{p_1^2}{p_2^2}\cdot\frac{m_2}{m_1} = \frac{4}{1} \times \frac{4}{1} = \frac{16}{1}$$

$$\frac{v_1}{v_2} = \frac{p_1}{p_2}\cdot\frac{m_2}{m_1} = \frac{2}{1} \times \frac{4}{1} = \frac{8}{1}$$

134. 10 lines $= 5C_2 \Rightarrow$ excited state is $n = 5$

$$\Rightarrow \quad V = 13.6\left(1 - \frac{1}{25}\right) = 13.6 \times \frac{24}{25}$$

$n = \infty \Rightarrow V' = 13.6$

$$\Delta V = 13.6\left(1 - \frac{24}{25}\right) = 13.6 \times \frac{1}{25}$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{1}{24}$$

135. $p = mv \Rightarrow p \propto \dfrac{1}{n}$

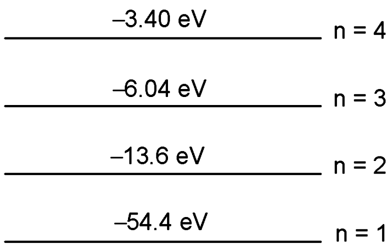
$\therefore$ Transitions are: $n_1 \rightarrow \dfrac{n_1}{4} \rightarrow 9\, n_1$

Minimum $\dfrac{n_1}{4} = 1 \Rightarrow$ minimum $n_1 = 4$

$\Rightarrow$ minimum $n_3 = 36$

136. The energy levels in He^+ are as shown

$$E_n = \frac{-Z^2 \times 13.6\,eV}{n^2} = \frac{-54.4\,eV}{n^2} \;(\because Z = 2)$$



$E_4 - E_3 = -3.40 + 6.04 = 2.64$ eV (visible)

Hence He^+ ion needs to be excited up to n = 4 state for emitting visible radiation.

$\therefore$ Excitation energy $= E_4 - E_1$

$= -3.40 + 54.4 = 51$ eV

137. $71.4 = 13.6\, Z^2 \left(\dfrac{1}{4} - \dfrac{1}{25}\right)$

$$\Rightarrow Z^2 = \frac{71.4}{13.6} \times \frac{25 \times 4}{21}$$

$$= 25 \times 4 \times \frac{7 \times 10.2}{7 \times 3 \times 13.6}$$

$$= 25 \times 4 \times \frac{3.4}{13.6} = \frac{25 \times 13.6}{13.6} = 25$$

$\Rightarrow Z = 5$

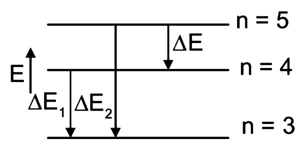
138. Continuous spectrum, with

$$\lambda_{min} = \frac{1240\,eV.nm}{62 \times 10^3\,eV}$$

$= 0.02$ nm

$= 0.2$Å

139.



$\Delta E \propto \dfrac{1}{\lambda}$

$\Delta E = \Delta E_2 - \Delta E_1$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \Rightarrow \lambda = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

140. 2 Å= 0.2 nm $\Rightarrow \dfrac{1240}{0.2} = 6200$ eV

$$6200 < 13.6 \times \frac{3}{4}(Z-1)^2 = 10.2\,(Z-1)^2$$

$\Rightarrow (Z - 1)^2 > 607$

$\Rightarrow Z - 1 \cong 25$

$\Rightarrow Z = 26$

141. $\dfrac{mv^2}{r} = qvB \Rightarrow mv = qBr$

$$\Rightarrow \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{qBr}$$

$$= \frac{6.6 \times 10^{-34}}{2 \times 1.6 \times 10^{-19} \times 0.1 \times 1.25 \times 10^{-2}}$$

$= 1.65 \times 10^{-12}$ m

142. $$\text{Ratio} = \frac{RhcZ^2\left(1 - \dfrac{1}{(5)^2}\right)}{RhcZ^2\left(1 - \dfrac{1}{(3)^2}\right)} = \frac{24/25}{8/9} = \frac{27}{25}$$

143. $E_3 \sim E_1 = 1.5$ eV ~ (13.6 eV) = 12.1 eV

144. $L = n\left(\dfrac{h}{2\pi}\right)$

Energy equation.

$$-0.85\,eV = \frac{Z^2}{n^2}(-13.6)\,eV$$

$$n^2 = \frac{13.6}{0.85} = 16$$

$n = 4$

$\therefore\; L = \dfrac{2h}{\pi}$

145. $\dfrac{e^2}{4\pi\varepsilon_0 r^2} = \dfrac{mv^2}{r}$ (centripetal force) —(1)

$mvr = \dfrac{nh}{2\pi}$—(2) (quantization of angular momentum)

$$v = \frac{nh}{2\pi mr} \text{ —(3) substitute in (1)}$$

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = m\frac{n^2h^2}{4\pi^2m^2r^3} \Rightarrow r = \frac{n^2h^2\varepsilon_0}{\pi me^2} \text{ —(4)}$$

$$\frac{1}{f} = \frac{2\pi r}{v}$$

Substitute (3) and (4)

$$\therefore \quad f = \frac{nh}{4\pi^2 m}\frac{\pi^2m^2e^4}{n^4h^4\varepsilon_0^{\,2}} = \frac{me^4}{4n^3h^3\varepsilon_0^{\,2}}$$

Aliter

$r \propto n^2$

$v \propto \frac{1}{n}$

$$\omega = \frac{v}{r} \propto \frac{1}{n} \times \frac{1}{n^2} \propto \frac{1}{n^3}$$

$$\Rightarrow \quad f \propto \omega \propto \frac{1}{n^3}$$

146. $U = -\frac{2Z^2}{n^2}(13.6)\text{ eV}$

U is negative and $U \to 0$ as $n \to \infty$

147. n = 6 to n = 2

$\Rightarrow$ Balmer series

$\Rightarrow$ visible

148. $f \propto \frac{1}{n^3} \Rightarrow f_m = \frac{f_n}{8}$

$$\Rightarrow \quad \frac{f_m}{f_n} = \frac{1}{8}$$

$$\rightarrow \left(\frac{n_n}{n_m}\right)^3 = \frac{1}{8}$$

$\Rightarrow \quad n_n = n;\ n_m = m$

$$\left(\frac{n}{m}\right)^3 = \frac{1}{8} \Rightarrow \frac{n}{m} = \frac{1}{2}$$

i.e., 2, 1 or 4, 2

149. $r = r_0\, A^{\frac{1}{3}}$

$$r' = r_0\left(\frac{A}{2}\right)^{\frac{1}{3}}$$

$$\therefore \quad \frac{r'}{r} = \frac{1}{2^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{2}}$$

150. $2m_pc^2 = 2hf$

$$f = \frac{m_p c^2}{h} = \frac{c}{\lambda}$$

$$\lambda = \frac{h}{m_p c}$$

$$= \frac{6.62 \times 10^{-34}}{1.67 \times 10^{-27} \times 3 \times 10^8} = 0.13 \times 10^{-4}\ \text{Å}$$

Aliter:

1 proton = 938 MeV = 938×10^6 eV

$$\therefore \quad \lambda = \frac{12422}{938 \times 10^6}\text{Å} = 0.13 \times 10^{-4}\ \text{Å}$$

151. Isobars – mass number same

152. In elastic collisions, maximum change in speed occurs if masses are comparable.

153. 1 microgram = 1×10^{-9} kg

Mass energy relation,

$E = Mc^2 = 1 \times 10^{-9} \times (3 \times 10^8)^2$

$= 1 \times 10^{-9} \times 9 \times 10^{16} = 9 \times 10^7$ J

154. α-particle emission $\rightarrow$ mass number less by 4 unit Atomic number less by 2 unit

β-particle emission $\rightarrow$ mass number, same. Atomic number increased by one unit

γ-emission: only photon emitted (em wave)

A = 180 – 4 – 0 – 4 – 0 = 172

Z = 72 – 2 + 1 – 2 – 0 = 69

155. $T_{\frac{1}{2}} = 1.37 \times 10^9$

Ratio = 1:7

$$\Rightarrow \quad \frac{N}{N_0} = \frac{1}{8} = \frac{1}{2^3} \Rightarrow \text{3 Half lives}$$

$\Rightarrow \quad 3 \times 1.37 \times 10^9 = 4.11 \times 10^9$ year

156. Let 'N' represent the nuclei

$${}_nN^m \xrightarrow[\alpha]{} {}_{n-2}N^{m-4} \xrightarrow[\beta^+]{} {}_{n-1}N^{m-4}$$

m – 4, n – 1

157. $\frac{N}{N_0} = \frac{1}{8} = \frac{1}{2^3}$

$\Rightarrow$ 3 Half lives = 60 min

$$\Rightarrow \quad \text{Half life} = \frac{60}{3} = 20\text{ min.}$$

158. 15 protons, 16 neutrons

Mass defect

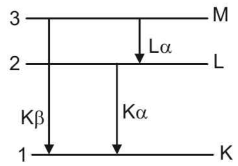
$= 15 \times 1.007825 + 16 \times 1.008665 - 30.973763$

$= 0.282252$ amu

$\Rightarrow$ Binding energy per nucleon

$$\frac{0.282252 \times 931}{31} = 8.47 \text{ MeV}$$

159.



$$\Delta E \, \alpha \, \frac{1}{\lambda}$$

$$\underset{(L\alpha)}{\Delta E_{3-2}} = \underset{(k\beta)}{\Delta E_{3-1}} - \underset{(k\alpha)}{\Delta E_{2-1}}$$

$$\therefore \quad \frac{1}{\lambda_{L\alpha}} = \frac{1}{\lambda_{K\beta}} - \frac{1}{\lambda_{K\alpha}} = \frac{1}{0.55} - \frac{1}{0.66}$$

$$\lambda_{L\alpha} = \frac{(0.55)(0.66)}{0.11} = 3.3\text{Å}$$

160. $\frac{1}{\lambda} = R(Z-b)^2 . \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Screening constant b = 1 for K lines and >1, usually 7.8 for L lines.

$$\frac{1}{\lambda_{K\alpha}} = R(Z-1)^2\left(1 - \frac{1}{4}\right) = R(Z-1)^2 . \frac{3}{4}$$

$$\frac{1}{\lambda_{K\beta}} = R(Z-1)^2\left(1 - \frac{1}{9}\right) = R(Z-1)^2 . \frac{8}{9}$$

$$\frac{1}{\lambda_{K\beta}} - \frac{1}{\lambda_{K\alpha}} = R(Z-1)^2\left[\frac{8}{9} - \frac{3}{4}\right]$$

$$= R(Z-1)^2 \frac{5}{36} \quad \text{----- (1)}$$

But $\frac{1}{\lambda_{L\alpha}} = R(Z-b)^2\left[\frac{1}{4} - \frac{1}{9}\right]$

$$= R(Z-b)^2 . \frac{5}{36} \quad \text{----- (2)}$$

b > 1 (usually 7.8 for Lα line);

$$\therefore \quad \frac{1}{\lambda_{L\alpha}} < \frac{1}{\lambda_{K\beta}} - \frac{1}{\lambda_{K\alpha}}$$

$\Rightarrow$ $\lambda_{L\alpha}$ value larger than previously estimated value.

161. Nuclear radius $\propto A^{1/3}$.

$$\because \frac{r_{A\ell}}{r_{Te}} = \left(\frac{A_{A\ell}}{A_{Te}}\right)^{1/3} = \left(\frac{13}{52}\right)^{1/3} = 0.63 \cong \frac{3}{5}$$

162. ${}_0^1x \Rightarrow$ neutron

163. ${}_0^1n \rightarrow {}_1^1p + {}_{-1}^{0}e$

164. Ratio of ^{14}C to ^{12}C in the fossil and that in living organism are used in the decay equation to calculate t.

165. $\frac{1240}{3} = 413$ nm < yellow

(b), (c) and (d) cannot induce photocurrent.

166. $\upsilon = 10^8 = \frac{c}{3} ms^{-1}$ $\quad \lambda_e = \frac{h}{p} = \lambda$(of photon)

Let E be the energy of photon, m- mass of electron and K its kinetic energy.

$$\text{Then } E = \frac{hc}{\lambda} = \frac{hc}{\frac{h}{\frac{mc}{3}}} = \frac{mc^2}{3} = \frac{1}{2}\frac{mc^2}{9}.6 = (K)6$$

$$\frac{E}{K} = 6$$

167. Negligible Rn $\because$ t > 11 half lives

negligible Po $\because$ t >> half life

Pb would have practically not decayed

$\because$ t << half life.

168. $n \rightarrow p + e + \bar{\nu}$

169. $E_{H \rightarrow H^+} = 13.6 \text{ eV} = E'$

$E_{He \rightarrow He^+} = E''$ (unknown)

$E_{He^+ \rightarrow He^{++}} = Z^2 (13.6) \text{ eV} = 4 \times 13.6 \text{ eV} = E'''$

TOTAL = E (given) = E' + E'' + E''' = E' + 5 (13.6)

$\therefore$ E'' = E – 5 × 13.6 = E – 54.4

170. Slows down neutrons.

171. If P.E at infinity is arbitrarily taken as zero, then total energy of an electron is negative.

172. (a)

173. Collision is elastic or inelastic; however an electron may undergo several collisions.

174. Self explanatory.

175. Volume $V \propto R^3 \propto A$

$\Rightarrow \frac{A}{V}$ = constant. $\left(\frac{A}{V} \propto \text{density}\right)$

176. I same, $h\nu$ decreases $\Rightarrow$ number of photons increases.

177. Time taken for half the population to decay is half life.

178. Self explanatory

179. Both statements are correct. When K-shell electron of the target element is knocked-off all characteristic K-series appear. To knock - off K-shell electron, energy required is Rhc $(Z - 1)^2$. If such an electron is fully stopped, it gives a cut-off wavelength

$$\frac{1}{\lambda_{\text{cut off}}} = R(Z-1)^2 > \frac{1}{\lambda_{k\beta}}$$

$$[\therefore \frac{1}{\lambda_{k\beta}} = R(Z - 1)^2 \frac{8}{9} \therefore \lambda_{\text{cut-off}} < \lambda_{k\beta}]$$

180. Usually the total energy is absorbed, which results in an absorption line spectrum. The excited atom then jumps from n = x to n = 1 or x to an intermediate state, which results in a larger wavelength.

Note: The exception is the Raman effect and the resulting Stokes and anti Stokes lines.

181. The λ_{min} will correspond to the K-shell ionization energy of the heaviest element i.e., C. The energy required for K-shell electron to be knocked out is (in eV):

$$E = 13.6\,(z - 1)^2 = 13.6 \times (45 - 1)^2 \text{ eV}$$

$$\therefore \lambda_{min} = \frac{1242}{13.6 \times 44^2} = 0.047 \text{ nm} = 0.47\text{Å}$$

182.

	A	B	C
K shell ionization	E_0	$\frac{8}{7}E_0$	$\frac{9}{7}E_0$
E Kα	$\frac{3}{4}E_0$	$\frac{3}{4}\frac{8}{7}E_0$	$\frac{3}{4}\frac{9}{7}E_0$
E Kβ	$\frac{8}{9}E_0$	$\frac{8}{9}\frac{8}{7}E_0$	$\frac{8}{9}\frac{9}{7}E_0$

for element C, $E_{k\beta}(C) = \frac{8}{9}E'$

[Where E' – K shell ionization energy]

$$\therefore \lambda_{k\beta}(C) = \frac{9}{8}\lambda_{min} = \frac{9}{8} \times 0.47\text{Å}$$

$$\frac{\lambda_{k\beta}(A)}{\lambda_{k\beta}(C)} = \frac{E_{0\,k\beta}(C)}{E_{0\,k\beta}(A)} = \frac{8}{9}\cdot\frac{9}{7}E_0\frac{9}{8E_0} = \frac{9}{7}$$

$$\Rightarrow \therefore \lambda_{K\beta(A)} = \frac{9}{7}\cdot\frac{9}{8}\,0.47 = 0.68\text{Å}$$

183. Taking data from previous solution

$$\frac{EK\alpha(B)}{EK\beta(C)} = \frac{3}{4}\cdot\frac{8}{7}E_0 \times \frac{7}{8E_0} = \frac{3}{4}$$

$$\therefore \frac{\lambda_{k\alpha(B)}}{\lambda_{k\beta(C)}} = \frac{4}{3} \Rightarrow \lambda_{K\alpha(B)} = \frac{4}{3} \times \frac{9}{8} \times 0.47 = 0.7\text{Å}$$

184. $A = A_0 e^{-\lambda t} \Rightarrow \ell_n A = \ell_n A_0 - \lambda t$

or $\log_{10} A = \log_{10} A_0 - \lambda t \log_{10}^{e}$

Both cases it is straight line, when semi log graph paper is used.

185. For any element let A'_0 be the activity at t = 0. Then

at $t = t_0 \Rightarrow A_0 = A_0' e^{-\lambda_1 t_0}$

at $t = 2t_0 \Rightarrow A_1 = A'_0 e^{-\lambda_1 2t_0}$

$$\Rightarrow \frac{A_0}{A_1} = \frac{A'_0 e^{-\lambda_1 t_0}}{A'_0 e^{-2\lambda_1 t_0}} = e^{\lambda_1 t_0}$$

$$\ell n \frac{A_0}{A_1} = \lambda_1 t_0 \text{; Given } \ell n \frac{A_0}{A_1} = 0.2$$

$\therefore$ Since at $t = t_0$, their activities are equal to A_0 at $t = 2t_0$ the activity of the least active element will be highest. Since $A_1 \frac{A_0}{e^{\lambda_x t_0}}$. Hence the activity ratio 3 : 2 : 1

$\Rightarrow A_1 : \frac{2}{3}A_1 : \frac{1}{3}A_1$ and the first element is least active:

$$\lambda_1 t_0 = \ell n \frac{A_0}{A_1} = 0.2$$

$$\lambda_2 t_0 = \ell n \frac{A_0}{A_1}\frac{3}{2} = \ell n \frac{A_0}{A_1} + \ell n\, 3 - \ell n\, 2$$

$$= 0.2 + 1.1 - 0.7 = 0.6$$

$$\lambda_3 t_0 = \ell n \frac{A_0}{A_1}3 = \ell n \frac{A_0}{A_1} + \ell n\, 3 = 0.2 + 1.1 = 1.3$$

$$\Rightarrow \lambda_3 = \frac{1.3}{t_0}$$

$$T_{\frac{1}{2}} \text{ for the element } = \frac{\ell n 2}{\lambda_3} = \frac{0.7 t_0}{1.3} = 0.54 t_0$$

186. Let $A'_{0(1)}$ be the initial activity of the first element

$$A_0 = A'_{0(1)}\, e^{-\lambda_1 t_0} \Rightarrow \frac{A'_{0(1)}}{A_0} = e^{\lambda_1 t_0} \qquad \text{— (1)}$$

We have seen in Q 185, $\dfrac{A_0}{A_1} = e^{\lambda_1 t_0}$ — (2)

From equation (1) and (2)

$$\frac{A'_{0(1)}}{A_0} = \frac{A_0}{A_1} \Rightarrow A'_{0(1)}.A_1 = A_0^2 = \text{constant, same for all}$$

$$\therefore \; A'_{0(1)} : A'_{0(2)} : A'_{0(3)}$$

$$= \frac{1}{A_1} : \frac{1}{A_2} : \frac{1}{A_3} = \frac{1}{3} : \frac{1}{2} : 1 = 2:3:6$$

187. In $0 < x < \ell$ $V = 0$

$$\therefore \quad \frac{-h^2}{8\pi^2 m}\frac{d^2\psi}{dx^2} = E\psi$$

$$\Rightarrow \quad \text{put } \frac{8\pi^2 mE}{h^2} = k^2 \text{ we get : } \frac{d^2\psi}{dx^2} + k^2\Psi = 0$$

This is simple harmonic

Solution to SHM equation is

$\psi = A \cos kx + B \sin kx$ or $\psi = p \sin (kx + Q)$

188. $\psi = 0$ at $x = 0$ and hence $A = 0 \;\Rightarrow \psi = B \sin kx$

$\psi = 0$ at $x = \ell \;\Rightarrow \therefore\; 0 = B \sin k\ell$

$B \neq 0;\; k\ell = n\pi \;\Rightarrow k = \dfrac{n\pi}{\ell}$

189. From above : $\dfrac{8\pi^2 mE}{h^2} = \dfrac{n^2\pi^2}{\ell^2}$

$$E = \frac{n^2h^2}{8m\ell^2}$$

$\Delta E = E_2 - E_1 = 0.05$ eV, m = electron mass

$$\Delta E = \frac{(2^2-1).(6.6\times10^{-34})^2}{8\times9.1\times10^{-31}\times\ell^2}$$

$$= 0.05\times1.6\times10^{-19} \qquad \therefore\; \ell = 4.8\text{ nm}$$

190. $v \propto \dfrac{Z}{n}$ $\quad \lambda = \dfrac{h}{p} = \dfrac{h}{mv}$

191. $\dfrac{hc}{\lambda} = h\nu_0 + K_{max} = \dfrac{hc}{\lambda_0} + K_{max}$

Intensity and photoelectric efficiency determine photocurrent.

192. Knowledge based

193. $-\dfrac{dN}{Ndt} = \lambda = 0.693\text{ year}^{-1}$

LHS represents probability of a nucleus disintegrating in unit time. Hence the probability that it will not disintegrate is 0.307 = 30.7%

$$t_{1/2} = \frac{\ell n2}{\lambda} = \frac{0.693}{0.693} = 1\text{ year}$$

$$\frac{N}{N_0} = e^{-\lambda t},\; t = \left(\frac{1}{0.693}\right)\text{year} \Rightarrow \frac{N}{N_0}$$

$$= e^{-1} = \frac{1}{e} = 0.37$$

$$\therefore \quad \frac{N}{N_0} = \frac{37}{100} \Rightarrow \frac{N_0 - N}{N_0} = \frac{63}{100} \Rightarrow \frac{\Delta N}{N_0} = 63\%$$

$\Rightarrow$ ΔN is the number that will disintegrate in the given time of $\dfrac{1}{0.693}$ year, out of total N_0

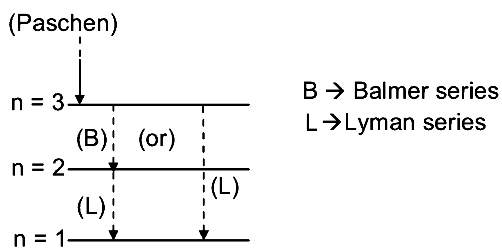
194. Knowledge based

195. Nuclei at rest. Momentum and energy conservation equations involve daughter nucleus and α particle, so the two kinetic energies will have unique values.

β decay yields daughter nucleus, β particle and antineutrino/neutrino. So two equations (momentum equation and K.E equation) will lead to no unique solution. β particles have a spectrum of K.E.

Often after α/β decay, nucleus will be in excited state $\Rightarrow \gamma$ decay follows.

196.



197. Half life

$$T_{1/2} = \ell n(2)\,(\text{Mean life}) = 300\text{ day}$$

$$\lambda = \frac{1}{T(\text{mean life})} = \frac{\ell n\, 2}{300} \sim 2.33\times10^{-3}\text{ per day.}$$

900 days $\rightarrow$ equal 3 half lives:

$\therefore \quad N = N_0 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{N_0}{8}$

$\Rightarrow \quad m = \frac{m_0}{8} = \frac{1}{8} g$

Probability of a nucleus existing after 300 days (i.e., 1 half life) = fraction remaining after 1 half life = $\frac{1}{2}$

Probability of 1 nucleus decaying with in 300 days

$= 1 - \frac{1}{2} = \frac{1}{2}$

198. The following results are easily derivable

$J = mvr = \frac{nh}{2\pi}$

$\therefore \quad J \propto n$ (a) $\to$ s

$\therefore \quad v_n = \frac{1}{4\pi\varepsilon_0} \frac{2\pi e^2}{nh}$

$\therefore \quad v \propto \frac{e^2}{n}$

$\therefore$ (b) $\to$ r, $r_n = 4\pi\varepsilon_0 \frac{n^2h^2}{4\pi^2 me^2}$

$\therefore \quad r_n \propto \frac{n^2}{e^2}$

$\therefore$ (c) $\to$ p

$\mu \propto = IA$

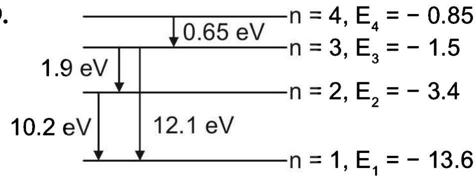
$= (f.e)\, \pi r^2 = \frac{v}{2\pi r} e.\, \pi r^2$

$= \frac{v}{2} er \propto \frac{e^2}{n} e. \frac{n^2}{e^2} \propto ne$

$\therefore \quad \mu \propto ne$

$\therefore$ (d) $\to$ q, s

199.



For photosensitive metal, emitted photoelectrons will have energies in range 0 to maximum kinetic energy

$\therefore$ (a) min $y_i = 1.9 \Rightarrow$ cannot be photoemission

$\therefore$ hydrogen $\Rightarrow$ n = 3 to n = 2

$\Rightarrow$ Excited state is n = 3 $\Rightarrow$ x = 12.1 eV

(b) Maximum $y_i = 1.9 \Rightarrow$ cannot be hydrogen

($\because$ the maximum would be 12.1 eV)

$\therefore$ metal $\Rightarrow$ max. KE = 1.9, x can be any value depending on ϕ.

(c) $y_i = 1.9$, neither maximum or minimum. If hydrogen, then $n_2 \geq 4 \Rightarrow x \geq 12.85$. If metal, x = any value.

(d) If no $y_i = 1.9$, then if hydrogen, then excited state 1, $\Rightarrow$ x = 10.2 eV. If metal, any x.

200. Self explanatory